CENTRE DE RECHERCHE
SUR LES STRATÉGIES ÉCONOMIQUES

Duopoly Competition and Regulation in a Two-Sided Health Care Insurance Market with Product Differentiation

Audrey Boilley

March 2013

Working paper No. 2013 – 02

CRESE
30, avenue de l'Observatoire
25009 Besançon
France
http://crese.univ-fcomte.fr/

UNIVERSITÉ DE FRANCHE-COMTÉ

Duopoly Competition and Regulation in a Two-Sided Health Care Insurance Market with Product Differentiation

Audrey Boilley
CRESE,
30 avenue de l'Observatoire,
BP 1559, 25009 Besançon Cedex,
France
audrey.boilley@univ-fcomte.fr

Abstract

We compare duopoly competition with a regulated public monopoly in the health care insurance sector using the two-sided market approach. Health plans allow policyholders and physicians to interact. Policyholders have a preference for one of two health plans and value the diversity of physicians. Physicians value the number of policyholders because they are paid on a fee-for-service basis. This is a positive network externality. We find that the resulting Nash equilibria are explained by the two standard effects of product differentiation: the price competition effect and the market share effect, and by two opposing effects related to the network externality. We call these the positive earning effect and the negative spending effect. Overall the comparison between the two types of organizations shows that regulation is preferred when the physicians' market is not covered and competition is preferred when it is covered. But each time the choice is made at the expense of one type of agent.

Keywords: Two-Sided Markets, Managed Care Competition, Network Effects, Product Differentiation, Hotelling, Public Policy

JEL-Classification: C72, D21, D43, I13, L11

1 Introduction

In Iceland, Norway, Sweden, and the United Kingdom, the government pays a large share of health care resources, whereas the United States together with Mexico and Chile are the only OECD countries where less than 50 percent of health spending is publicly financed. The public share of health expenditure in the United States was 48.2 percent in 2010, much lower than the OECD average of 72.2 percent (OECD, 2012 in [18]). Today, a specific type of organization dominates the United States health care insurance marketplace: managed care. There is a wide range of institutional arrangements for managed care and no single definition of them. For example, managed care practices can consist in denial of care, limiting choice of provider, or monitoring service utilization, all in order to limit health care costs. But the majority of managed care organizations (MCO) use networks of providers (Ma and McGuire, 2002), through exclusive or non-exclusive contracts. Thus they combine the functions of health insurance, delivery of care, and administration.

Managed care health plans can be seen as "two-sided markets" because they are platforms which allow two groups of agents to interact. A two-sided market is defined by Rochet and Tirole (2006) as a market in which interactions between end-users are enabled by one or several platforms competing for the two sides of the market.[1] This platform may also be called a network. In a two-sided market there are some cross-group externalities because an agent on one side of the market exerts a positive (or negative) externality on each agent in the other group.

Our paper builds on this recent literature on two-sided markets and on the literature on product differentiation. For public policy purposes, we seek to compare two types of health care insurance system: a duopoly competitive system and a regulated system with a public monopoly. The choice between a competitive organization and a regulated monopoly is a recurrent issue of economic policy in the health care insurance sector. Belgium, Germany, the Netherlands, and Switzerland have regulated competitive systems and it is more competitive in the United States. In France health insurance is provided by a regulated public monopoly. But each country tries to implement the relevant regulation, the main objective being to ensure universal service for everybody to improve efficiency in health care production.

Following the literature on two-sided markets, we assume that health plans are two-sided platforms which provide interaction services between two groups of agents: policyholders and health care providers. First we consider duopoly competition and we determine non-cooperative Nash equilibria. Then we consider a regulated public monopoly and we characterize the optimal situation. We also compare the resulting surpluses in each situation. The externality from policyholders to physicians (and from physicians to policyholders) is positive because a policyholder becoming sick provides more payment to a physician when the latter is paid on a fee-for-service basis. And a supplementary physician affiliated to the platform exerts a positive externality on policyholders because they value the diversity of physicians. But contrary to most papers on two-sided markets (see for example

[1] The "side" of the market is one of the groups of agents who interact through the platform. If there are several sides, it is a multi-sided market.

Caillaud and Jullien (2003), Rochet and Tirole on the credit card industry (2003, 2008), Gabszewicz, Laussel and Sonnac (2004), Peitz and Valletti (2004), Anderson and Gabszewicz (2006) on the media industry and Armstrong (2006) on shopping malls), this network externality is of particular nature. Indeed, health plans do not make profits on both sides of the market. They make profits on the policyholders' side only, while they have to pay physicians to attract them. Bardey and Rochet (2010) also use this framework to study health plan competition. They consider a Health Maintenance Organization (HMO) and a Preferred Provider Organization (PPO). They assume that there are more affiliated physicians to the PPO than to the HMO, and therefore the PPO attracts riskier policyholders. They also assume that both health plans have access to two identical but distinct groups of physicians and the physicians' market is not covered. Thus there is a hidden assumption: physicians are single-home.[2] Moreover health plans do not compete on this side of the market because of their access to two different pools of physicians.

Our paper is also built on the Hotelling literature of product differentiation. We consider exogenously horizontally differentiated health plans but we do not have a single attribute of the product as in Hotelling (1929). We consider two attributes of the "health insurance" service offered by health plans represented through the location of health plans (exogenous horizontal differentiation) and the number of affiliated physicians (endogenous vertical differentiation) like Neven and Thisse (1989) who consider a model with both endogenous horizontal and vertical differentiation. They show that firms try to reduce their differentiation on some attributes of the product only if they are differentiated enough on the other attributes. A similar conclusion appears in Ansari, Economides and Steckel (1998). They consider a variant of Hotelling's model with an n-attribute product model.[3] In the two-dimensional model, they show that when consumers assign a high weight to one attribute of the product, both competitive firms are maximally differentiated on that attribute and minimally differentiated on the other attribute. Thus there can be what they call a max-min equilibrium or a min-max equilibrium. And when consumers assign roughly comparable weights to each attribute, Ansari et al. (1998) show that both types of equilibria exist. Our results partly confirm these analyses within the framework of exogenously horizontally differentiated two-sided platforms. Besides being maximally horizontally differentiated, we find that health plans are minimally vertically differentiated when policyholders have a low preference for the diversity of physicians. This confirms both the results of Neven and Thisse (1989) and Ansari et al. (1998). When both the policyholders' preference parameters are intermediate, we find both symmetric and asymmetric equilibria. This point is more like the analysis of Ansari et al. (1998). And when policyholders have a high preference for the diversity of physicians, again health plans tend to be minimally vertically differentiated. This is a novel aspect in comparison to the model of product differentiation of Ansari et al. (1998) and it is explained by the network externality. More specifically, the Nash equilibria depend on four effects with two effects directly depending on the network externality. In addition to the standard effects of product differentiation: the price competition effect which

[2] Single-home agents are affiliated to one network only, whereas multi-home agents are affiliated to two or more networks.
[3] They analyze two- and three-attribute product models only.

drives health plans to be differentiated (Chamberlinian incentive) and the market share effect which drives them to play symmetric strategies, we find a positive "earning effect" and a negative "spending effect". On the one hand, when health plans decide to hire more physicians, the network externality makes it possible to set a higher premium and thus to make more profits. On the other hand, this raises costs through payments to physicians. Therefore the double effect of the network externality has to be taken into account when considering a model of product differentiation within the particular framework of a two-sided market.

A model of a regulated public monopoly considered as a two-sided market providing health care insurance services has not been studied yet. But Armstrong (2006) and Rochet and Tirole (2003, 2006) establish a general framework of a monopoly platform. The main difference between their articles is the tariff charged by the platform. Armstrong assumes fixed fees, whereas Rochet and Tirole assume per-transaction charges in their 2003 paper, and both types of charges in their 2006 paper. So there is pure membership pricing in Armstrong's model, and pure usage pricing in Rochet and Tirole (2003). In our paper under regulation as under duopoly competition, health plans set a charge on the policyholders' side and pay a fee-for-service rate on the physicians' side. Under regulation, we find that only the two effects related to the network externality remain. The regulated public monopoly decides to hire all physicians only when the policyholders' preference parameter is high enough. Moreover, under the maximization expected welfare criterion, the comparison between competition and regulation shows that when the physicians' market is not covered and when the social cost of public funds is relatively low, the regulated situation is preferred to the competitive situation to the detriment of health plans. But when the physicians' market is covered, the competitive situation will be preferred to the regulated situation to the detriment of policyholders.

The rest of the paper is organized as follows. Section 2 describes the model. Section 3 defines the equilibria under duopoly competition. The situation under a regulated public monopoly is derived in section 4. The policy comparison is presented in section 5. And section 6 concludes.

2 The model

Three categories of agents interact: health plans, policyholders, and physicians.

2.1 Health plans

First we consider duopoly competition between two MCOs providing health insurance to policyholders and buying physicians' services. We assume that health plans are horizontally differentiated. They can differ in the range of devices they offer to policyholders and physicians. They can also differ in working hours, in values they support which contribute to their reputation, or in the organization's size. We use a Hotelling specification to model horizontal differentiation among health plans. The location of health plans is exogenous on the segment of unit length, representing the town. Health plan 1 is located at one end of the town, at 0, and health plan 2 is at the other end, at 1. Both health plans have access to the same group of physicians. Then we assume that each health plan charges a fixed insurance premium to policyholders and pays its affiliated physicians on a fee-for-service basis. We assume that price discrimination is forbidden: the insurance premium is not a risk-based premium.

The profit function of health plan i $\forall i = 1, 2$ is then given by:

$$\pi^i = p^i n^i - TP^i, \tag{1}$$

where p^i is the insurance premium paid by policyholders to health plan i, n^i is the number of policyholders joining health plan i, and TP^i is the total payment paid by health plan i to physicians. It equals the fee-for-service rate set by health plan i multiplied by the expected number of patients in health plan i.

2.2 Policyholders

Insurance is mandatory for subscribers who are single-home. This means that they can choose only one health plan. We assume that there is a mass N of policyholders that we normalize to one hereafter. Policyholders are characterized by their probability of illness θ, which is uniformly distributed over the interval [0,1]. They also have an ideal health plan represented by their location x in the town, with x distributed over $[0, 1]$ according to a cumulative distribution function $F(.)$ and a density f that is everywhere positive. Their preferences are about the range of devices offerred by health plans, opening hours, or values claimed by the organizations. We assume that policyholders are risk-neutral [4] and they can interact with physicians through the network only. They value the diversity of physicians in a health plan according to the parameter β. All policyholders are assumed to have the same parameter. Moreover, they have to pay a fixed insurance premium p^i to health plan i in exchange for the insurance service provided.

Their utility in health plans 1 and 2 is given by: $U^1(\beta, x) = \omega + \beta n_B^1 - \theta B + \theta R - p^1 - tx$ and $U^2(\beta, x) = \omega + \beta n_B^2 - \theta B + \theta R - p^2 - t(1 - x)$, where ω is the policyholders' initial wealth, n_B^i is the number of physicians affiliated to health plan i, B represents the loss due to illness, R is the insurance's reimbursement, and t is the policyholder's "transportation cost" parameter. We assume full insurance coverage so that $R = B$. Therefore the policyholders' utility depends on their preference parameter β and their location x. It does not depend on their probability of illness θ because of the full insurance coverage.

2.3 Physicians

Using the Hotelling's framework, we assume that physicians are characterized by their location y in the town represented by a unit segment. We assume that y is uniformly distributed on the interval $[0, 1]$ and corresponds to the ideal health plan of physicians.[5] This means that if health plans offer the same fee-for-service rate to

[4] We assume that insurance is mandatory (so policyholders cannot avoid paying the insurance premium). And because we also assume that insurance premiums never drive policyholders out of the market, we can make the assumption that policyholders are risk-neutral, like Glazer and Mc Guire (2000).

[5] Like policyholders, it refers to the horizontal characteristics of health plans.

physicians, there will be a positive demand from physicians for each of them. And in this case, physicians will go to the health plan which is closest to them. The mass of physicians is N_B, which we normalize to one hereafter. We assume that physicians can choose to be affiliated to health plan 1, or to health plan 2, or to both health plans at the same time, given their utility when they join one or other health plan, or both. So physicians are not necessarily single-home like policyholders, they may be multi-home.[6] And as they can join both health plans at the same time, they do not sign an exclusive contract with their employer. Moreover, we assume that the physicians' decision to join one health plan is independent of their decision to join the other health plan, i.e. it is not an "either-or" decision as described in Armstrong (2006). This assumption of independent decisions means that in fact health plans do not compete to attract physicians. Finally, in our paper there is a positive cross-group externality between policyholders and physicians. Policyholders value the diversity of physicians. And the more affiliated policyholders there are falling ill, the higher physicians' income will be because they are paid on a fee-for-service basis.

The utility functions of physicians under health plan 1 and health plan 2 are respectively: $U_B^1(y) = TR - t_B \cdot y$ and $U_B^2(y) = TR - t_B \cdot (1-y)$, where TR is physicians' total revenue and t_B is the "transportation cost" parameter that physicians incur because they are not affiliated to their preferred health plan. Physicians' total revenue equals the fee-for-service rate net of the unit treatment cost, multiplied by the average expected number of patients.

3 Equilibria under duopoly competition

We define the policyholders' and physicians' demands for each health plan. Then we determine Nash equilibria which are such that $({n_B^i}^*, p^{i*})$ is an optimal response to the health plan j's strategies $\forall i \neq j$. If at the resulting equilibria the number of physicians affiliated to health plan 1 is different from the number of physicians affiliated to health plan 2 (${n_B^1}^* > {n_B^2}^*$ or ${n_B^1}^* < {n_B^2}^*$), there are asymmetric Nash equilibria and the model incorporates both horizontal and vertical differentiation. But if ${n_B^1}^* = {n_B^2}^*$ there are symmetric equilibria and health plans are horizontally differentiated only.

3.1 Demands

Policyholders' side

A policyholder prefers to join health plan 1 rather than health plan 2 if: $U^1(\beta, x) \geq U^2(\beta, x)$, then if:

$$x \leq \frac{1}{2} + \frac{\beta(n_B^1 - n_B^2) - p^1 + p^2}{2t} = \bar{x}. \tag{2}$$

Conversely a policyholder prefers to join health plan 2 rather than health plan 1 if $x \geq \bar{x}$. Assuming full market

[6] The situation in which one of the two groups of agents who interact through the health plan single-homes and the other group multi-homes is called a "competitive bottleneck" situation. See Armstrong (2006). Our model approaches this model because we have assumed that policyholders single-home and physicians can multi-home.

coverage and $N = 1$, policyholders' demands are defined by $n^1 = F(\bar{x})$ and $n^2 = 1 - F(\bar{x})$.

Physicians' side

As we have assumed that physicians do not make an "either-or" decision as regards their choice of health plan, to define the marginal physician we have to compare the utility of physicians under health plan i $\forall i = 1, 2$, to their reservation utility set to zero. Hence physicians opt for health plan 1 (independently of their decision to join health plan 2), if $U_B^1(y) \geq 0$, then if: $\frac{(s^1-c)F(\bar{x})}{n_B^1} \cdot \int_0^1 \theta f(\theta) d\theta - t_B y \geq 0$, where s^1 is the fee-for-service rate set by health plan 1 and c is the unit cost of treatment. Moreover, as the probability of illness θ is uniformly distributed on the interval $[0, 1]$, the expected number of patients under health plan 1 is: $F(\bar{x}) \cdot \int_0^1 \theta f(\theta) d\theta = \frac{F(\bar{x})}{2}$, yielding: $y \leq \frac{(s^1-c)F(\bar{x})}{2n_B^1 t_B} = \bar{y}$. Hence physicians' demand for health plan 1 n_B^1 corresponds to the number of physicians who choose this health plan. Physicians whose location y is lower than the threshold $\bar{y}$ opt for health plan 1. As $N_B = 1$ we obtain:

$$n_B^1 = \frac{(s^1 - c)F(\bar{x})}{2n_B^1 t_B}. \tag{3}$$

In the same way as for health plan 1, physicians whose location y is higher than the threshold $\bar{y}'$ opt for health plan 2 if: $y \geq 1 - \frac{(s^2-c)(1-F(\bar{x}))}{2n_B^2 t_B} = \bar{y}'$. Hence the market share of health plan 2 on the physicians' side is:

$$n_B^2 = \frac{(s^2 - c)(1 - F(\bar{x}))}{2n_B^2 t_B}. \tag{4}$$

3.2 Equilibria

The profit function of health plan i $\forall i = 1, 2$ is given by equation (1), which can also be written:

$$\pi^1 = p^1 F(\bar{x}) - \frac{s^1 F(\bar{x})}{2} \qquad \pi^2 = p^2(1 - F(\bar{x})) - \frac{s^2(1 - F(\bar{x}))}{2}.$$

We can rewrite equations (3) and (4) to express profit functions as functions of the insurance premiums p^i and of the numbers of physicians n_B^i:

$$(n_B^1)^2 t_B + \frac{cF(\bar{x})}{2} = \frac{s^1 F(\bar{x})}{2}, \tag{5}$$

and

$$(n_B^2)^2 t_B + \frac{c(1 - F(\bar{x}))}{2} = \frac{s^2(1 - F(\bar{x}))}{2}. \tag{6}$$

Because physicians opt for one health plan (or both) independently of their choice to join the other health plan, there is competition on the policyholders' side only. Health plans have two instruments to attract policyholders:

the insurance premium p^i and the number of physicians n_B^i, because policyholders value the diversity of physicians. Using equations (5) and (6), profit functions can be rewritten as:

$$\pi^1 = p^1 F(\bar{x}) - (n_B^1)^2 t_B - \frac{cF(\bar{x})}{2}, \tag{7}$$

$$\pi^2 = p^2(1 - F(\bar{x})) - (n_B^2)^2 t_B - \frac{c(1 - F(\bar{x}))}{2}. \tag{8}$$

Each health plan i maximizes its profit function π^i with respect to p^i and n_B^i, subject to the following constraints:

$$n_B^i \leq 1 \tag{9}$$

$$\bar{x} \leq 1 \tag{10}$$

We show in Appendix 1 that constraints (9) and (10) for each firm may be simultaneously not binding. They may also be simultaneously binding, or constraint (9) may be the single binding constraint for one of the two health plans only. The resulting Nash equilibria of the non-cooperative game between both health plans are given in the following proposition.

Proposition 1 *Under duopoly competition, with $\bar{x}$ defined by (2),*

- *when $1 - \frac{2t_B}{\beta} < F(\bar{x}) < \frac{2t_B}{\beta}$, i.e. $\beta < 4t_B$, i.e. no constraints are binding, the Nash equilibrium is symmetric and characterized by:*

$$p^{1*} = \frac{c}{2} + \frac{F(\bar{x})}{f(\bar{x})} 2t, \tag{11}$$

$$p^{2*} = \frac{c}{2} + \frac{(1 - F(\bar{x}))}{f(\bar{x})} 2t, \tag{12}$$

$$n_B^{1\,*} = \frac{\beta F(\bar{x})}{2t_B}, \tag{13}$$

$$n_B^{2\,*} = \frac{\beta(1 - F(\bar{x}))}{2t_B}. \tag{14}$$

- *when $F(\bar{x}) < Min\{\frac{2t_B}{\beta}, 1 - \frac{2t_B}{\beta}\} = 1 - \frac{2t_B}{\beta}$ if $\beta < 4t_B$, $Min\{\frac{2t_B}{\beta}, 1 - \frac{2t_B}{\beta}\} = \frac{2t_B}{\beta}$ if $\beta > 4t_B$, i.e. constraint (9) is binding for health plan 2 only, the Nash equilibrium is asymmetric and characterized by: $n_B^{2\,*} = 1$, and equations (11), (12), (13), with $\alpha = \beta(1 - F(\bar{x})) - 2t_B > 0$.*

- *when* $F(\bar{x}) > Max\{\frac{2t_B}{\beta}, 1-\frac{2t_B}{\beta}\} = \frac{2t_B}{\beta}$ *if* $\beta < 4t_B$, $Max\{\frac{2t_B}{\beta}, 1-\frac{2t_B}{\beta}\} = 1-\frac{2t_B}{\beta}$ *if* $\beta > 4t_B$, *i.e. constraint (9) is binding for health plan 1 only, the Nash equilibrium is asymmetric and characterized by:* ${n_B^1}^* = 1$, *and equations (11), (12), (14), with* $\mu = \beta F(\bar{x}) - 2t_B > 0$.

- *when* $\frac{2t_B}{\beta} < F(\bar{x}) < 1 - \frac{2t_B}{\beta}$, *i.e.* $\beta > 4t_B$, *i.e. condition (9) is binding for both health plans, the Nash equilibrium is symmetric and characterized by:* ${n_B^1}^* = {n_B^2}^* = 1$, *and equations (11) and (12), with the Kuhn-Tucker multipliers* μ *and* $\alpha > 0$.

Moreover the second-order conditions are locally satisfied,[7] *thus the profit functions are locally quasi-concave, which ensures local maxima.*

Uniform distribution

In the case of a uniform distribution of the location of policyholders we obtain four equilibria (equilibria 1, 2, 3 and 4 hereafter):

1. when no constraints are binding, the second-order conditions are satisfied and the profits of health plans are strictly positive, *i.e.* $\beta < 4t_B$ and $\beta^2 < 8tt_B$, there is a symmetric Nash equilibrium: $F(\bar{x}) = 1 - F(\bar{x}) = \frac{1}{2}$, $p^{1*} = p^{2*} = \frac{c}{2} + t$, and ${n_B^1}^* = {n_B^2}^* = \frac{\beta}{4t_B}$;
2. when constraint (9) is binding for health plan 2 only, the second-order conditions are satisfied and the profits of health plans are strictly positive, *i.e.* $\beta < 4t_B$, $6tt_B < \beta^2 < 8tt_B$ and $\beta < 3t$, there is an asymmetric Nash equilibrium:

$$F(\bar{x}) = \frac{2t_B(\beta - 3t)}{\beta^2 - 12tt_B}, \tag{15}$$

$$1 - F(\bar{x}) = \frac{\beta^2 - 2t_B(\beta + 3t)}{\beta^2 - 12tt_B}, \tag{16}$$

$$p^{1*} = \frac{c}{2} + \frac{4tt_B(\beta - 3t)}{\beta^2 - 12tt_B}, \tag{17}$$

$$p^{2*} = \frac{c}{2} + \frac{2t[\beta^2 - 2t_B(\beta + 3t)]}{\beta^2 - 12tt_B}, \tag{18}$$

$${n_B^1}^* = \frac{\beta(\beta - 3t)}{\beta^2 - 12tt_B}, \tag{19}$$

$${n_B^2}^* = 1; \tag{20}$$

[7] See Appendix 1.

3. when constraint (9) is binding for health plan 1 only, under the same conditions, the results of equilibrium 2 are reversed. $F(\bar{x})$ is defined by equation (16) and $1 - F(\bar{x})$ by equation (15), p^{1*} is defined by equation (18) and p^{2*} by equation (17), ${n_B^1}^* = 1$ whereas ${n_B^2}^*$ is defined by equation (19) ;
4. when constraint (9) is binding for both health plans, the second-order conditions are satisfied and the health plans' profits are strictly positive, *i.e.* $\beta > 4t_B$, and $\beta^2 < 8tt_B$, there is a symmetric Nash equilibrium with: $F(\bar{x}) = 1 - F(\bar{x}) = \frac{1}{2}$, $p^{1*} = p^{2*} = \frac{c}{2} + t$, and ${n_B^1}^* = {n_B^2}^* = 1$.

Existence of Nash equilibria

We have a multiplicity of equilibria as in Ansari et al. (1998). When the physicians' market is not covered: $\beta < 2t_B$, [8] equilibrium 1 is the sole equilibrium. But when the physicians' market is covered and β is not too high ($\beta < 4t_B$), both equilibria 1 and 2 (or 3) exist. We do not know which of equilibrium 2 or equilibrium 3 will arise. In this case, we have the contrary of a coordination game (like the battle of the sexes game). The outcome of the game is unpredictable because health plans simultaneously choose the number of affiliated physicians and prices. And we cannot consider a two-step game because the choice of the number of physicians is directly determined through the choice of a fee-for-service rate as in Bardey and Rochet (2010). When equilibria 1 and 2 (or 3) coexist, it can be shown that equilibrium 1 is suboptimal relative to equilibrium 2 (or 3).

In this paper, we consider two attributes of the "health insurance" service: the location of health plans, which corresponds to the horizontal differentiation, and the number of affiliated physicians, which corresponds to the vertical differentiation. Thus given the firms' location, even when the diversity of physicians between both firms is not great, they do not have incentives for undercutting[9] as explained in Economides (1984)[10] because they are already differentiated on one attribute. Moreover the insurance premium cannot be lowered indefinitely. Because of the existence of a network externality of a particular nature, it cannot be too small because the profits made by health plans from policyholders are used to pay physicians. And without physicians, there is no demand from policyholders and thus no market. Finally, for the existence of Nash equilibria, we have verified that the second-order conditions are satisfied locally.[11]

Differentiation of health plans

The first attribute of the "health insurance" service offered by health plans is their location. This corresponds to their own characteristics.[12] The weight that policyholders assign to this attribute is t. The

[8] The physicians' market is covered when ${n_B^1}^* + {n_B^2}^* \geq 1$. At equilibrium 1, ${n_B^1}^* + {n_B^2}^* = \frac{\beta}{2t_B} < 1$ for $\beta < 2t_B$.

[9] Here undercutting would consist in lowering the insurance premium to attract more and more policyholders.

[10] Economides (1984) explains that a Nash equilibrium in prices does not always exist when firms are located close together because in this case undercutting is always optimal. And this is caused by the non-quasiconcavity of the profit functions. The rival firm always benefits from lowering its price to poach the other firm's market share.

[11] See Appendix 1.

[12] In section I, as examples of health plans' characteristics, we have mentionned the range of devices offered to policyholders and physicians, working/opening hours, values claimed by the organization and its size.

second attribute is the number of physicians affiliated to the health plan, which conveys the diversity of physicians offered by health plans. The weight that policyholders place on this attribute is β. Thus our model can incorporate both horizontal and vertical differentiation when health plans have different numbers of physicians.

The profit variation of each health plan divides into four effects. The total differential of the profit function can be used to underline these effects: $d\pi i = \frac{\partial \pi^i}{\partial n_B^i} dn_B^i + \frac{\partial \pi^i}{\partial p^i} dp^i \; \forall i = 1, 2$, yielding:

$$d\pi i = \left[\left(p^i - \frac{c}{2} \right) \frac{\beta}{2t} - 2n_B^i t_B \right] dn_B^i + \left[\left(p^i - \frac{c}{2} \right) \left(\frac{-1}{2t} \right) + n^i \right] dp^i \quad \forall i = 1, 2. \tag{21}$$

The first term of (21): $\left(p^i - \frac{c}{2}\right) \frac{\beta}{2t} \cdot dn_B^i$, is the positive effect of vertical differentiation because it corresponds to $\frac{\partial \pi^i}{\partial (n_B^i - n_B^j)}$, meaning that when the difference between n_B^i and n_B^j increases, π^i increases too.[13] It can be linked to the price competition effect which drives firms to become differentiated. The second term of (21): $-2n_B^i t_B \cdot dn_B^i$, is the negative effect of the network externality. We call it the "spending effect" because when the number of physicians increases, even if this also has a positive effect, charges for the physicians' payment increase for health plans, which lowers their profit. The third term of (21): $\left(p^i - \frac{c}{2}\right) \left(\frac{-1}{2t}\right) \cdot dp^i$, is the negative effect of vertical differentiation corresponding to $\frac{\partial \pi^i}{\partial (p^i - p^j)}$. It is comparable to the market share effect which drives firms to play similar strategies. And the last term of (21): $n^i \cdot dp^i$, is the positive effect of the network externality. We call it the "earning effect". Indeed, hiring more physicians makes it possible to set a higher premium on the policyholders' side and to make more profits. These four effects will determine equilibria.

Our paper comes close to the conclusions of Neven and Thisse (1989) and Ansari et al. (1998). At equilibrium 1 when β is low: $\beta < 2t_B$ and t is high: $t > \frac{\beta^2}{8t_B}$, health plans are minimally vertically differentiated and (exogenously) maximally differentiated as expected in the models of Neven and Thisse (1989) and Ansari et al. (1998). Thus equilibrium 1 is a max-min equilibrium following Ansari et al. (1998). When β and t are intermediate: $2t_B < \beta < 4t_B$ and $\frac{\beta^2}{8t_B} < t < \frac{\beta^2}{6t_B}$, there are two equilibria: equilibrium 1 and equilibrium 2 (or 3). In the two-dimensional model, when the attributes' weights are nearly the same, Ansari et al. (1998) also find two equilibria: a max-min equilibrium and a min-max equilibrium. In our case, whereas equilibrium 1 is a max-min equilibrium, equilibrium 2 (or 3) cannot be a min-max equilibrium, nor a max-max equilibrium. Indeed, because we assume an exogenous maximal horizontal differentiation, we necessarily have a "max-... equilibrium". Moreover, health plans cannot be maximally vertically differentiated because of the specific "diversity of physicians" attribute which cannot be zero for one of the two health plans. Thus the equilibrium cannot be a "...-max equilibrium", and therefore not a "max-max equilibrium". We can say that we obtain a "max-intermediate equilibrium" at equilibrium 2 (or 3) because health plans are vertically differentiated but not maximally so.

[13] This effect is positive because for all equilibria $p^i > \frac{c}{2} \; \forall i = 1, 2$.

Finally when β and t are high: $\beta > 4t_B$ and $t > \frac{\beta^2}{8t_B}$, we obtain a max-min equilibrium with both health plans hiring all physicians due to the positive effect of the network externality. Following Neven and Thisse (1989), health plans are maximally differentiated on one attribute (horizontal differentiation), and they are minimally differentiated on the other attribute (vertical differentiation).

This analysis can be summarized as follows. In the case of a uniform distribution on the interval $[0, 1]$ of the location of policyholders, health plans' strategies depend on the two standard effects of product differentiation. The price competition effect, which drives health plans to be vertically differentiated to soften competition, and the market share effect, which drives them to play symmetric strategies to increase their market share on the policyholders' side. Health plans' strategies also depend on two effects related to the particular network externality: a spending effect which drives health plans to hire fewer physicians because of charges, and an earning effect which drives them to hire more physicians to set a higher premium for policyholders, so as to make more profits.

4 Regulated public monopoly health plan

We now consider a single health plan which is a regulated public monopoly. Regulation concerns activities whose organization is a network and public utilities, like health care production. In this type of sector, there is often a natural monopoly because of the existence of network externalities and risk mutualization. Because, in most of these cases, market failures can arise, regulation may be useful to attempt to prevent socially undesirable outcomes and to direct market activity toward desired outcomes.

We assume that the health plan is located at $x = 0$ in the town represented by the unit segment. Full insurance coverage is still mandatory for policyholders. Therefore they all join the regulated health plan whereas physicians can choose between joining the market or staying outside. The model described in section 2 also applies here taking account of changes in market shares. As all policyholders are affiliated to the health plan and as their mass is normalized to one, $n = 1$ policyholders join the health plan. The system is funded through taxes and therefore policyholders pay a tax T to finance their insurance, and not a fixed premium any more. They bear the social cost of the tax λ, because a tax system implies distortions. We assume that the health plan seeks to maximize social welfare under the balanced-budget constraint, by adequately fixing the tax T paid by policyholders and by choosing the right number of physicians employed. It also has to take into account that the number of physicians cannot exceed one.

With a full insurance coverage, the utility of policyholders is given by: $U(\beta, x) = \omega + \beta n_B^M - T(1 + \lambda) - tx$. And the physicians' utility function is:

$$U_B(y) = \frac{(s^M - c)}{n_B^M} \cdot \int_0^1 \theta f(\theta) d\theta - t_B \cdot y. \quad (22)$$

As $N = 1$ and the probability of illness θ is uniformly distributed on the interval $[0, 1]$, the expected number of consultations in the network is $\frac{1}{2}$. Thus (22) can be rewritten as: $U_B(y) = \frac{(s^M - c)}{2n_B^M} - t_B \cdot y$.

We can define the physicians' demand for the health plan from the physicians' participation constraint: $U_B(y) \geq 0$, then if: $y \leq \frac{(s^M - c)}{2n_B^M t_B} = \bar{y}$. Therefore the physicians whose location y is lower than the threshold $\bar{y}$ decide to join the regulated public monopoly. And the number of physicians affiliated to the monopoly health plan is: $n_B^M = \frac{(s^M - c)}{2n_B^M t_B}$, yielding:

$$(n_B^M)^2 t_B + \frac{c}{2} = \frac{s^M}{2}. \tag{23}$$

Expected welfare can be written as:

$$EW^M = \int_0^1 U(\beta, x)\, dx + \int_0^{\bar{y}} U_B(y) dy \tag{24}$$

$$= \omega + \beta n_B^M - T(1 + \lambda) - \frac{t}{2} + \frac{(s^M - c)^2}{8(n_B^M)^2 t_B}.$$

The regulator's constraints are the balanced-budget constraint and the constraint on the number of physicians: $T \cdot n = s^M \cdot \int_0^1 \theta f(\theta) d\theta$, then:

$$T = \frac{s^M}{2} = (n_B^M)^2 t_B + \frac{c}{2}, \tag{25}$$

from equation (23), and

$$n_B^M \leq 1. \tag{26}$$

Therefore the regulator has to maximize (24) under (25) and (26).

Proposition 2 *Social welfare is maximized*[14] *when constraint (26) is not binding, i.e.* $\beta < t_B(1 + 2\lambda)$*, if* $n_B^{M*} = \frac{\beta}{t_B(1+2\lambda)}$*. Otherwise constraint (26) is binding and* $n_B^{M*} = 1$*.*

Then when $\beta < t_B(1 + 2\lambda)$: $T^* = \frac{\beta^2}{t_B(1+2\lambda)^2} + \frac{c}{2}$ and $s^{M*} = \frac{2\beta^2}{t_B(1+2\lambda)^2} + c$. Otherwise, $T^* = t_B + \frac{c}{2}$ and $s^{M*} = 2t_B + c$. The policy choice of the regulator depends on the earning and the spending effects only. There is a trade-off between hiring more physicians to offer a higher diversity to policyholders, and charges due to the physicians' wages. When β is high enough, the regulator hires all physicians.

[14]See Appendix 2 for more details.

5 Welfare comparison

We compare the agents' equilibrium surpluses and expected welfare under duopoly competition with a uniform distribution on the interval $[0, 1]$ of the location of policyholders, and under the regulated public monopoly when it is relevant, i.e. when the physicians' market is not covered and when it is covered by all platforms. Let S be the policyholders' surplus, S_B be the physicians' surplus, and W be expected welfare under duopoly competition. Let S^M be the policyholders' surplus, S_B^M be the physicians' surplus, and W^M be expected social welfare under the regulated public monopoly.

Under duopoly competition, one of the two health plans hires all physicians from $\beta > 2t_B$ and both health plans hire all physicians from $\beta > 4t_B$. For $\lambda = \frac{1}{2}$, i.e. $2t_B = t_B(1 + 2\lambda)$, and $\beta > 2t_B$, one of the two competitive health plans and the regulated monopoly hire all physicians. For $\lambda = \frac{9}{2}$, i.e. $4t_B = t_B(1+2\lambda)$, and $\beta > 4t_B$, all health plans hire all physicians. The social cost of public funds is assessed to be around 0.3 for most of the developed economies (see Ballard, Shoven and Whalley, 1985) thus we can assume that $\lambda < \frac{1}{2}$ for the rest of the comparison. $\lambda < \frac{1}{2}$ implies that $\beta > 2t_B > t_B(1 + 2\lambda)$, thus the regulated public monopoly hires all physicians at a lower threshold of β than competitive health plans because only the effects of the network externality matter for it.

The physicians' market is not covered under each type of organization

When $\beta < t_B(1 + 2\lambda) < 2t_B$, the total number of physicians at equilibrium 1 is: $n_B^{1*} + n_B^{2*} = n_B^* = \frac{\beta}{2t_B}$. Then, given the value of λ, the diversity of physicians offered to policyholders is higher under regulation than under duopoly competition: $n_B^{M*} = \frac{\beta}{t_B(1+2\lambda)} > \frac{\beta}{2t_B} = n_B^*$. But the tax paid by policyholders under regulation is decreasing with λ. If $\lambda > \bar{\lambda} = \frac{\beta}{2\sqrt{tt_B}} - \frac{1}{2}$, they will pay a lower charge under regulation than under duopoly competition: $T^* = \frac{\beta^2}{t_B(1+2\lambda)^2} + \frac{c}{2} < \frac{c}{2} + t = p^*$.

Assuming that $\frac{\beta}{2\sqrt{tt_B}} - \frac{1}{2} < \lambda < \frac{1}{2}$, which is true only if $t > \frac{\beta^2}{4t_B}$, or else if the importance that policyholders attribute to horizontal differentiation is high, competitive health plans take advantage of the policyholders' preference to set a higher premium. Therefore, in this case, policyholders benefit from a higher diversity of physicians and pay a lower premium under regulation than under duopoly competition.[15] More broadly, the comparison of the policyholders' surplus under duopoly competition and regulation shows that it is better under the regulated monopoly: $S^* - S^{M*} = \frac{\beta^2}{t_B}\left[\frac{1}{4} - \frac{\lambda}{(1+\lambda)^2}\right] - c\left(1 + \frac{\lambda}{2}\right) - \frac{14t}{8} < 0$, i.e. if:

$$\frac{\beta^2}{4t_B} < \frac{\beta^2\lambda}{t_B(1+\lambda)^2} + c\left(1 + \frac{\lambda}{2}\right) + \frac{14t}{8}. \tag{27}$$

Given the assumptions required for the parameters' values, condition (27) is verified. Moreover the fee-for-service rate set by the regulated monopoly for physicians is higher than the one set by competitive

[15] But the physicians' market is not covered in each case.

health plans: $s^{M*} = \frac{2\beta^2}{t_B(1+2\lambda)^2} + c > \frac{\beta^2}{4t_B} + c = s^*$. The expected number of patients[16] ($\frac{1}{2}$ vs $\frac{1}{4}$) is also higher under regulation. But competition between physicians is fiercer under regulation, which tends to lower their total revenue. Accordingly the physicians' total revenue will depend on these two opposing forces. A comparison of their expected total revenue in each situation shows that it is higher under regulation than under duopoly competition given the assumed value of the social cost of public funds λ: $\frac{s^{M*}}{2n_B^{M*}} - \frac{s^*}{2n_B^*} = 8\beta^2 + (1+2\lambda)(8ct_B\lambda - \beta^2) > 0$. There is only a drawback for physicians whose location $y > \frac{1}{2}$ because their transportation cost will be higher under regulation. As a whole, the comparison of the physicians' surplus in each type of organization gives: $S_B^* - S_B^{M*} = \frac{(\beta - t_B)}{2} - \frac{\beta^2}{2t_B}\left[\frac{1}{8} + \frac{1}{(1+\lambda)^2}\right] < 0$, which is true in this case of uncovered physicians' market. Thus the physicians' surplus is higher under regulation than under duopoly competition even if the competition is fiercer. Concerning expected welfare, if we assume that $\beta = t_B = 4t$, or else that $\beta = \frac{t_B}{2} = 5t$[17], and $c > 0$ the comparison[18] between EW^* and EW^{M*} shows that $EW^* > EW^{M*}$ if λ is high and $EW^{M*} > EW^*$ if λ is low. This result is due to the distortionary taxation and is consistent with the observations made on the surplus of each type of agent. Therefore for a relatively low social cost of public funds ($\lambda < \frac{1}{2}$), the expected welfare is likely to be greater under regulation even if health plans make zero profit.

The physicians' market is covered under each type of organization and by all health plans

When $\beta > 4t_B > t_B(1+2\lambda)$, or when all health plans hire all physicians, under duopoly competition physicians can choose which health plan they join and are better paid: $s^* = 4t_B + c > 2t_B + c = s^{M*}$. Their surplus and expected welfare are always higher than under regulation: $S_B^* - S_B^{M*} = t_B - \frac{t_B}{2} = \frac{t_B}{2} > 0$ and $EW^* - EW^{M*} = \frac{t}{4} + \frac{\lambda(c+t_B) - t_B}{2} > 0$, only if: $\lambda > \frac{2t_B - t}{2(c+t_B)} = \bar{\lambda}$, with $\bar{\lambda} < 0$ because $t > 2t_B$ at equilibrium 4. Competition is always preferred to regulation by health plans because they make profits. The policyholders' surplus is also higher under duopoly competition than under regulation only if λ is high. In this case, insurance is too costly for policyholders under regulation. But even when λ is low and thus the policyholders' surplus is higher under regulation, the gain they make does not offset the lesser gains of physicians and health plans. Financing health insurance by taxation implies distortions which are not offset by gains when the situations are identical under duopoly competition and under regulation.[19]

Using the expected welfare maximization criterion, this comparison shows that the choice between a duopoly and a regulated monopoly depends on the size of the physicians' market. When the physicians' market is not covered, if $\lambda \simeq 0.3$ it is more likely that expected welfare under regulation will be higher than under duopoly competition. Indeed, the policyholders' surplus is maximized under regulation and is higher than

[16]The expected number of patients on each HP is given by: $N\int_0^1 \theta f(\theta)d\theta$.

[17]In the case that we study, we must have: $\beta < 2t_B$, $\beta < t_B(1+2\lambda)$ and $t > \frac{\beta^2}{6t_B}$.

[18]In the uncovered physicians' market, $EW^* - EW^{M*} = \frac{\beta - t_B + c\lambda}{2} + \frac{t}{4} + \frac{\beta^2}{2t_B}\left[\frac{1}{8} - \frac{1}{(1+2\lambda)}\right]$.

[19]When insurance is mandatory for policyholders and when all physicians are affiliated.

under duopoly competition, the physicians' surplus is also maximized under regulation and is greater than under duopoly competition, but the profits of health plans are only maximized under duopoly competition. And expected welfare is higher under duopoly competition than under regulation when the physicians' market is covered by all health plans. All physicians are multi-home under duopoly competition. The sum of the surpluses that they receive is higher than their maximized surplus under regulation ($S_B^* = 2S_B^{M*}$). The profits of health plans are maximized under duopoly competition, and the policyholders' surplus is maximized under regulation but turns out to be smaller than under duopoly competition if the social cost of public funds becomes too high. But given that this latter $\simeq 0.3$, they are more likely to obtain a greater surplus under regulation. Thus, the choice of a way of financing health care insurance is made to the detriment of one type of agent.

Armstrong (2006) shows that in a competitive bottleneck model, the interests of the multi-homing group are always ignored at equilibrium. By contrast the joint interests of the competitive platform and its single-homing group are maximized. Indeed, the platform maximizes its profit subject to delivering a required utility to the single-homing group. Our model is not a real competitive bottleneck model because under duopoly competition physicians can multi-home but they are not necessarily multi-home. But we find that given the policyholders' preference parameter for the diversity of physicians β and if $\lambda \simeq 0.3$, if a regulated system is preferred, the health plans are unfavoured while if a competitive system is chosen, policyholders are unfavoured.

6 Conclusion

In this article, we have used the two-sided market approach to characterize a duopoly competition between two exogenously horizontally differentiated MCOs where policyholders are single-home and physicians can be multi-home. We also studied a regulated situation with a public monopoly.

We derived non-cooperative Nash equilibria under duopoly competition and characterized the optimal situation under regulation. We have found symmetric equilibria where health plans play symmetric strategies and asymmetric equilibria where health plans choose to be vertically differentiated besides being horizontally differentiated. Overall we observed that the strategy chosen by health plans at each equilibrium depends on four effects with two effects related to product differentiation and two effects related to the network externality. The first two effects are the standard effects of product differentiation: the price competition effect and the market share effect. And the other two effects are the earning effect and the spending effect. Each time there is a trade-off because differentiation makes it possible to soften competition but similar strategies can capture a greater share of the market. And offering a greater diversity of physicians makes it possible to set a higher premium on the policyholders' side but it also gives rise to more expenses through the physicians' payment. This latter point extends the results of product differentiation to a two-sided market structure with a network externality which differs from most of the modeled externalities in the two-sided market literature.

Under regulation, only the earning and the spending effects remain. This is consistent with the market structure. The regulated public monopoly will hire all physicians only for a relatively high policyholders' preference parameter for the diversity of physicians. Moreover, the comparison of expected welfare under duopoly competition and under regulation shows that when the policyholders' preference parameter and the social cost of public funds are low and the physicians' market is not covered, the expected welfare is higher under regulation. Whereas when the importance that policyholders place on the diversity of physicians is high and when the physicians' market is covered, expected welfare is higher under competition. But in each case, the interests of one group of agents are ignored.

Appendix

Appendix 1

In Appendix 1, we detail the maximization programs of health plans 1 and 2 under duopoly competition. Health plan 1 maximizes equation (7) and health plan 2 maximizes equation (8), under constraints (9) and (10). As these constraints are linear, the constraint qualification condition is automatically verified. Since $-\frac{\partial \bar{x}}{\partial p^1} = \frac{\partial \bar{x}}{\partial p^2} = \frac{1}{2t}$ and $\frac{\partial \bar{x}}{\partial n_B^1} = -\frac{\partial \bar{x}}{\partial n_B^2} = \frac{\beta}{2t}$ and with μ and λ the Kuhn-Tucker mutipliers, the Kuhn-Tucker conditions for platform 1 are:

$$\frac{\partial L^1}{\partial n_B^1} = \frac{\beta}{2t}\left[f(\bar{x})\left(p^1 - \frac{c}{2}\right) - \lambda\right] - 2n_B^1 t_B - \mu \leq 0 \quad (28)$$

$$\frac{\partial L^1}{\partial p^1} = F(\bar{x}) + \frac{\lambda}{2t}\left[1 - f(\bar{x})\left(p^1 - \frac{c}{2}\right)\right] \leq 0 \quad (29)$$

$$n_B^1\left[\frac{\beta}{2t}\left[f(\bar{x})\left(p^1 - \frac{c}{2}\right) - \lambda\right] - 2n_B^1 t_B - \mu\right] = 0 \quad (30)$$

$$p^1\left[F(\bar{x}) + \frac{\lambda}{2t}\left[1 - f(\bar{x})\left(p^1 - \frac{c}{2}\right)\right]\right] = 0 \quad (31)$$

$$\frac{\partial L^1}{\partial \mu} = 1 - n_B^1 \geq 0 \quad (32)$$

$$\mu(1 - n_B^1) = 0 \quad (33)$$

$$\frac{\partial L^1}{\partial \lambda} = 1 - \bar{x} \geq 0 \quad (34)$$

$$\lambda(1 - \bar{x}) = 0 \quad (35)$$

$$n_B^1 \geq 0, p^1 \geq 0, \mu \geq 0, \lambda \geq 0. \quad (36)$$

As for health plan 1, with α and γ the Kuhn-Tucker multipliers, the Kuhn-Tucker conditions for health plan 2 are:

$$\frac{\partial L^2}{\partial n_B^2} = \frac{\beta}{2t}\left[\gamma - f(\bar{x})\left(\frac{c}{2} - p^2\right)\right] - 2n_B^2 t_B - \alpha \leq 0$$

$$\frac{\partial L^2}{\partial p^2} = 1 - F(\bar{x}) + \frac{1}{2t}\left[f(\bar{x})\left(\frac{c}{2} - p^2\right) - \gamma\right] \leq 0 \quad (37)$$

$$n_B^2\left[\frac{\beta}{2t}\left[\gamma - f(\bar{x})\left(\frac{c}{2} - p^2\right)\right] - 2n_B^2 t_B - \alpha\right] = 0 \quad (38)$$

$$p^2\left[1 - F(\bar{x}) + \frac{1}{2t}\left[f(\bar{x})\left(\frac{c}{2} - p^2\right) - \gamma\right]\right] = 0 \quad (39)$$

$$\frac{\partial L^2}{\partial \alpha} = 1 - n_B^2 \geq 0 \quad (40)$$

$$\alpha(1 - n_B^2) = 0 \quad (41)$$

$$\frac{\partial L^2}{\partial \gamma} = 1 - \bar{x} \geq 0 \quad (42)$$

$$\gamma(1-\bar{x}) = 0 \quad (43)$$

$$n_B^2 \geq 0, p^2 \geq 0, \alpha \geq 0, \gamma \geq 0. \quad (44)$$

If we consider both health plans simultaneously, we find four equilibria:

1. In the first case, $\mu = \lambda = \alpha = \gamma = 0$, *i.e.* there are no binding constraints, and from the first-order conditions we obtain:

$$p^{1*} = \frac{c}{2} + \frac{F(\bar{x})}{f(\bar{x})}2t, \quad (45)$$

$$p^{2*} = \frac{c}{2} + \frac{(1-F(\bar{x}))}{f(\bar{x})}2t, \quad (46)$$

$$n_B^1{}^* = \frac{\beta F(\bar{x})}{2t_B} < 1, \quad (47)$$

$\leftrightarrow$

$$\frac{2t_B}{\beta} > F(\bar{x}),$$

$$n_B^2{}^* = \frac{\beta(1-F(\bar{x}))}{2t_B} < 1, \quad (48)$$

$\leftrightarrow$

$$F(\bar{x}) > 1 - \frac{2t_B}{\beta},$$

$$\bar{x} = \frac{1}{2} + (2F(\bar{x})-1)\left[\frac{\beta^2}{4tt_B} - \frac{1}{f(\bar{x})}\right] < 1, \quad (49)$$

$\leftrightarrow$

$$2 < 4F(\bar{x}) + f(\bar{x})\left[1 + \frac{\beta^2}{tt_B}\left(\frac{1}{2} - F(\bar{x})\right)\right],$$

Equilibrium profits are then given by:

$$\pi^{1*} = \frac{(F(\bar{x}))^2 2t}{f(\bar{x})} - \frac{(F(\bar{x}))^2\beta^2}{4t_B} > 0 \quad (50)$$

$\leftrightarrow$

$$\frac{8tt_B}{\beta^2} > f(\bar{x}),$$

and

$$\pi^{2*} = \frac{(1-F(\bar{x}))^2 2t}{f(\bar{x})} - \frac{(1-F(\bar{x}))^2\beta^2}{4t_B} > 0 \quad (51)$$

$\leftrightarrow$

$$\frac{8tt_B}{\beta^2} > f(\bar{x}).$$

Thus this equilibrium exists only if $1 - \frac{2t_B}{\beta} < F(\bar{x}) < \frac{2t_B}{\beta}$, i.e. if $\beta < 4t_B$. And if $\frac{8tt_B}{\beta^2} > f(\bar{x})$.

2. In the second case, $\mu = \lambda = \gamma = 0$, $\alpha > 0$, *i.e.* only constraint (9) is binding for health plan 2. Then $n_B^2{}^* = 1$ and p^{1*}, p^{2*} and $n_B^1{}^*$ are respectively defined by (45), (46) and (47). Moreover,

$$\bar{x} = \frac{1}{2} + \frac{\beta(\beta F(\bar{x}) - 2t_B)}{4tt_B} + \frac{(1-2F(\bar{x}))}{f(\bar{x})} < 1,$$

$\leftrightarrow$

$$2 < f(\bar{x})\left[1+\frac{\beta}{t}\left(1-\frac{\beta F(\bar{x})}{2t_B}\right)\right]+4F(\bar{x}),$$

$$\alpha = \beta(1-F(\bar{x})) - 2t_B > 0,$$

$\leftrightarrow$

$$1-\frac{2t_B}{\beta} > F(\bar{x}).$$

The equilibrium profit of health plan 1 is given by (50) and that of health plan 2 equals:

$$\pi^{2*} = \frac{(1-F(\bar{x}))^2 2t}{f(\bar{x})} - t_B > 0 \qquad (52)$$

$\leftrightarrow$

$$\frac{(1-F(\bar{x}))^2}{f(\bar{x})} > \frac{t_B}{2t}.$$

Thus at this equilibrium we must have: $F(\bar{x}) < Min\{\frac{2t_B}{\beta}, 1-\frac{2t_B}{\beta}\} = 1-\frac{2t_B}{\beta}$ if $\beta < 4t_B$ and $Min\{\frac{2t_B}{\beta}, 1-\frac{2t_B}{\beta}\} = \frac{2t_B}{\beta}$ if $\beta > 4t_B$. And for positive profit functions we must also have: $\frac{8tt_B}{\beta^2} > f(\bar{x})$ and $\frac{(1-F(\bar{x}))^2}{f(\bar{x})} > \frac{t_B}{2t}$.

3. In the third case, $\lambda = \alpha = \gamma = 0$, $\mu > 0$, *i.e.* only constraint (9) is binding for health plan 1. Then ${n_B^1}^* = 1$, p^{1*} and p^{2*} are respectively given by (45) and (46) and:

$$ {n_B^2}^* = \frac{\beta(1-F(\bar{x}))}{2t_B} < 1,$$

$\leftrightarrow$

$$F(\bar{x}) > 1-\frac{2t_B}{\beta},$$

$$\bar{x} = \frac{1}{2}+\frac{\beta[2t_B-\beta(1-F(\bar{x}))]}{4tt_B}+\frac{(1-2F(\bar{x}))}{f(\bar{x})} < 1,$$

$\leftrightarrow$

$$2 < f(\bar{x})\left[1+\frac{\beta}{t}\left(\frac{\beta}{2t_B}(1-F(\bar{x}))-1\right)\right]+4F(\bar{x}),$$

$$\mu = \beta F(\bar{x}) - 2t_B > 0,$$

$\leftrightarrow$

$$\frac{2t_B}{\beta} < F(\bar{x}).$$

The equilibrium profit of health plan 2 is given by (51) and that of health plan 1 equals:

$$\pi^{1*} = \frac{(F(\bar{x}))^2 2t}{f(\bar{x})} - t_B > 0 \qquad (53)$$

$\leftrightarrow$

$$\frac{t_B}{2t} < \frac{(F(\bar{x}))^2}{f(\bar{x})}.$$

Then this equilibrium exists only if: $F(\bar{x}) > Max\{\frac{2t_B}{\beta}, 1-\frac{2t_B}{\beta}\} = \frac{2t_B}{\beta}$ if $\beta < 4t_B$ and $Max\{\frac{2t_B}{\beta}, 1-\frac{2t_B}{\beta}\} = 1-\frac{2t_B}{\beta}$ if $\beta > 4t_B$. And profits are strictly positive only if: $\frac{8tt_B}{\beta^2} > f(\bar{x})$ and $\frac{t_B}{2t} < \frac{(F(\bar{x}))^2}{f(\bar{x})}$.

4. Finally in the fourth case, $\lambda = \gamma = 0$, $\mu > 0$ and $\alpha > 0$, *i.e.* constraint (9) is binding for both health plans. Then ${n_B^1}^* = 1$ and ${n_B^2}^* = 1$. p^{1*} and p^{2*} are respectively given by (45) and (46) and:

$$\bar{x} = \frac{1}{2} + \frac{(1-2F(\bar{x}))}{f(\bar{x})} < 1$$

$\leftrightarrow$

$$2 < f(\bar{x}) + 4F(\bar{x}),$$

$$\mu = \beta F(\bar{x}) - 2t_B > 0$$

$\leftrightarrow$

$$F(\bar{x}) > \frac{2t_B}{\beta},$$

$$\alpha = \beta(1-F(\bar{x})) - 2t_B > 0$$

$\leftrightarrow$

$$1 - \frac{2t_B}{\beta} > F(\bar{x}).$$

Equilibrium profits are given by (53) and (52). This equilibrium exists only if: $\frac{2t_B}{\beta} < F(\bar{x}) < 1 - \frac{2t_B}{\beta}$, i.e. if $\beta > 4t_B$. For strictly positive profits, we must also have: $\frac{t_B}{2t} < \frac{(F(\bar{x}))^2}{f(\bar{x})}$ or $\frac{t_B}{2t} < \frac{(1-F(\bar{x}))^2}{f(\bar{x})}$.

Now we verify that the problem is locally concave by calculating the second-order conditions at each equilibrium. At equilibrium 1, $({n_B^1}^*, p^{1*})$ is a strict local maximum for health plan 1 if:

$$\frac{\partial^2 \pi^1(.)}{\partial({n_B^1}^*)^2} < 0 \tag{54}$$

then if

$$2 > \frac{f'(\bar{x})F(\bar{x})}{(f(\bar{x}))^2}, \tag{55}$$

and if the determinant of the Hessian matrix is strictly positive:

$$\frac{\partial^2 \pi^1(.)}{\partial({n_B^1}^*)^2} \cdot \frac{\partial^2 \pi^1(.)}{\partial(p^{1*})^2} - \left[\frac{\partial^2 \pi^1(.)}{\partial(p^{1*})^2}\right]^2 > 0 \tag{56}$$

then if

$$2 > \frac{f'(\bar{x})F(\bar{x})}{(f(\bar{x}))^2} + \frac{\beta^2 f(\bar{x})}{4tt_B}. \tag{57}$$

$({n_B^2}^*, p^{2*})$ is a strict local maximum for health plan 2 if:

$$\frac{\partial^2 \pi^2(.)}{\partial({n_B^2}^*)^2} < 0 \tag{58}$$

then if

$$-2 - \frac{f'(\bar{x})(1-F(\bar{x}))}{(f(\bar{x}))^2} < 0, \tag{59}$$

and if the determinant of the Hessian matrix is strictly positive:

$$\frac{\partial^2 \pi^2(.)}{\partial({n_B^2}^*)^2} \cdot \frac{\partial^2 \pi^2(.)}{\partial(p^{2*})^2} - \left[\frac{\partial^2 \pi^2(.)}{\partial(p^{2*})^2}\right]^2 > 0 \tag{60}$$

then if

$$2 > \frac{\beta^2 f(\bar{x})}{4tt_B} - \frac{f'(\bar{x})(1-F(\bar{x}))}{(f(\bar{x}))^2}. \tag{61}$$

Equation (59) is always satisfied and equations (55) and (61) are also satisfied whenever equation (57) is satisfied.

At equilibria 2, 3 and 4, we have to consider the bordered Hessian matrix H because the problem is constrained. If the determinant of the Hessian is positive at a Lagrange critical point, then it is a maximum, and if the determinant is negative, it is a minimum.

At equilibrium 2, only the optimization problem of health plan 2 is constrained with (9) binding. The determinant of the bordered Hessian matrix is:

$$\left|H\right| = \frac{f'(\bar{x})}{2t} \cdot \frac{(1 - F(\bar{x}))}{f(\bar{x})} + \frac{f(\bar{x})}{t} > 0. \quad (62)$$

(62) can be rewritten as:

$$2 > \frac{-f'(\bar{x})(1 - F(\bar{x}))}{(f(\bar{x}))^2} \quad (63)$$

which is true. Moreover this condition is the same as condition (59). Thus equilibrium 2 is a local maximum if (57) alone is satisfied.

At equilibrium 3, the optimization problem of health plan 1 is constrained because (9) is binding. The determinant of the bordered Hessian matrix is:

$$\left|H'\right| = \frac{-f'(\bar{x})}{2t} \cdot \frac{F(\bar{x})}{f(\bar{x})} + \frac{f(\bar{x})}{t} > 0. \quad (64)$$

(64) can be rewritten as:

$$2 > \frac{f'(\bar{x})F(\bar{x})}{(f(\bar{x}))^2} \quad (65)$$

and this condition is the same as the condition (55). Thus equilibrium 3 is a local maximum if (55) and (61) are satisfied.

At equilibrium 4, constraint (9) is binding for health plan 1 and health plan 2. Thus, this equilibrium is a local maximum only if (63) and (65) are satisfied. And (63) is always satisfied.

Appendix 2

In this appendix, we solve the maximization problem of the regulated public monopoly. Expected welfare is given by (24). The regulator maximizes this welfare function subject to n_B^M under the constraint that the number of physicians is not higher than one (condition (26)). As the constraint is linear, the constraint qualification condition is automatically verified. With δ the Kuhn-Tucker multiplier, the Kuhn-Tucker conditions are:

$$\frac{\partial L}{\partial n_B^M} \leq 0$$

then

$$\beta - 2n_B^M t_B(\frac{1}{2} + \lambda) - \delta \leq 0 \quad (66)$$

$$n_B \cdot \frac{\partial L}{\partial n_B^M} = 0$$

then

$$n_B^M \left[\beta - 2n_B^M t_B(\frac{1}{2} + \lambda) - \delta\right] = 0 \quad (67)$$

$$\frac{\partial L}{\partial \delta} \geq 0$$

then

$$n_B^M \leq 1 \quad (68)$$

$$\delta \cdot \frac{\partial L}{\partial \delta} = 0$$

then

$$\delta(1 - n_B^M) = 0 \quad (69)$$

$$n_B^M \geq 0, \delta \geq 0. \quad (70)$$

Therefore, if constraint (26) is not binding, $n_B^M < 1$, then $\delta = 0$. With $n_B^M > 0$, $\frac{\partial L}{\partial n_B^M} = 0$. We find one critical point: $n_B^M = \frac{\beta}{t_B(1+2\lambda)}$.

If constraint (26) is binding, $\delta > 0$, then $n_B^M = 1$ and $\frac{\partial L}{\partial n_B} = 0$. Thus the critical point is $n_B^M = 1$ and $\delta = \beta - t_B(1 + 2\lambda)$ which is strictly positive if $\beta > t_B(1 + 2\lambda)$.

Now we verify that the critical points are local or global maxima or minima. When (26) is not binding, the second-order condition is:

$$\frac{\partial^2 EW}{\partial (n_B^M)^2} = -t_B(1 + 2\lambda) < 0.$$

Thus as the expected welfare is strictly concave, $n_B^{M*} = \frac{\beta}{t_B(1+2\lambda)}$ is a global maximum. And when (26) is binding, because EW is concave and the constraint (26) is linear, then $n_B^{M*} = 1$ is also a global maximum. Therefore there are two solutions:

$$\begin{cases} n_B^* = \frac{\beta}{t_B(1+2\lambda)} & \text{if} \quad \beta < t_B(1 + 2\lambda) \\ n_B^* = 1 & \text{otherwise.} \end{cases}$$

References

[1] Simon P Anderson and Jean J Gabszewicz. The media and advertising: A tale of two-sided markets. *Handbook of the Economics of Art and Culture*, 1:567–614, 2006.

[2] Asim Ansari, Nicholas S Economides, and Jan Steckel. The max-min-min principle of product differentiation. *Journal of Regional Science*, 38(2):207–230, 1998.

[3] Mark Armstrong. Competition in two-sided markets. *RAND Journal of Economics*, 37(3):668–691, 2006.

[4] Mark Armstrong and Julian Wright. Two-sided markets, competitive bottlenecks and exclusive contracts. *Economic Theory*, 32(2):353–380, 2007.

[5] Charles L Ballard, John B Shoven, and John Whalley. General equilibrium computations of the marginal welfare costs of taxes in the united states. *The American Economic Review*, 75(1):128–138, 1985.

[6] David Bardey and Jean-Charles Rochet. Competition between hmo and ppo: A two-sided market approach. *Journal of Economics & Management Strategy*, 19(2):435–451, 2010.

[7] Bernard Caillaud and Bruno Jullien. Chicken and egg: Competition among intermediation service providers. *RAND Journal of Economics*, 34(2):309–328, 2003.

[8] Helmut Cremer and Jacques-François Thisse. Location models of horizontal differentiation: A special case of vertical differentiation models. *The Journal of Industrial Economics*, 39(4):383–390, 1991.

[9] Anthony J. Culyer and Joseph P. Newhouse. *Handbook of Health Economics*. Elsevier, 2000.

[10] Nicholas S Economides. The principle of minimum differentiation revisited. *European Economic Review*, 24(3):345–368, 1984.

[11] Jean J Gabszewicz, , and Xavier Wauthy. Two-sided markets and price competition with multihoming. *Mimeo*, 2004.

[12] Jean J Gabszewicz. *La différenciation des produits*. La Découverte, 2006.

[13] Jean J Gabszewicz, Didier Laussel, and Nathalie Sonnac. Programming and advertising competition in the broadcasting industry. *Journal of Economics and Management Strategy*, 13(4):657–669, 2004.

[14] Harold Hotelling. Stability in competition. *Economic Journal*, 39:41–57, 1929.

[15] Ching-to Albert Ma and James F Burgess. Quality competition, welfare, and regulation. *Journal of Economics*, 58(2):153–173, 1993.

[16] Ching-to Albert Ma and Thomas G McGuire. Network incentives in managed health care. *Journal of Economics & Management Strategy*, 11(1):1–35, 2002.

[17] Damien Neven and Jacques-François Thisse. Choix des produits : concurrence en qualité et en variété. *Annales d'Economie et de Statistiques*, 15/16:85–112, 1989.

[18] OECD. Oecd health data 2012, how does the united states compare. 2012.

[19] Martin Peitz and Tommaso Valletti. Content and advertising in the media: Pay-tv versus free-to-air. *Mimeo*, 2004.

[20] Mario Pezzino and Giacomo Pignataro. Competition in the health care markey: a "two-sided" approach. *Working paper.*

[21] Jean-Charles Rochet and Jean Tirole. Platform competition in two-sided markets. *Journal of the European Economic Association*, 1:990–1029, 2003.

[22] Jean-Charles Rochet and Jean Tirole. Two-sided markets: A progress report. *RAND Journal of Economics*, 37(3):645–667, 2006.

[23] Jean-Charles Rochet and Jean Tirole. Tying in two-sided markets and the honor all cards rule. *International Journal of Industrial Organization*, 26(6):1333–1347, 2008.

[24] Takatoshi Tabuchi and Jacques-François Thisse. Asymmetric equilibria under spatial competition. *International Journal of Industrial Organization*, 13:213–227, 1995.

[25] Jean Tirole. *Théorie de l'organisation industrielle*, volume 2. Economica, 1993.

[26] Jean Tirole. *Théorie de l'organisation industrielle*, volume 1. Economica, 1993.