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The Value of a Right of First Refusal Clause in a Procurement First-Price Auction

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Abstract

In a procurement first-price sealed-bid auction with risk-averse suppliers, we determine the conditions under which the buyer has an incentive to grant a supplier a right of first refusal. We show that this clause can lower the buyer's expected cost when suppliers (the incumbent and new suppliers) are risk-neutral or slightly risk-averse. We also show that the incumbent's expected utility is higher when he is granted a right of first refusal than when he competes under a first-price auction. So, this clause may benefit both the buyer and the favored supplier.

JEL classification: D44; D81.

Keywords: auctions; right of first refusal; risk-aversion.

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1 Introduction

In private procurement auctions, a buyer may give preferential treatment to an incumbent supplier. He may grant him a right of first refusal (hereafter ROFR), i.e. a contract clause that provides its holder with the right to supply goods or services at the lowest price the buyer is able to get from another supplier. It gives the incumbent supplier the possibility of winning the procurement auction by matching the best offer in the first-price reverse auction organized among his rivals. The purchaser of a private firm often favors an incumbent supplier by such a clause (in order to reward long term business partners). This clause is also known as a meet-the-competition clause. The ROFR clause is simple to implement. It is transparent for all bidders and easy to take into consideration when bidding (Lee (2008)). For a professional purchaser, no information about incumbent supplier's real cost is required.

The ROFR is also frequently used in sports contracts (employment contracts, especially those of athletes and entertainers may empower the current employer with the right of first refusal as encouragement to support unproven talents early in their careers (Chouinard (2005), in the National Football League (NFL), the incumbent team has the right to match the best offer a player has once he is eligible to change teams (Lee 2008)), in broadcasting rights (in 2001, the National Broadcasting Company negotiated the broadcasting rights for the hit show "Frasier" and enjoyed the right of first refusal from Paramount Studios (Grosskoft and Roth (2009)), in real estate sales (in France, the law protects the tenant by granting him an ROFR in the sale of property), and in monopoly concession rights (Chouinard (2005)). Most papers on this subject show that this mechanism is better for the favored right holder but not always for the auctioneer, absent any legal or illegal side-payments, and never for the competing bidders' profits. The ROFR may also increase the risk of inefficient allocation (the supplier with the lowest cost does not necessarily win). As described by Lee (2008), motivation for granting an ROFR in procurement is often political, in order to simply reward long term business partners. Within a single-object private-value sealed-bid auction with symmetric and risk-neutral bidders, Arozamena and Weinschelbaum (2009) show that an ROFR cannot increase the auctioneer's expected revenue in "regular cases". However, it increases the colluded surplus of the auctioneer and the favored bidder, while generating a negative externality for all other bidders. Similarly Burguet and Perry (2008) show that the ROFR may increase the expected joint surplus when the buyer cannot design the rules of the final procurement auction. They also show that the ROFR never benefits the auctioneer if he is not compensated in return for the preferential treatment granted. Choi (2009) also concludes that, compared to standard auctions, the ROFR increases the joint profit of the seller and the right-holder by reducing the third party's profit. This result is independent of whether the third party is aware of the ROFR's existence and of whether the ROFR creates a welfare loss. Assuming correlation in bidders' values in a second price auction, Bikhchandani et al. (2005) conclude that an ROFR never benefits the auctioneer without a side-payment from the favored bidder. However, in some settings, two papers show that the ROFR may be better for the auctioneer i.e. may raise the expected revenue from the auction or decrease the expected cost of procurement. In an asymmetric procurement first-price auction with two bidders, Lee (2008) shows that the buyer prefers to grant the ROFR to the ex-ante weaker bidder and that granting this right can decrease the buyer's expected cost. The ROFR gives the stronger bidder incentive to elicit more aggressive bids than in a simple procurement first-price auction and reduces his original advantage. In an independent private values (hereafter IPV) procurement first-price single auction, Elmaghraby et al. (2011) confirm that the ROFR increases the buyer's expected cost. However, they show that with a second auction in the future (with the same participating suppliers), the buyer can lower his total procurement cost by granting the ROFR to a supplier in the first of the two sequential auctions. The non-preferred supplier has an incentive to bid extremely aggressively in the first auction with ROFR. So, it may decrease the buyer's total procurement cost compared to a benchmark case of running both auctions without ROFR.

In this paper, in a specific informational framework, we provide a new argument for the use of an ROFR in a single procurement first-price auction. Following Von Ungern-Sternberg (1991) (hereafter VUS), we adopt a simpler informational paradigm which, in our opinion, does not entail any great loss of realism compared with the standard IPV model. As first described by VUS,

> "*the model we shall use combines the simplifying properties of the standard IPV and the common value models. As in the IPV model, we assume that each bidder can predict his own cost of completing the contract with certainty. As in the simplest version of the common value model, we assume that each bidder has no grounds for believing his own cost estimate to be higher or lower on average than his competitor's costs. Formally, we*

model this by assuming that the different producers' costs are independent drawings from a known distribution with an unknown mean."

This prior belief is depicted by Biais and Bossaerts (1998) as the "average opinion rule". In a more general setting, Biais and Bossaerts (1998) also present an informational structure where each agent believes that the private values of the others are i.i.d. drawn from a distribution indexed by a parameter. Accordingly, the agents know the functional form of the distribution but are uncertain about the parameter. They will infer this parameter from their own private valuation. In such a context, the observation of a producer's private cost tells him nothing about his relative position compared with his competitors. So, one of the key features of this assumption is that there is no reason why a producer should let his strategic markup depend on his own cost. This enables us to explicitly compute the equilibrium bidding strategies and shed light on strategic issues. This framework seems very credible in the context of an electronic procurement auction, particularly for commodities. Generally, suppliers do not know their competitors' identity or the market's average clearing price (especially when commodity prices are changing rapidly).

In the special case of a uniform distribution and within the IPV model, Arozamena and Weinschelbaum (2009) show that bidding behavior in a first-price sealed-bid auction remains unaltered by the presence of an ROFR. In this paper, we assume that potential suppliers may be risk-averse. In contrast to Arozamena and Weinschelbaum (2009), we show that, under a uniform distribution, a bidder's strategy is always more aggressive in the presence of an ROFR, whatever his risk-aversion level. Then, we show that an ROFR clause can lower the buyer's expected cost when suppliers (incumbent and new suppliers) are risk-neutral or slightly risk-averse. Using a numerical simulation, we also show that the incumbent's expected utility is higher when he is granted an ROFR than he competes under an FPA. So, an ROFR clause can benefit both the buyer and the favored supplier. This result is in contrast with known results about the impact of ROFR on the buyer's procurement cost in single-auction settings. It shows that the performance of a mechanism for a purchaser strongly depends on bidders' beliefs and information about their competitors' costs.

The paper is organized as follows. In the next section, we outline the model. Section 3 analyzes the first-price sealed-bid auction (hereafter FPA) with n bidders. In section 4, we consider a setting with $n-1$ bidders in the auction and one supplier who is granted an ROFR. Section 5 compares bidding strategies. In section 6 and

7, we compare the buyer's expected revenues and the incumbent's expected utilities from both mechanisms. Section 8 offers some concluding remarks. All proofs are in the Appendix.

2 The outline of the model

We analyze competition among n bidders for the award of a procurement contract. We assume that each bidder, say i, has a private cost c_i. All bidders are ex ante symmetric relative to the informational knowledge and believe that their private costs are drawn from a distribution F over $[\mu - a, \mu + a]$, where a and F are *common knowledge* whereas the mean μ is *unknown*. In order to highlight the impact of risk-aversion on bidding strategies, we assume that each bidder i is characterized by a constant relative risk-aversion (CRRA) utility function $U_i(x) = x_i^{\rho}$ (with $0 < \rho \leq 1$), where $1 - \rho$ is the CRRA parameter of each bidder and x_i is bidder i's income. Our goal is to compare two procedures in terms of minimization of expected costs for the buyer and also in terms of maximization of expected utilities for the incumbent. The first procedure is an FPA with n bidders. In the second procedure, $(n - 1)$ bidders compete in an FPA and also compete with another player (i.e. the incumbent), say I, who is granted an ROFR.

3 The FPA with n bidders

Let us first consider an FPA with n bidders. Since the bidders know the functional form of the distribution but are uncertain about the mean, they will infer this mean from their own private cost. Thus, when a bidder, say i, learns his own cost c_i, knowing that $c_i \in [\mu - a, \mu + a]$, he can infer that $\mu \in [c_i - a, c_i + a]$ according to the cumulative F_μ with corresponding density f_μ. Since all the bidders are ex ante symmetric relative to the informational knowledge, bidder i then infers that the cost of an opponent, say j, $c_j \in [c_i - 2a, c_i + 2a]$ according to the cumulative distribution F_j with corresponding density f_j. Within this informational framework, the fact that a bidder privately knows his own cost does not reveal anything to him about his relative position and so does not affect his probability of winning. Therefore, as noted by VUS,

> "*there is no reason why he should let his strategic markup depend on his own cost.*"

Hence, considering e.g. the case of bidder i, we can assume that his equilibrium bid takes the form

$$b_i(c_i) = c_i + m,$$

where m represents his markup (or profit). In an FPA, bidder i's probability of winning can be written as $P(m, m^*)$ when he chooses a strategic markup m, while the other bidders choose m^*. Then, bidder i's expected utility can be written as

$$EU_i = (m)^\rho P(m, m^*), \tag{1}$$

Differentiating (1) with respect to m yields the optimal markup for bidder i

$$\begin{aligned} \frac{\partial EU_i}{\partial m} &= (m)^\rho P'(m, m^*) + \rho m^{(\rho-1)} P(m, m^*) = 0 \\ &\Leftrightarrow mP'(m, m^*) + \rho P(m, m^*) = 0 \\ &\Leftrightarrow m = -\frac{\rho P(m, m^*)}{P'(m, m^*)}. \end{aligned} \tag{2}$$

Let us now derive the winning probability in the auction. Bidder i wins against bidder j if $c_i + m < c_j + m^*$, *i.e.* if

$$c_j > c_i + m - m^*,$$

which occurs with probability $1 - F(c_i + m - m^*)$. Then, ex ante, bidder i wins against $(n-1)$ bidders with probability P (defined in expectation over the unknown mean μ) such that

$$P = \int_\mu (1 - F(c_i + m - m^*))^{n-1} f_\mu(\mu) d\mu.$$

In order to provide an explicit form of the probability of winning, let us now consider the special case of a uniform distribution for F. We have

$$P = \int_\mu \left(\frac{a - m + m^* - c_i + \mu)}{2a}\right)^{n-1} \frac{1}{2a} d\mu. \tag{3}$$

To compute this probability, assume[1] e.g. that $m \geq m^*$.

Then $\left(\frac{a-m+m^*-c_i+\mu)}{2a}\right) \geq 0$ if $\mu \geq c_i - a + m - m^*$ and $\left(\frac{a-m+m^*-c_i+\mu)}{2a}\right) \leq 1$ if $\mu \leq c_i + a + m - m^*$. Since $m \geq m^*$ and given that μ lies over the interval $[c_i - a, c_i + a]$, one thus has to integrate only over the interval $[c_i - a + m - m^*, c_i + a]$. Then P becomes

$$\begin{aligned} P &= \int_{c_i-a+m-m^*}^{c_i+a} \left(\frac{a-m+m^*-c_i-\mu)}{2a}\right)^{n-1} \frac{1}{2a} d\mu \quad (4) \\ &= \frac{1}{n}\left(\frac{2a-(m-m^*)}{2a}\right)^n. \quad (5) \end{aligned}$$

Given (5) and the first order condition (2), the unique symmetric equilibrium for the strategic markup satisfies[2]

$$m = m^* = \frac{2a\rho}{n}.$$

Then, bidder i bids

$$b_i = c_i + \frac{2a\rho}{n}. \quad (6)$$

Note that the optimal strategic markup of a bidder is increasing with respect to the uncertainty parameter a, decreasing with his risk-aversion level and decreasing with the number of bidders. These results are consistent with intuition and conventional results in auction theory.[3] We believe that, in many situations of practical interest, the behavioral rule by which a supplier demands a constant absolute margin is relevant. Such behavior is even observed where bidders should use more sophisticated equilibrium strategies. This is confirmed by a recent laboratory experiment. In the framework of a private-value first-price sealed-bid auction, Shachat and Wei (2011) estimate that one-fourth of the subjects follow a simple markup rule and approximately two-thirds follow a strategic Nash equilibrium strategy.

4 The FPA with an ROFR

Consider now that player I is granted an ROFR and that $(n-1)$ bidders now compete in the FPA. Consider e.g. the case of a bidder, say i, in the auction. In order to

[1] The same reasoning can be applied with $m < m^*$.

[2] We check that the second order conditions are satisfied.

[3] See among others Klemperer (2001) or Krishna (2002).

win, bidder i not only has to defeat $(n-2)$ bidders in the auction but has also to defeat player I. As in the previous section, the fact that a bidder privately knows his own cost does not reveal anything to him about his relative position and so does not affect his probability of winning. Therefore, there is no reason why he should let his strategic markup depend on his own cost. Hence, still considering the case of bidder i, we can assume that his equilibrium bid takes the form (where upperscript R reflects the procedure with an ROFR)

$$b_i^R\left(c_i\right) = c_i + r,$$

where r represents his markup (or profit).

Bidder i's probability of winning the auction can be written as $P(r, r^*)$ when he chooses a strategic markup r, while the other bidders choose r^*. Since I is granted an ROFR, this provides him with the chance to win the procurement contract by matching the best offer of the $(n-1)$ bidders competing in the FPA. So i knows that I will win the procurement contract if the best offer of the $(n-1)$ bidders is higher than c_I, the cost of player I. Thus when i chooses a strategic markup r, he can only win if his bid is lower than c_I, which occurs with probability $Q(r, c_I)$.

Then, bidder i's expected utility can be written as

$$EU_i^R = (r)^\rho P(r, r^*) Q(r, c_I), \tag{7}$$

Differentiating (7) with respect to r yields the optimal markup for bidder i

$$r = \frac{-\rho P(r, r^*) Q(r, c_I)}{P'(r, r^*) Q(r, c_I) - P(r, r^*) Q'(r, c_I)}. \tag{8}$$

In the previous section, (5) yields bidder i's probability of winning the auction with n bidders. Obviously, with $(n-1)$ bidders, this probability becomes

$$P = \frac{1}{n-1}\left(\frac{2a - (r - r^*)}{2a}\right)^{n-1}. \tag{9}$$

Let us now derive $Q(r, c_I)$. The optimal strategy for player I is to accept the contract if the lowest bid submitted by his competitors in the auction is higher than his own cost c_I. So Bidder i wins against player I if $c_i + r < c_I$, which occurs with probability $1 - F(c_i + r)$. Then, ex ante, bidder i wins with probability Q (defined in expectation over the unknown mean μ) such that

$$Q = \int_{\mu} (1 - F(c_i + r)) f_{\mu}(\mu) d\mu.$$

Under the special case of a uniform distribution for F, Q becomes

$$Q = \int_{\mu} \left(\frac{a - r - c_i + \mu)}{2a} \right) \frac{1}{2a} d\mu. \tag{10}$$

Notice that $\left(\frac{a-r-c_i+\mu)}{2a}\right) \geq 0$ if $\mu \geq c_i - a + r$ and $\left(\frac{a-r-c_i+\mu)}{2a}\right) \leq 1$ if $\mu \leq c_i + a + r$. Given that μ lies over the interval $[c_i - a, c_i + a]$, one thus has to integrate only over the interval $[c_i - a + r, c_i + a]$. Then we have

$$Q = \int_{c_i - a + r}^{c_i + a} \left(\frac{a - r - c_i - \mu)}{2a} \right) \frac{1}{2a} d\mu \tag{11}$$

$$= \frac{(2a - r)^2}{8a^2}. \tag{12}$$

Given (12) and the first order condition (8), the unique symmetric equilibrium is reached[4] when

$$r = r^* = \frac{a(1 + n + \rho) - a\sqrt{(n+1)^2 - 2\rho(n-3) + \rho^2}}{(n-1)}.$$

Remark that r and r^* tend to zero when ρ tends to zero, i.e. when bidders are infinitely risk-averse.

5 Comparison between bidding strategies

Comparing bidding strategies under both procedures, the following proposition can be stated[5]

Proposition 1 *The bidding strategies are more aggressive under the ROFR procedure than under the FPA.*

[4] We check that the second order conditions are satisfied.

[5] See Appendix A for a proof.

Notice that the result of proposition 1 differs from Arozamena and Weinschelbaum (2009). Under the IPV paradigm, those authors show that the bids under both procedures are the same in the special case of a uniform distribution. Their result may seem to be counter-intuitive but the interpretation given by Arozamena and Weinschelbaum (2009) is the following. Consider e.g. the two-bidder case where i submits a bid whereas I is granted an ROFR. Does i have an incentive to bid more aggressively than in a two-bidder FPA? Actually, i knows that I is ready to be more aggressive in an ROFR than in an FPA since I is ready to reduce his bid to his own cost. Thus i has an incentive to bid more aggressively. However, there is a counteracting effect. Since I is ready to bid his true cost, his inverse bidding function has a slope equal to 1, while the slope of i's inverse bidding function is steeper than 1. This change in the marginal behavior of I provides i with incentives to become less aggressive. In the special case of a uniform distribution, both effects exactly offset one another.

In our model, the counteracting effect vanishes since bidders use constant margins whatever their valuations. So the slopes of the inverse bidding functions are the same in both procedures. Thus, in the ROFR procedure, while choosing his bid, a bidder faces $n-2$ other bidders who choose the same markup and another player whose cost is drawn from the same distribution but who is ready to choose a markup of zero. In the FPA procedure, a bidder faces $n-1$ other bidders who choose the same markup. So, it seems intuitive that bids are more aggressive in the ROFR than in the FPA since a bidder in the auction is replaced by a bidder with a markup of zero in the ROFR.

Notice that bidders' attitude toward risk does not affect the result of proposition 1 since it holds whatever the value of ρ. However, when ρ increases, an interesting question to investigate is whether this effect impacts a bidder's strategy more in the FPA or in the ROFR procedure. Then we have the following corollary[6]

Corollary 1 *An increase of ρ impacts a bidder's strategy more in the FPA than in the ROFR procedure.*

We can provide an interpretation of corollary 1. In the ROFR procedure, a bidder faces $n-2$ other bidders exhibiting the same level of risk-aversion as him and an incumbent who is ready to choose a markup of zero. So an increase of ρ impacts

[6]See Appendix B for a proof.

the bidding strategy of $n-1$ bidders in the auction against a "strong" risk-averse incumbent. In the FPA, an increase of ρ impacts the bidding strategy of n bidders who all behave as "symmetric" risk-averse bidders. So when bidders become less risk-averse, the FPA bidding strategy increases more than the ROFR one.

6 Comparison between buyer's expected costs

Given the bidders' optimal strategies, we can now compute the buyer's expected cost, EC^A (where upperscript A reflects the auction procedure), from an FPA when the costs are drawn from a uniform distribution. Define $EC_1^{(n)}$ as the lowest expected cost among n bidders; We have

$$\begin{aligned} EC^a &= EC_1^{(n)} + m && (13) \\ &= \int_{\mu-a}^{\mu+a} cnf(c)(1-F(c))^{n-1}\, dc + \frac{2a\rho}{n} \\ &= \mu - a + \frac{2a}{n+1} + \frac{2a\rho}{n}, && (14) \end{aligned}$$

where $nf(c)(1-F(c))^{n-1}$ is the frequency distribution of the lowest cost among n bidders.

Consider now the ROFR procedure. The best strategy for player I, who holds the ROFR, is to accept the contract if the lowest bid submitted by his competitors in the auction is higher than his own cost c_I. Then, the buyer's expected cost, EC^R, will equal the lowest bid among $n-1$ bidders in the auction. Thus, we can compute

$$\begin{aligned} EC^R &= EC_1^{(n-1)} + r \\ &= \int_{\mu-a}^{\mu+a} c(n-1)f(c)(1-F(c))^{n-2}\, dc + r \\ &= \mu - a + \frac{2a}{n} + r \\ &= \mu + \frac{a\left(-2 + n\left(4 + \rho - \sqrt{(n+1)^2 - 2\rho(n-3) + \rho^2}\right)\right)}{n(n-1)}, && (15) \end{aligned}$$

where $(n-1)f(c)(1-F(c))^{n-2}$ is the frequency distribution of the lowest cost among

$(n-1)$ bidders. Then, comparing the buyer's expected costs under both procedures, we have the following proposition[7]

Proposition 2 *The buyer is better off (resp. worse off) granting a player an ROFR if*

$$\rho > (resp. <)\rho^* = \frac{2 + n\left(\sqrt{8 + n(n+8)} - n - 2\right)}{2(n+1)}. \tag{16}$$

We can give an intuition of this result. In neither procedure does the markup depend on bidders' costs. In an FPA, the seller's expected cost is equal to the lowest expected cost among n bidders, $EC_1^{(n)}$, plus the markup m. In an ROFR procedure, the seller's expected cost is equal to the lowest expected cost among $(n-1)$ bidders, $EC_1^{(n-1)}$, plus the markup r. Obviously, $EC_1^{(n)} < EC_1^{(n-1)}$. This competition effect tends to favor the FPA in terms of minimizing expected costs. However, from proposition 1, $r < m$, i.e. bidders are more aggressive under the ROFR procedure. This "aggressiveness " effect may offset the competition effect. From corollary 1, we have stated that when bidders become more risk-averse, this impacts the bidding strategies in the FPA more than in the ROFR. So the gap between the markup decreases as bidders become more risk-averse. The more risk-averse the bidders, the lower the "aggressiveness " effect. This is why the buyer should use the FPA procedure when bidders are sufficiently risk-averse (i.e. $\rho < \rho^*$). In this case the aggressiveness effect cannot offset the competition effect. Notice from proposition 2 that when bidders are risk-neutral (i.e. $\rho = 1$), the buyer is better off granting a player an ROFR.

Another interesting question to investigate is whether the cut-off point ρ^* increases or decreases when the number of bidders increases. Tackling this issue, the following corollary can be stated[8]

Corollary 2 *The cut-off point ρ^* increases with the number of bidders.*

When the number of bidders increases, the difference between $EC_1^{(n)}$ and $EC_1^{(n-1)}$ decreases and thus the impact of the "competition effect" on the comparison between both procedures decreases. Then, the aggressiveness effect can offset the competition effect even if bidders are slightly risk-averse.

[7]See Appendix C for a proof. We also show in Appendix C that $0 < \rho^* < 1 \ \forall n > 2$.

[8]See Appendix C for a proof.

7 Comparison between the incumbent's expected utilities

Let us now compare both procedures in terms of expected utilities for the incumbent. In the ROFR, the incumbent may win more often than in the FPA (where he is in a symmetric position with the other competitors). On the other hand, when he wins the contract, he may receive a lower payment in the ROFR than in the FPA since he is ready to reduce his offer to his own cost. Therefore, a comparison of expected utilities for the incumbent between both procedures is not clear cut. In order to yield a comparison, let us firstly compute the incumbent's expected utility in an FPA. Under this procedure, at equilibrium, all n bidders have the same winning probability $\frac{1}{n}$. Then the incumbent's expected utility can be written as

$$EU_I^A = \frac{m^\rho}{n}$$

Under the ROFR procedure, the incumbent can only win if its cost c_I is lower than the lowest bid of $n-1$ bidders in the FPA. Define c_1 as the lowest expected cost among $n-1$ bidders. The lowest expected bid is $c_1 + r$. Then the incumbent can only win if $c_I < c_1 + r$ i.e. if $c_1 > c_I - r$. Thus, one has to integrate c_1 over $[c_I - r, \mu + a]$. Then we have to distinguish two cases. When $c_I - r > \mu - a$ i.e. $c_I > \mu - a + r$, we have to integrate c_I over $[\mu - a + r, \mu + a]$. When $c_I - r < \mu - a$ i.e. $c_I < \mu - a + r$, we have to integrate c_I over $[\mu - a, \mu - a + r]$. Thus, the incumbent's expected utility can be written as

$$\begin{aligned} EU_I^R &= \int\limits_{\mu-a+r}^{\mu+a} \int\limits_{c_I-r}^{\mu+a} (c_1 + r - c_I)^\rho \, (n-1)(1-F(c_1))^{(n-2)} f\,(c_1)\, dc_1 f\,(c_I)\, dc_I \\ &+ \int\limits_{\mu-a}^{\mu-a+r} \int\limits_{\mu-a}^{\mu+a} (c_1 + r - c_I)^\rho \, (n-1)(1-F(c_1))^{(n-2)} f\,(c_1)\, dc_1 f\,(c_I)\, dc_I \end{aligned}$$

The first term is the incumbent's expected utility when $\mu - a + r \leq c_I$. The second term is his expected utility when $c_I \leq \mu - a + r$. We did not success to compare both expected utilities for all n and for all ρ. To simplify, we only provide some numerical results. Using a mathematical software, we can compute the ratio EU_I^R / EU_I^A for a number of competitors $n \in \{2, ..., 10\}$ and for many values of $\rho \in (0, 1]$. Appendix D displays the results. Even though we have no formal proof, by virtue of the continuity of this ratio with respect to ρ, we formulate the following proposition:

Proposition 3 *For $n \in \{2, 10\}$, the incumbent's expected utility is higher in the ROFR mechanism than in the FPA one, whatever ρ.*

We can see in the numerical computations that the difference between both utilities increases with n. So, we will expect this result to be true for all n.

8 Conclusion

In this paper, we have analyzed the economic impact of an ROFR clause on both the strategic behavior of unfavored bidders and the purchaser's expected cost. We have shown that bidding strategies are more aggressive under the ROFR procedure than under the FPA. We have also shown that an ROFR clause can lower the buyer's expected cost when suppliers (incumbent and new suppliers) are risk-neutral or slightly risk-averse. Using a numerical simulation, we have also shown that the incumbent's expected utility is higher when he is granted an ROFR than when he competes under an FPA. So, under our specific framework about bidders' beliefs and information about their competitors' costs, an ROFR clause can benefit both the buyer and the favored supplier.

9 Appendix

Appendix A

Proof of proposition 1

$$r - m = \frac{a\left(n(1-\rho) + n^2 + 2\rho - n\sqrt{(n+1)^2 - 2\rho(n-3) + \rho^2}\right)}{n(n-1)}.$$

Then

$$\begin{aligned} r - m < 0 \quad &\Leftrightarrow \quad n(1-\rho) + n^2 + 2\rho < n\sqrt{(n+1)^2 - 2\rho(n-3) + \rho^2} \\ &\Leftrightarrow \quad \left(n(1-\rho) + n^2 + 2\rho\right)^2 < n^2\left((n+1)^2 - 2\rho(n-3) + \rho^2\right) \\ &\Leftrightarrow \quad -4(n-1)\rho(n+\rho) < 0, \end{aligned}$$

which is satisfied $\forall n \geqslant 2$ and $\forall \rho > 0$. $\square$

Appendix B

Proof of corollary 1

The RHS of (17) evaluated at $\rho = 0$ equals $\frac{2a}{n(n+1)}$, which is positive $\forall n \geqslant 2$ and $\forall a > 0$.

Besides, we can compute

$$\frac{\partial m}{\partial \rho} - \frac{\partial r}{\partial \rho} = \frac{2a}{n} - \frac{a}{n-1}\left(1 + \frac{n-3-\rho}{\sqrt{(n+1)^2 - 2(n-3)\rho + \rho^2}}\right). \tag{17}$$

The derivative of the RHS of (17) with respect to ρ is equal to

$$\frac{8a}{((n+1)^2 - 2(n-3)\rho + \rho^2)^{\frac{3}{2}}}$$

which is positive. Then, the difference $\frac{\partial m}{\partial \rho} - \frac{\partial r}{\partial \rho}$ increases with ρ. So, we can conclude that $\frac{\partial m}{\partial \rho} > \frac{\partial r}{\partial \rho}$. $\square$

Appendix C

Proof of proposition 2

$$EC^a - EC^R = a\left(\frac{2}{n+1} + \frac{2(\rho-1)}{n} + \frac{2+\rho}{1-n} - 1 + \frac{\sqrt{(n+1)^2 - 2(n-3)\rho + \rho^2}}{n-1}\right)$$

Solving $EC^A - EC^R = 0$ yields a single positive root $\rho^* = \frac{2+n\left(\sqrt{8+n(n+8)}-n-2\right)}{2(n+1)}$ with $0 < \rho^* < 1$. When $\rho = 0$, we have $EC^A - EC^R = \frac{-2a}{n(n+1)} < 0$ $\forall n > 0$. When $\rho = 1$, we have $EC^A - EC^R = a\left(\frac{2}{n+1} - \frac{3}{n-1} - 1 + \frac{\sqrt{n^2+8}}{n-1}\right) > 0$ $\forall n > 1$. Then, since $EC^A - EC^R$ is continuous in ρ, we can conclude than $EC^A < (resp. >)EC^R$ for $\rho > (resp. <)\rho^*$. $\square$

Proof of $\rho^* > 0$.

$$\begin{aligned}\rho^* > 0 &\Leftrightarrow 2 + n\left(\sqrt{8+n(n+8)} - n - 2\right) > 0 \\ &\Leftrightarrow n\sqrt{8+n(n+8)} > n^2 + 2n - 2 \\ &\Leftrightarrow 4(n^3 + 2n^2 + 2n - 1) > 0,\end{aligned}$$

which is satisfied $\forall n \geqslant 2$. $\square$

Proof of $\rho^* < 1$.

$$\begin{aligned}\rho^* < 1 &\Leftrightarrow 2 + n\left(\sqrt{8+n(n+8)} - n - 2\right) < 2 + 2n \\ &\Leftrightarrow n\sqrt{8+n(n+8)} < n^2 + 4n \\ &\Leftrightarrow 8 < 16,\end{aligned}$$

which is satisfied. $\square$

Proof of corollary 2

Let us denote $A = \sqrt{8+n(n+8)}$. We have

$$\frac{\partial \rho^*}{\partial n} = \frac{(n+2)^3 - A\,(4+n(n+2))}{2A(n+1)^2}.$$

Since $A > 0$,

$$\begin{aligned}\frac{\partial \rho^*}{\partial n} > 0 &\Leftrightarrow A < \frac{(n+2)^3}{A\,(4+n(n+2))} \\ &\Leftrightarrow -8(n^4 + 2n^3 - 8n - 8) < 0,\end{aligned}$$

which is obviously satisfied $\forall n \geqslant 2$. $\square$

Appendix D

Using a mathematical software, we can compute the ratio EU_I^R/EU_I^A (i.e. the incumbent's expected utility in the ROFR divided by his expected utility in the FPA) for a number of competitors $n \in \{2, ..., 10\}$. The following table displays the results for $\rho \in \{0.00000001, 0.05, 0.10, ..., 0.95, 1\}$. It appears clearly that the ratio is always larger than 1 and that it is increasing in n for each of the computed values of ρ. $\square$

Table 1: Incumbent's ratio of expected utility (ROFR/FPA)

	n								
ρ	2	3	4	5	6	7	8	9	10
0.00000001	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
0.05	1.15494	1.16415	1.16922	1.17248	1.17476	1.17644	1.17773	1.17875	1.17958
0.1	1.24786	1.26820	1.27924	1.28635	1.29131	1.29496	1.29777	1.30000	1.30180
0.15	1.31694	1.34978	1.36748	1.37888	1.38683	1.39269	1.39719	1.40075	1.40364
0.2	1.36997	1.41634	1.44130	1.45736	1.46858	1.47684	1.48318	1.48820	1.49227
0.25	1.41072	1.47143	1.50417	1.52525	1.53996	1.55081	1.55913	1.56571	1.57105
0.3	1.44152	1.51719	1.55817	1.58457	1.60301	1.61660	1.62703	1.63528	1.64196
0.35	1.46396	1.55506	1.60468	1.63670	1.65907	1.67557	1.68822	1.69822	1.70633
0.4	1.47923	1.58610	1.64474	1.68263	1.70913	1.72868	1.74366	1.75551	1.76511
0.45	1.48828	1.61113	1.67910	1.72312	1.75393	1.77666	1.79409	1.80787	1.81903
0.5	1.49185	1.63081	1.70839	1.75875	1.79405	1.82010	1.84008	1.85587	1.86866
0.55	1.49060	1.64567	1.73311	1.79003	1.82998	1.85947	1.88209	1.89997	1.91445
0.6	1.48505	1.65617	1.75367	1.81735	1.86209	1.89515	1.92051	1.94056	1.95679
0.65	1.47569	1.66271	1.77044	1.84104	1.89073	1.92747	1.95566	1.97795	1.99599
0.7	1.46292	1.66564	1.78372	1.86140	1.91617	1.95670	1.98781	2.01241	2.03232
0.75	1.44712	1.66524	1.79378	1.87869	1.93867	1.98309	2.01720	2.04418	2.06602
0.8	1.42862	1.66180	1.80085	1.89311	1.95841	2.00683	2.04403	2.07346	2.09727
0.85	1.40773	1.65556	1.80516	1.90487	1.97561	2.02811	2.06848	2.10041	2.12627
0.9	1.38472	1.64675	1.80687	1.91413	1.99042	2.04710	2.09071	2.12521	2.15315
0.95	1.35984	1.63557	1.80618	1.92107	2.00298	2.06393	2.11085	2.14799	2.17806
1	1.33333	1.62220	1.80324	1.92581	2.01343	2.07873	2.12903	2.16887	2.20113

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