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Freezeout, Compensation Rules and Voting Equilibria*

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Abstract

A single proposer has the opportunity to generate a surplus by taking the assets of a group of individuals. These individuals are called upon to vote for accepting or rejecting the monetary offer made to them by the proposer, who needs the agreement of a qualified majority. The voters who rejected the offer while the qualified majority is met are frozen out but they can claim a compensation in exchange for their asset. This article analyses how compensation rules influence both the votes and the offer made by the proposer. We find that unanimity rule or compensation equals to the proposal or voters' initial wealth maximize the expected social surplus that entirely accrues to the proposer. We show that increasing the offer does not always increase the probability of acceptance, in sharp contrast to many close models. We identify the optimal offer when the compensation does not depend on the proposal. Increasing the compensation always reduces the expected social surplus and the expected profit of the proposer, but does not always benefit to the voters. Reinforcing the qualified majority always increases the expected profit of the proposer, and can decrease both the expected social surplus and the expected utility of the voters. When the compensation is based on the proposal we find that the success or the failure of the proposition depends crucially of the compensation's shape.

Keywords: Voting games, Compensations, Fairness, Freezeout, Regulatory takings, Debt restructuring.
JEL Classification number: D72, K2.

1 Introduction

Few years ago, Argentina's government, facing a major economic crisis, tried to restructure its sovereign debt. They offered new bonds, paying less than 30 cents for each dollar owed in default, and by 2010, 93% of the original bondholders agreed to the swaps. Many analysts have described the debt relief granted by these exchange bondholders as a model for Greece and other debt-burdened countries to consider. But, the minority holdouts refused the swaps, insisting on payment in full plus interest, and started to litigate. In November 2012, a federal judge in New York ordered Argentina to pay immediately and in full everything it owes to the holdouts. As specified in the judge's decision, this remedy is "fair". Fairness is also a key argument of the exchange bondholders who claim that "it's not fair [...] that a few holdouts can earn up to 200% on debt they bought for pennies on

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the dollar after Argentina's collapse". Beside, if allowed to stand, this kind of remedy will make it impossible for other countries to get critical debt relief, they argued. Clearly fairness criteria applied to litigating holdouts via the compensation rule may ex-ante impact the decision to agree to the proposed swaps, and by the way the probability of success.

This issue is the main concern of this article. Nonetheless, if current sovereign debt crisis sheds a particular light on debt restructuring process, such a phenomenon appears in numerous other relevant economic situations such as freezeouts, land assembly projects and aggregate settlements in class-action. We propose a model which is general enough to grasp these different situations. A single proposer has the opportunity to generate a surplus by taking the assets (or property) of a group of individuals. These individuals are called upon to vote for accepting or rejecting the monetary offer made to them by the proposer, who needs the agreement of a qualified majority of voters. A voter will receive this offer as a payment in exchange for his asset provided that he voted in favor and that the qualified majority is reached. Such situations are common in real life as pointed out below, but it is rarely the case that all voters accept the offer. The natural question is therefore to specify what happens to the voters who rejected the offer if the qualified majority is met. In spite of the fact that these voters cannot prevent the proposal from being implemented, they still can legitimately claim a payoff or compensation in exchange for their asset. The determination of a fair compensation is at the heart of the decision making process since it will influence both the votes and the offer made by the proposer. For instance, if the offer is large enough to secure a qualified majority and if the compensation is lower, then a voter who is *a priori* not favorable to the proposal, will easily understand that accepting the offer is his least worse option.

The difficulty raised by the above-mentioned question of law is to find an equitable calibration of the "fair requirements". This question is one of the main two issues adressed in this article. Firstly, we provide various formulations of what could be considered as fair compensations. In this first main issue, we do not take strategic considerations into account. On the contrary, these formulations relies on equity principles such as treating equally two minority shareholders if they own the same number of shares and if they take the same voting decision. Strategic considerations are incorporated to the second main issue that we address. More specifically, we analyze the equilibrium of the strategic voting game. In particular, we examine the impact of the suggested fair compensations on the two sides of the decision making process, that is on the optimal offer made by the proposer as well as on the optimal probability of acceptance.

A first result deals with situations in which unanimity is required to pass the proposal. We show that the equilibrium offer is just the initial wealth of each voter, that the equilibrium probability is 1 and that the expected social surplus is maximal and goes to the proposer (Proposition 1). More generally, it is not guaranteed that the relationship between the equilibrium offer and the equilibrium probability of acceptance is not a one-to-one mapping. In particular, Proposition 2 shows that increasing the offer does not always increase the probability of acceptance, in sharp contrast to many close models. Then, the other results are split in two categories. Firstly, we focus on offer-invariant compensations, *i.e.* compensations that do not depend on the proposer's offer. Proposition 3 characterizes the associated equilibrium probability of acceptance. Based on this result, various relevant effects are pointed out. Increasing the compensation always reduces the expected social surplus and the expected profit of the proposer, and does not always benefit to the voters (Proposition 4). Reinforcing the qualified majority always increases the expected profit of the proposer, and can decrease both the expected social surplus and the expected utility of the voters (Proposition 5). The optimal probability of acceptance increases when the voters' initial wealth increases if and only if this probability is large enough to guarantee that the proposal is implemented on average, and otherwise it is decreasing (Proposition 6). Secondly, we analyze compensation rules that can be influenced by the proposer's offer. If the compensation is equal to the proposer's offer, then there exists a dominant pure-strategy such that the proposal is accepted with probability 1 and the optimal offer coincides with the voter's initial wealth, so that all the surplus accrues to the proposer (Proposition 7). Finally, Proposition 8 examines, for a fixed level of compensation, the relationships between the optimal probabilities of acceptance when the compensation is offer-invariant and when it can rely on the proposer's offer. It is worth to note that the phenomena studied in this article rely on the simplest forms of compensation rules. Our results are therefore best regarded as suggestive and other plausible types of compensations are discussed by way of conclusion.

The rest of the article is organized as follows. The general model is described in section 2 while section 3 comes back to its most significant interpretations. Section 4 contains a basic equilibrium analysis. The design of fair compensations and the study of their impact of the equilibrium decisions are provided in section 5. Finally, section 6 suggests some extensions of our work.

2 Model

We consider a single proposer and a finite even number $n > 2$ of symmetric voters. We denote by $N = \{1, \ldots, n\}$ the set of n voters, each of whom owns exactly one vote. The timing can be described as follows.

- $t = 0$: the proposer makes a proposal $b \in \mathbb{R}_+$ that generates a revenue $\Delta > 0$.
- $t = 1$: the voters simultaneously and non-cooperatively cast ballots to accept or reject the proposal.
- $t = 2$: if at least $\sigma > n/2$ voters accept, the proposal is implemented and the accepting voters each have $U(b) \geq 0$, otherwise the status quo prevails and they each have the utility $U_0 \geq 0$. For the sake of simplicity, we assume that $U(b) = b$. Note that σ, or equivalently σ/n, represent the qualified majority.
- $t = 3$: if at least σ voters have accepted the proposal, the $n - \sigma$ voters who have rejected it are expropriated and each of them receives a compensation $c(\zeta) \geq 0$ determined by a court, where ζ is the set of parameters on which the court's decision is based.

Throughout this article, we make the following assumptions:

Assumption 1 *Implementing the proposal yields a positive social surplus:* $\Delta > nU_0$.

Assumption 2 *A court cannot punish a voter for rejecting the offer:* $U_0 \leq c(\zeta)$.

Assumption 1 means that we rule out the situations in which the implementation of the proposal is socially sub-optimal compared to the initial situation. Combined with Assumption 1, Assumption 2 basically states that each compensated voter should get some share of the generated surplus. Since we are primary interested in fair compensations, this assumption only excludes situations which are not very appealing. Moreover, allowing $U_0 > c(\zeta)$ would not change the nature of our results.

3 Interpretations

Our framework is rich enough to model various economic situations.

- *Freezeouts*: A freezeout is a transaction in which a controlling shareholder buys out the minority shareholders in a publicly traded corporation. Freezeouts often occur when a bidder has accumulated most but not all of the shares in a preceding tender offer and wants to acquire the remainder. In our model, the proposer is the controlling shareholder and the voters are the minority shareholders in a publicly traded corporation. In this context, the proposal is called a freezeout merger. The offer made by the proposer is thus interpreted as the tender offer proposed by the controlling shareholder for buying any single share owned by the minority shareholders. This problem is examined by [Maug, 2006] and [Subramanian, 2005] among others. A typical problem comes from the valuation of freezeouts for the dissenting shareholders (valuation of $c(.)$ under our notations). The compensation comes up when a court has to evaluate the rights of the dissenting shareholders. Minority freezeout have been scrutinized by Delaware courts since they involve inherent conflict of interest between the controlling and minority shareholders. Under Delaware law:

  > "Shareholders controlling at least 90% of a target's stock can utilize a short-form merger under Del. Corp Code 253, which obviates the fairness requirements applied to freeze-out bids but grants appraisal rights to minority shareholders regardless of the consideration granted."

  A first possibility for the court is to choose the market price, which does not distinguish the voters on the basis of their decision. Another option is to select a compensation that relies on previous purchases or a valuation by appraisal where a court-appointed appraiser seeks fairness. The reader is referred to [Subramanian, 2005] for numerous examples and an analysis of the US doctrine.

- *Land Assembly projects*: Land assembly projects are frequently delayed or blocked by holdout landowners attempting to capture a greater share of the gains from trade. Hence, assemblers pressure local governments to condemn land on their behalf. As [Heller and Hills, 2008] note, eminent domain overcomes the holdout problem, but only at the expense of introducing other fairness and efficiency concerns, especially because the compensation's valuation is made by the court. Within the framework of our model, the proposer is the group of assemblers and the voters are the landowners. The offer is the price proposed for each land plot. Various views of what can be considered as a fair compensation for the unwilling sellers can be found in [Miceli and Segerson, 2007], which provides a comprehensive survey on the closely connected problem of the government's right to regulate private property.

- *Debt restructuring with a prepackaged bankruptcy plan*: A debtor seeks to restructure debt claims held by n identical creditors. If a prepackaged plan is accepted by the requisite percentages of creditors, the plan will become binding on all parties, including parties that did not vote or voted against the plan (this eliminates any economic advantage to non-participation). Besides the situation of Argentina mentioned in the introduction, the regulation of debt restructuring differs from country to country. Under the Bankruptcy Code of the U.S., two-thirds of the creditors who vote must accept the plan before it can be implemented (only a majority is required in Australia, and Canada has reduced the needed proportion from three-quarters to two-thirds in 1997). Using our notations, this is equivalent to set $\sigma = 2/3$. Usually, we observe $c(\zeta) = b$ is chosen very often as a compensation, but other compensations may occur, as pointed out in the Argentina's example. An empirical investigation of this question for U.S. firms is contained in [Tashjian et al., 1996].

- *Aggregate settlements in class-action*: In an aggregate settlement, all plaintiffs must consent after consultation before the agreement can be final and binding. Otherwise, a majority vote ($\sigma = 1/2$) can be organized to decide whether to settle (in New Jersey for example), but cannot bind all to a settlement. In other words, the dissenting plaintiffs receive $c(\zeta)$ from the trial's result. For a recent study of this problem, we refer the reader to [Erichson and Zipursky, 2011] and the references therein.

4 Equilibrium

We begin with a basic equilibrium analysis in which the compensation is not yet specified. The formal framework shares similarities with the one considered in [Bagnoli and Lipman, 1988]. As in the latter article, we look for the symmetric mixed-strategy subgame perfect Nash equilibrium, in which each voter accepts the proposal with a probability $p \in [0, 1]$ to be determined. Let v be the number of voters who have accepted the proposal and, for each $i \in N$, let v_{-i} denote the number of positive votes among the set $N\backslash\{i\}$ of voters other than i. Each voter randomizes with probability p in response to a proposal $b \in \mathbb{R}_+$. For a voter to be randomizing he must be indifferent between accepting and rejecting. Therefore, the proposal must satisfy:

$$b\mathbb{P}_i^s + U_0\left(1 - \mathbb{P}_i^s\right) = c(\zeta)\mathbb{P}_{-i}^s + U_0\left(1 - \mathbb{P}_{-i}^s\right) \tag{1}$$

or equivalently

$$b = U_0 + \frac{\mathbb{P}_{-i}^s}{\mathbb{P}_i^s}(c(\zeta) - U_0) \tag{2}$$

where

$$\mathbb{P}_i^s = \mathbb{P}(v_{-i} \geq \sigma - 1) = \sum_{j=\sigma-1}^{n-1} \binom{n-1}{j} p^j (1-p)^{n-1-j} \tag{3}$$

is the probability that the proposal succeeds when a given voter decides to accept and

$$\mathbb{P}_{-i}^s = \mathbb{P}(v_{-i} \geq \sigma) = \sum_{j=\sigma}^{n-1} \binom{n-1}{j} p^j (1-p)^{n-1-j} \tag{4}$$

is the probability that the proposal succeeds when a given voter decides to reject it.

The proposer's objective is to maximize his expected surplus. In order to determine this surplus, let us first consider the expected social surplus, that is:

$$E(S) = \mathbb{P}^s(\Delta - nU_0) \tag{5}$$

where

$$\mathbb{P}^s = \mathbb{P}(v \geq \sigma) = \sum_{j=\sigma}^{n} \binom{n}{j} p^j (1-p)^{n-j}$$

is the probability that the proposition is successful. The voters' cumulated expected surplus $E(U)$ writes:

$$\begin{aligned} E(U) &= \mathbb{P}_i^s\,(b-U_0)\,pn + \mathbb{P}_{-i}^s\,(c(\zeta)-U_0)\,(1-p)n \\ &= \mathbb{P}_{-i}^s\,(c(\zeta)-U_0)\,n. \end{aligned} \tag{6}$$

Therefore, the proposer's expected surplus $E(\pi)$ is defined as the difference between the expected social surplus and the expected surplus of the voters:

$$E(\pi) = E(S) - E(U) = \mathbb{P}^s(\Delta - nU_0) - \mathbb{P}_{-i}^s\,(c(\zeta)-U_0)\,n. \tag{7}$$

We may establish a first result when unanimity is required as a qualified majority, that is when $\sigma = n$. Under unanimity, each voter is pivotal, which means that any voter has the power to prevent the implementation of the proposal by voting against it. The decision of each voter relies on the comparison between his status-quo utility U_0 and the offer b. Hence, all voters are willing to accept any offer at least as large as U_0. It is also clear that the expected social surplus derived from the implementation of the proposal can only be achieved if all voters vote in favor. To sum up:

Proposition 1 *Under unanimity rule, the proposal is accepted with probability 1 and the optimal offer is $b^\star = U_0$. The expected social surplus is maximal and goes to the proposer.*

The unanimity voting rule unveils the traditional trade-off between efficiency and equity. On the one hand, this rule ensures the success of any proposal that gives a positive social surplus. On the other hand, this efficient outcome can clearly be considered as unfair since the proposer captures all the generated surplus.

More generally, the proposer chooses the offer b that maximizes his expected profit, which in turn determines the corresponding equilibrium probability of acceptance p. As observed in [Bagnoli and Lipman, 1988], [Holmström and Nalebuff, 1992] and [Ferguson, 1994], the previous implication can be inverted as follows. The proposer targets the probability p so that the expected number of positive votes is just the needed number, determining thereby the equilibrium offer b. In other words, the proposer maximizes the probability with which any voter will be pivotal. These two ways at looking the same problem are equivalent if there exists a one-to-one mapping from p to b. However, such a relationship does not always hold in our model depending on the compensation's form.

Proposition 2 *Increasing the offer does not always increase the probability of acceptance.*

Proof. Note that the compensation can depend on b and/or the expected number of voters who have accepted the proposal $v = pn$. Differentiating (2) yields:

$$\frac{db}{dp} = \frac{(c(\zeta)-U_0)\mathcal{P}(p)' + \mathcal{P}(p)\dfrac{\partial c(\zeta)}{\partial p}}{1 - \dfrac{\partial c(\zeta)}{\partial b}\mathcal{P}(p)} \tag{8}$$

where subscripts are derivatives and $\mathcal{P}(p) \equiv \mathbb{P}_{-i}^s/\mathbb{P}_i^s$.

Firstly, let us study $\mathcal{P}(p)'$. We know that if $X \sim \mathcal{B}(n,p)$, then it holds that

$$\mathbb{P}(X \geq x) = I_p(n-x+1,x) = x \begin{pmatrix} n \\ n-x \end{pmatrix} \int_0^p (1-\epsilon)^{n-x}\epsilon^{x-1}d\epsilon.$$

Hence, we can write

$$\mathcal{P}(p) = \frac{I_p(n-\sigma,\sigma)}{I_p(n-\sigma+1,\sigma-1)}.$$

Some tedious calculations give

$$\mathcal{P}(p)' = \frac{(n-\sigma)(n+1-\sigma)}{n(\sigma-1)} \frac{\displaystyle\int_0^p (1-\epsilon)^{n-\sigma-1}\epsilon^{\sigma-2}(p-\epsilon)d\epsilon}{\left(\displaystyle\int_0^p (1-\epsilon)^{n-\sigma}\epsilon^{\sigma-2}d\epsilon\right)^2} > 0.$$

Moreover, we have $\mathcal{P}(p) \in [0,1]$. Since the inequality $(c(\zeta) - U_0)\mathcal{P}(p)' > 0$ holds whatever the compensation, the sign of db/dp depends on the signs and levels of both $\partial c(\zeta)/\partial b$ and $\partial c(\zeta)/\partial p$ (note that the latter derivative can be non null when the compensation depends on v). ■

5 Compensation's rules

The equilibrium analysis provided in the previous section leaves the problem of finding an equitable or fair compensation unresolved. This section is devoted to this problem. Several types of compensation are proposed in order to evaluate their impact on the equilibrium probability of acceptance for the voters and on the agents surpluses. To that end, we envisage the compensation as a function of both the strategic variable b and the parameters of the model.[1] The parameters of the model are the threshold σ, the surplus Δ, the status-quo utility U_0, the total number n of voters and the number v of accepting voters. Thus, we have $\zeta = \{b, \sigma, \Delta, U_0, n, v\}$. These various possibilities are grouped into two categories depending on the impact of b on c. In order to save on notations, we shall write $\bar{c}$ when the strategic variable b is not involved. To begin with, compensations which do not rely on the proposal are studied. Such compensations are called "offer-invariant". Secondly, the other cases, in which the compensation is influenced by the value of the offer (as well as the other parameters of the model), are examined.

5.1 Offer-invariant compensations $\bar{c}$

To begin with, we consider compensation's rules that do not depend on b. The proposition below provides the optimal probability of voting in favor of the proposal as a function of the level of the offer-invariant compensation.

[1] We discuss in the concluding remarks some more complex compensation rules depending also on the number v of voters having accepted the proposal.

Proposition 3 *If the compensation is offer-invariant, then the optimal probability of acceptance is given by*

$$p^{\star} = \begin{cases} 0 & \text{if } \bar{c} \geq \dfrac{\Delta - \sigma U_0}{n - \sigma}, \\ 1 - \dfrac{(n-\sigma)(\bar{c} - U_0)}{\Delta - nU_0} & \text{if } \bar{c} < \dfrac{\Delta - \sigma U_0}{n - \sigma}, \\ 1 & \text{if } \bar{c} = U_0. \end{cases}$$

Proof. Since the compensation is offer-invariant, we both have $\partial c(\zeta)/\partial b = 0$ and $\partial c(\zeta)/\partial p = 0$. From Proposition (2) we know that b is increasing in p. Thus, we can maximize the proposer's expected surplus according to p. We get the following first order condition:

$$\begin{aligned} \frac{dE(\pi)}{dp} &= \frac{(1-p)^{n-\sigma}p^{\sigma-1}(\Delta - nU_0)}{Beta(n+1-\sigma,\sigma)} - \frac{n(1-p)^{n-1-\sigma}p^{\sigma-1}(\bar{c}-U_0)}{Beta(n-\sigma,\sigma)} \\ &= (1-p)^{n-\sigma-1}p^{\sigma-1}\sigma\left((\Delta - nU_0)(1-p)\begin{pmatrix} n \\ n-\sigma \end{pmatrix} - n\begin{pmatrix} n-1 \\ n-\sigma-1 \end{pmatrix}(\bar{c}-U_0)\right) \\ &= (1-p)^{n-\sigma-1}p^{\sigma-1}n\sigma\begin{pmatrix} n-1 \\ \sigma \end{pmatrix}\left[(1-p)\frac{\Delta - nU_0}{n-\sigma} - \bar{c} + U_0\right] = 0. \end{aligned} \tag{9}$$

This yields three candidates for the optimal probability: 0, 1 and

$$p_0 := 1 - \frac{(n-\sigma)(\bar{c}-U_0)}{\Delta - nU_0}.$$

The second derivative valued at $p = 0$ and $p = 1$ gives

$$\left.\frac{d^2E(\pi(p))}{dp^2}\right|_{p=0} = \left.\frac{d^2E(\pi(p))}{dp^2}\right|_{p=1} = 0,$$

so that only p_0 remains. We have:

$$\left.\frac{d^2E(\pi(p))}{dp^2}\right|_{p=p_0} = -n\sigma(\bar{c}-U_0)\begin{pmatrix} n-1 \\ \sigma \end{pmatrix}\left(1 - \frac{(n-\sigma)(\bar{c}-U_0)}{\Delta - nU_0}\right)^{\sigma-1}\left(\frac{(n-\sigma)(\bar{c}-U_0)}{\Delta - nU_0}\right)^{n-2-\sigma} < 0.$$

There exists an interior solution $0 < p_0 < 1$ if and only if $U_0 < \bar{c} < (\Delta - \sigma U_0)/(n-\sigma)$. The candidate p_0 is a local maximum and also a global one since it the unique. ■

First, it is interesting to know the behavior of $p^{\star}$ when n goes to infinity. This analysis can only be done if σ grows at the same time sufficiently to maintain $\sigma > n/2$. Therefore, it is better to replace σ by a given proportion of the total number of voters, which we denote by $\alpha \in (1/2, 1]$. Proceeding in this fashion, we get that

$$\lim_{n \to +\infty} \frac{\Delta - \alpha n U_0}{n - \alpha n} = -\frac{\alpha}{1-\alpha}U_0.$$

Hence, we have always $\bar{c} > (\Delta - \alpha n U_0)/(n - \alpha n)$ giving $p^{\star} = 0$ as shown in Proposition 3. In such a situation, the proposer's surplus also tends to zero. This is an illustration of an extreme free-riding:

the choice of a voter has no impact on whether the proposal is accepted or not. Voters do not perceive themselves as pivotal, so that each of them has an incentive to wait for a compensation, but as a result the proposal eventually fails to be implemented.

The legislator or the central planner has two tools to regulate freezeout: the qualified majority σ and the compensation $\bar{c}$. Therefore, it can be useful to analyze the influence of these tools on the results of the vote and on the expected surplus' distribution. Each tool has a predictable complex impact on the proposer and voters' choices. This is the reason why they deserve two distinct paragraphs.

IMPACT OF c

While the probability of acceptance is decreasing in the compensation, the relation between the optimal proposal $b^\star(\bar{c})$ and the compensation is non monotonic. From equation (2), we have:

$$\frac{db}{d\bar{c}} = \underbrace{\mathcal{P}(p)'\frac{\partial p^\star}{\partial \bar{c}}(\bar{c} - U_0)}_{<0} + \underbrace{\mathcal{P}(p)}_{>0}.$$

It also holds that

$$\left.\frac{db}{d\bar{c}}\right|_{\bar{c}=U_0} = \mathcal{P}(1) > 0 \quad \text{and} \quad \left.\frac{db}{d\bar{c}}\right|_{\bar{c}\to\frac{\Delta-\sigma U_0}{n-\sigma}} < 0.$$

For small compensation, this quantity is increasing but for high compensations it becomes decreasing. In other words, when the compensation is high, too many voters prefer to reject the offer to benefit from a high compensation and the proposer prefers to bear the risk not to reach a qualified majority (on average) than paying too much to induce the voters to accept. The impact of the compensation on the surplus distribution are summarized in the following proposition.

Proposition 4 *Increasing the compensation always reduces the expected social surplus and the expected profit of the proposer, and does not always benefit to the voters.*

Proof. Some comparative statics give:

- $\left.\frac{\partial E(S)}{\partial \bar{c}}\right|_{p=p^\star} = \frac{\partial E(S)}{\partial p}\frac{\partial p}{\partial \bar{c}} < 0.$
- $\left.\frac{\partial E(\pi)}{\partial \bar{c}}\right|_{p=p^\star} = \frac{\partial E(\pi)}{\partial p}\frac{\partial p}{\partial \bar{c}} + \frac{\partial E(\pi)}{\partial \bar{c}} < 0.$
- $\left.\frac{\partial E(U)}{\partial \bar{c}}\right|_{p=p^\star} = \frac{\partial E(U)}{\partial p}\frac{\partial p}{\partial \bar{c}} + \frac{\partial E(U)}{\partial \bar{c}}$ has an indeterminate sign.

■

Increasing $\bar{c}$ does not affect the social surplus since it is just a monetary transfer between the proposer and some voters, even if this influences the number of compensated voters. Nevertheless, this decreases the probability of a success. The same negative effect is incurred by the proposer in addition to the higher compensation he has to pay. According to the voters, an increase in the compensation involves two opposite effects, a straightforward positive one and an indirect negative one since an increase in the compensation reduces the probability of success and so, the possibility of claiming the compensation. A direct consequence is that if a central planner prefers to reduce the expected social surplus in order to give more to the voters, he can increase the compensation but this can also lead to the opposite effect. This aspect is illustrated by the following numerical example.

Numerical example. Suppose that $n = 10$, $\sigma = 7$, $U_0 = 1$ and $\Delta = 15$. By choosing the compensation $\bar{c} = U_0$ we have $p^\star = 1$ and $E(S) = E(\pi) = 5$ and $E(U) = 0$. Now, assume that the central planner wants to increase the expected utility of the voters by increasing the compensation. For a 40% increase, the compensation becomes $\bar{c} = 1.4$ (that represents 80% of Δ), and we have $p^\star = 0.76$, $E(S) = 4$, $E(\pi) = 1.5$ and $E(U) = 2.5$. For a 50% increase, the compensation becomes $\bar{c} = 1.5$ (that represents 100% of Δ), and this yields a reduced probability $p^\star = 0.7$, and $E(S) = 3.25$, $E(\pi) = 0.94$ and $E(U) = 2.31$.

Remark. The special case $\bar{c} = \Delta/n$, in which each individual who votes against the proposal gets an equal share of the surplus, enables to recover a well-known result in a connected literature. In fact, we get $p^\star = \sigma/n$ as in [Bagnoli and Lipman, 1988] and [Holmström and Nalebuff, 1992] within the framework of a takeover.

Impact of σ

The probability of acceptance is increasing in the qualified majority. As σ increases, each voter is more likely to be pivotal and tends to accept more frequently the proposal (recall that in the limit case under unanimity, the probability of acceptance is equal to one). However, the relation between the optimal proposal $b^\star(\sigma)$ and the qualified majority is non monotonic. A change in σ has significant consequences that are summarized in the following proposition.

Proposition 5 *Reinforcing the qualified majority always increases the expected profit of the proposer, and can decrease both the expected social surplus and the expected utility of the voters.*

Proof. In this proof, we shall use subscripts $_\sigma$ and $_{\sigma+1}$ in order to distinguish among situations in which the qualified majorities require σ and $\sigma+1$ votes in favor respectively. For instance $E(\pi(p_\sigma))_{\sigma+1}$ will stand for the expected surplus of the proposer with qualified majority $\sigma + 1$ and when the voters choose the optimal equilibrium probability when the qualified majority is σ.

Firstly, we prove that raising the majority increases the proposer's expected surplus. So suppose that the qualified majority increases from σ to $\sigma + 1$. In the new situation, the expected profit of the proposer is thus maximized for

$$p_{\sigma+1} = 1 - \frac{(n - \sigma - 1)(\bar{c} - U_0)}{\Delta - nU_0},$$

which yields the following maximum:

$$E(\pi(p_{\sigma+1}))_{\sigma+1} = \sum_{j=\sigma+1}^{n} \binom{n}{j} p_{\sigma+1}^j (1 - p_{\sigma+1})^{n-j} A - \sum_{j=\sigma+1}^{n-1} \binom{n-1}{j} p_{\sigma+1}^j (1 - p_{\sigma+1})^{n-1-j} B,$$

where $A = \Delta - nU_0$ and $B = (\bar{c} - U_0)\, n$. It holds that:

$$
\begin{aligned}
E(\pi(p_{\sigma+1}))_{\sigma+1} >& E(\pi(p_\sigma))_{\sigma+1}\\
=&\sum_{j=\sigma+1}^{n}\binom{n}{j}p_\sigma^j(1-p_\sigma)^{n-j}A-\sum_{j=\sigma+1}^{n-1}\binom{n-1}{j}p_\sigma^j(1-p_\sigma)^{n-1-j}B\\
=&\sum_{j=\sigma}^{n}\binom{n}{j}p_\sigma^j(1-p_\sigma)^{n-j}A-\sum_{j=\sigma}^{n-1}\binom{n-1}{j}p_\sigma^j(1-p_\sigma)^{n-1-j}B\\
&-\binom{n}{\sigma}p_\sigma^\sigma(1-p_\sigma)^{n-\sigma}A+\binom{n-1}{\sigma}p_\sigma^\sigma(1-p_\sigma)^{n-1-\sigma}B\\
=&E(\pi(p_\sigma))_\sigma-\binom{n}{\sigma}p_\sigma^\sigma(1-p_\sigma)^{n-\sigma}A+\binom{n-1}{\sigma}p_\sigma^\sigma(1-p_\sigma)^{n-1-\sigma}B\\
=&E(\pi(p_\sigma))_\sigma+\binom{n}{\sigma}p_\sigma^\sigma(1-p_\sigma)^{n-1-\sigma}\left(-(1-p_\sigma)A+\frac{n-\sigma}{n}B\right).
\end{aligned}
$$

From Proposition 3, we know that for a majority σ, the optimal probability is

$$p_\sigma = 1-\frac{(n-\sigma)(\bar{c}-U_0)}{\Delta-nU_0}.$$

Using the latter expression in the previous inequality gives the desired result $E(\pi(p_{\sigma+1}))_{\sigma+1} > E(\pi(p_\sigma))_\sigma$.

Secondly, we know that both $E(U)$ and $E(S)$ are increasing in p and since p is increasing in σ. As a consequence:

$$E(U(p_{\sigma+1}))_{\sigma+1} > E(U(p_\sigma))_{\sigma+1} = E(U(p_\sigma))_\sigma - \binom{n-1}{\sigma}p_\sigma^\sigma(1-p_\sigma)^{n-1-\sigma}$$

and

$$E(S(p_{\sigma+1}))_{\sigma+1} > E(S(p_\sigma))_{\sigma+1} = E(S(p_\sigma))_\sigma - \binom{n}{\sigma}p_\sigma^\sigma(1-p_\sigma)^{n-\sigma}.$$

We can conclude that the impact of σ is not monotonic. The following numerical example shows that both the expected social surplus and utility of the voters can decrease when σ increases: suppose that $n = 10$, $U_0 = 0.1$, $\Delta = 20$ and $\bar{c} = 1.2$. Then, it holds that$E(S)|_{\sigma=6} = 17.89 > E(S)|_{\sigma=7} = 17.5$ and $E(U)|_{\sigma=6} = 9.54 > E(U)|_{\sigma=7} = 8.86$. ■

Two opposite effects are observed: a higher σ increases the probability of acceptance but at the same time this makes it more difficult to obtain enough positive votes. Hence, we observe ambiguous consequences on both the expected social surplus and the expected utility of voters. However, we get the non intuitive result on the proposer's expected profit found by [Holmström and Nalebuff, 1992]: raising the qualified majority increases the proposer's expected profit. The reason is that when a greater number of votes is required for success, this raises the probability with which each voter is pivotal.

The impact of the status quo utility U_0 on the optimal probability of acceptance is detailed in the following result.

Proposition 6 *The optimal probability of acceptance increases when the voters' initial wealth increases (a larger U_0) if and only if this probability is large enough to guarantee that the proposal is implemented on average. Otherwise it is decreasing.*

Proof. By Proposition 3, we can focus on the situations which satisfy $U_0 \le \bar{c} \le (\Delta - \sigma U_0)/(n-\sigma)$ since the optimal probability of acceptance is constant otherwise. In such situations, recall that the optimal probability of acceptance is given by

$$p^\star = 1 - \frac{(n-\sigma)(\bar{c} - U_0)}{\Delta - nU_0}$$

The partial derivative of $p^\star$ according to U_0 is positive if and only if $\bar{c} < \Delta/n$. Next, let us look for an optimal probability of acceptance which is large enough to ensure that the proposal is implemented on average. In other words, since all voters are identical, we need that $p^\star$ satisfies $np^\star \ge \sigma$. Replacing $p^\star$ by its above-mentioned expression, one can easily check that the inequality $np^\star \ge \sigma$ is indeed equivalent to $\bar{c} \le \Delta/n$. As a consequence, we conclude that $p^\star$ is increasing in U_0 if and only if $np^\star \ge \sigma$. ■

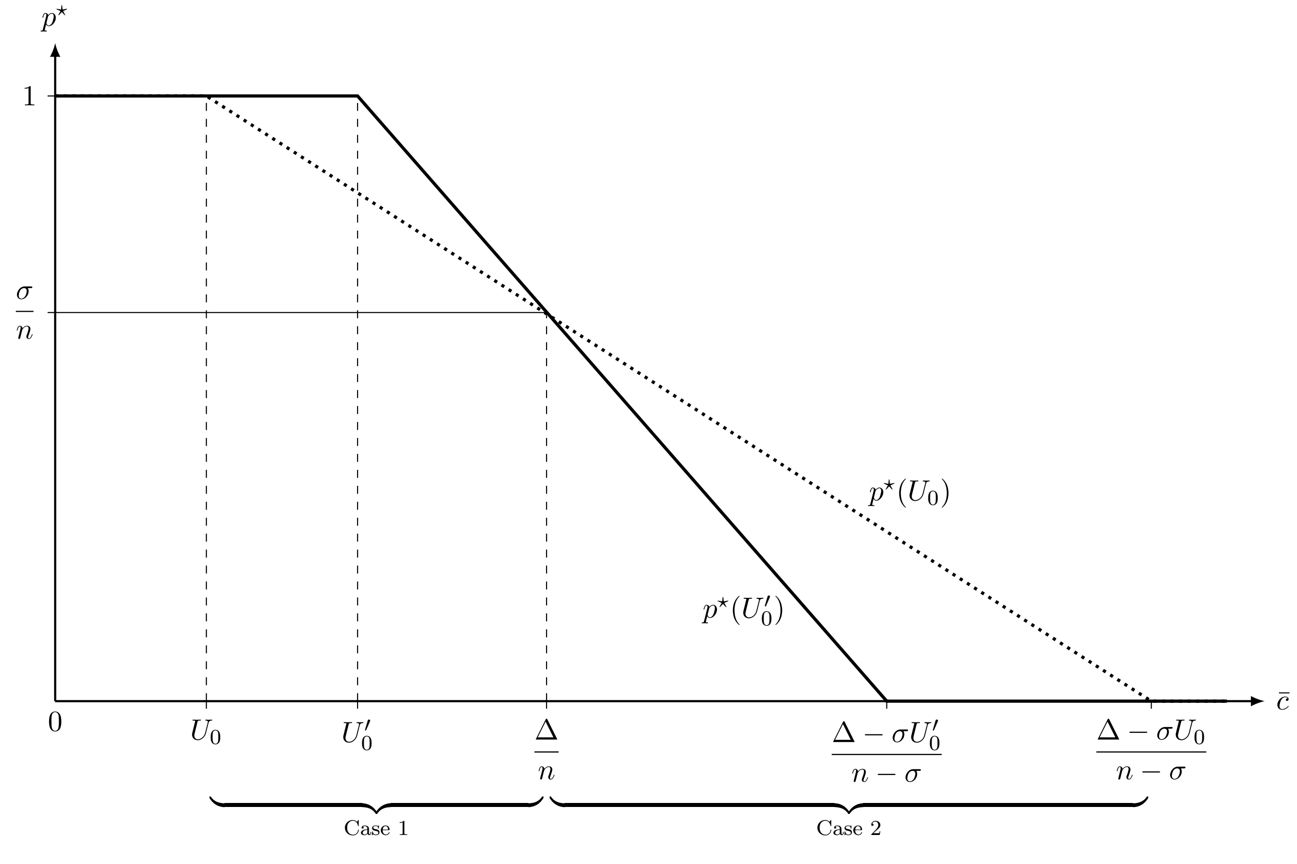

Figure 1: Evolution of the optimal probability of acceptance with the compensation levels for two values $U_0' > U_0$ of the status-quo utility.

The phenomena pointed out by Proposition 6 can be represented in Figure 1. We consider two situations, which only differ in that U_0' is greater than U_0. Obviously, the impact of the status-quo utility U_0 on the optimal probability is not monotone. Let us distinguish two cases.

Firstly, consider case 1 in which $p^\star > \sigma/n$ (or equivalently $\bar{c} < \Delta/n$ as shown in the proof of Proposition 6 and as represented on the picture), which means that in average the proposal is implemented. If a voter rejects the offer, then the probability to obtain the compensation $\bar{c}$ is larger than the probability to get the status quo utility U_0. The expected payoff of such a rejection therefore decreases if the voters are richer in the initial situation ($U_0' > U_0$). In a sense, a voter has more to win by rejecting the offer when he is poor (U_0) than when he is rich (U_0'). This stems from the fact that when a voter becomes richer, the compensation becomes less interesting, which in turn implies that it is less in the interest of the voter to reject the offer, *i.e.* the optimal probability of acceptance is increasing in U_0.

Secondly, consider case 2 in which $p^\star < \sigma/n$, which means that in average the proposal is not implemented. The reasoning is opposite. If a voter rejects the offer, then the probability to obtain the compensation $\bar{c}$ is lesser than the probability to get the status quo utility U_0. The expected payoff of rejecting the offer therefore increases if the voters are richer in the initial situation ($U_0' > U_0$). As such, a voter has more to loose by rejecting the offer when he is poor (U_0) than when he is rich (U_0'), *i.e.* the optimal probability of acceptance is decreasing in U_0.

To sum up, rich voters are more inclined to accept the proposal than poor voters when the compensation is small, while opposite incitations arise when the compensation is large. As shown by the slope of the two functions in Figure 1, the optimal probability of acceptance is more and more decreasing in the compensation as the voters become richer in the status quo situation.

An alternative exposition of Proposition 6 in terms of surplus is as follows:

Corollary 1 *For a given Δ, the likelihood of implementing the proposal decreases with the generated surplus ($\Delta - nU_0$) if and only if the expected number of accepting voters is large enough to implement the proposal. Otherwise it is decreasing.*

5.2 Compensation rules depending on the offer b

This section examines compensation rules which are not offer-invariant. A judge can based his compensation's choice on the proposer's offer. In a takeover environment, [Amihud et al., 2004] argue that the current practice in the US corresponds to a freezeout price that is the higher of the market price before the tender offer and the price paid in the tender offer (b in our framework).

The most simple compensation rule is to give the proposer's offer to the minority voters, $c(\zeta) = b$. It is worth to note that setting $c(\zeta) = b$ does not amount to the situation of absence of compensation.

Proposition 7 *Under the compensation $c(\zeta) = b$, there exists a dominant pure-strategy such that the proposal is accepted with probability 1 and the optimal offer is $b^\star = U_0$. All the surplus accrues to the proposer.*

Since the proof is elementary, let us focus on two comments on the choice of such a compensation. Firstly, it is not harmful for the voters since none of them is worse off after the implementation of the proposal than before. Secondly, it is not very equitable since the whole created surplus $\Delta - nU_0$ goes to the proposer. Observe that the same result can be obtained by setting the compensation equal to U_0. The difference is that when $c(b) = b$, the compensation is eventually equal to $\bar{c} = U_0$ as a result of the strategic choice of the proposer and not fixed at U_0 right from the beginning.

Under more general compensation rules, we may establish the following result:

Proposition 8 *Fix a compensation's level $c(\zeta)$. Denote by p_{oi} and p_{noi} the corresponding optimal probabilities of acceptance under offer-invariant compensations and compensations influenced by b respectively. It holds that:*

- *if $c_b < 0$, then $p_{oi} < p_{noi}$,*
- *if $c_b \in (0, 1]$, then $p_{oi} > p_{noi}$,*
- *if $c_b \geq 1$, then the result is ambiguous.*

Proof. We have $\partial c(\zeta)/\partial p = 0$. From Proposition 2, it holds that:

$$\frac{db}{dp} = \frac{(c(\zeta) - U_0)\mathcal{P}(p)'}{1 - \dfrac{\partial c(\zeta)}{\partial b}\mathcal{P}(p)}. \tag{10}$$

Recall the notation $c_b := \partial c(\zeta)/\partial b$. Since $\operatorname{sign}\big(db^\star/dp\big) = \operatorname{sign}\left(1 - c_b(b)\mathcal{P}(p)\right)$ and $\mathcal{P}(p) \in [0, 1]$, this implies three cases:

- if $c_b < 0$, then $db^\star/dp > 0$,
- if $c_b \in (0, 1]$, then $db^\star/dp > 0$,
- if $c_b \geq 1$, then the sign of $db^\star/dp$ rests on the level of $\mathcal{P}(p)$ and there does not exist a one-to-one mapping from p to b.

Maximizing the expected surplus of the proposer gives the following FOC:

$$(1-p)^{n-\sigma-1}p^{\sigma-1}n\sigma \begin{pmatrix} n-1 \\ \sigma \end{pmatrix} \left[(1-p)\frac{\Delta - nU_0}{n-\sigma} - c(\zeta) + U_0\right] - n\mathbb{P}^s_{-i}\frac{\partial c(\zeta)}{\partial b}\frac{db}{dp} = 0 \tag{11}$$

We know that:

$$\frac{dE(\pi)}{dp}|_{p=p_{\text{oi}}} = -n\mathbb{P}^s_{-i}\frac{\partial c(\zeta)}{\partial b}\frac{db}{dp}|_{p=p_{\text{oi}}}, \tag{12}$$

from which we conclude that:

- if $c_b < 0$, then $p_{\text{oi}} < p_{\text{noi}}$,
- if $c_b \in (0, 1]$, then $p_{\text{oi}} > p_{\text{noi}}$,
- if $c_b \geq 1$, then result is ambiguous,

as desired. ■

Both situations considered in the above proposition are interesting from economic and fairness perspectives. Firstly, assume that the compensation is increasing in the offer made by the proposer, which means that that the greater the offer of the proposer, the greater the compensation a rejecting voter can expect. In such a situation, Proposition 8 states that a voter is less likely to accept the proposal than in a situation where the compensation is offer-invariant. The intuitive explanation is that a rejecting voter has a greater incentive to reject the proposal in order to claim the compensation. Secondly, assume that the compensation is decreasing in the offer made by the proposer. This second type of situations also makes sense because one can consider that rejecting a larger offer should not be stimulated by an increasing compensation. Proposition 8 then states that a voter has a smaller incentive to reject the proposal.

Proposition 8 also states that for a given compensation's level, the optimal probability of acceptance is greater when the compensation is decreasing in the offer than when the offer is weakly increasing, i.e. increases but less than the offer. Since the expected social surplus is increasing in the probability of acceptance, a judge seeks to induce a probability as high as possible. According to Proposition 8, the shape of the compensation is just as much crucial as its simple level. If a social planner selects a compensation that depends on the offer, he must calibrate carefully the gradient of the compensation. A numerical illustration of these comments is given below.

Numerical example. Suppose that $n = 10$, $\sigma = 7$, $U_0 = 1/2$, $\Delta = 10$ and consider the compensations $c_{dec}(b) = -1/2b + 3/2$ and $c_{inc}(b) = 1/2b + 0.68$. The resulting equilibrium offers are $b^\star_{dec} = 0.86$ and $b^\star_{inc} = 0.79$ respectively. At equilibrium, observe that the same compensation $c = 1.07$ is obtained and is greater than the offers. Nevertheless, this setup yields different probabilities of acceptance $p_{inc} = 0.61$ and $p_{dec} = 0.69$ respectively.

6 Concluding remarks

The results provided in section 5.2 are incomplete, mainly for technical reasons. In particular, we have restricted the analysis to compensations which are not influenced by important parameters such as the number of accepting voters. The next objective is to extend our current work to such compensation rules. In order to construct more sophisticated compensation rules, one may want to have a better evaluation of the real surplus which is achieved when the proposal is implemented, but before distributing the compensations to the unwilling voters. The success of the proposal generates a value Δ, which accrues to the proposer. However, the final payoff of the proposer is not equal to this entire value. In fact, the $v \geq \sigma$ voters who have accepted the proposal are each paid b by the proposer, meaning that only $\Delta - vb$ is still available to the proposer. It makes sense to think that the level of compensation should be evaluated in light of the loss $-vb$ borne by the proposer to ensure the implementation of its proposal. Since the v accepting voters have left the situation with their payoffs, we only have to study the smaller $n - v - 1$ problem in which the $n - v$ remaining voters bargain with the proposer. Since each remaining voters enjoys a utility U_0 for owning his single share, the total surplus still available can therefore be measured by $\Delta - vb - (n - v)U_0$. Recall that in this problem, the proposer already owns v shares. If we accept the principle that each bargaining entity has to get a portion of the above-mentioned surplus which is proportional to its number of shares, then it is reasonable to give to each remaining voter a compensation that consists of his initial wealth

U_0 plus a fraction $1/n$ of the available surplus. Formally, we get:

$$c(\zeta) = U_0 + \frac{\Delta - vb - (n-v)U_0}{n}.$$

Other similar compensation rules can be constructed by using different fair requirements, for instance by assuming that the available surplus is divided equally among the $n - v - 1$ bargainers instead of proportionally to their respective number of shares. If such sophisticated compensation rules are incorporated into the model, then it is likely that the analysis would become intractable, preventing from stating general results. Nonetheless, numerical simulations can help to understand their impact on the choices of the decision makers. The discussion of these aspects is left for future works.

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