CENTRE DE RECHERCHE
SUR LES STRATÉGIES ÉCONOMIQUES

# Compulsory Non-binding ADR

Nathalie Chappe

October 2013

**Working paper No. 2013 – 07**

CRESE
30, avenue de l'Observatoire
25009 Besançon
France
http://crese.univ-fcomte.fr/



UFC
UNIVERSITÉ DE FRANCHE-COMTÉ

# Compulsory Non-binding ADR

Nathalie Chappe *

22 octobre 2013

**Résumé**

**Abstract :** This article focuses on compulsory non-binding ADR when third parties have reputation concerns that make them dislike their decision be rejected by the adversary parties who in this event decide to insist on a court verdict. The third party is assumed to trade-off deviations from her favored decision and the possibility that her recommendation might be rejected by the parties. Given that non binding ADR takes place in the shadow of a court verdict, the third party's award might be diverted from her preferred outcome in the direction of the likely court ruling.

**Keywords :** litigation, non-binding, arbitration, reputation, Alternative Dispute Resolution

**JEL : K31, K41**

*CRESE, University of Franche-Comté ; 45, avenue de l'observatoire, 25030 Besançon, France. E-mail : nathalie.chappe@univ-fcomte.fr

# 1 Introduction

Commercial contracts (franchisors and franchisees, producers and consumers, etc.) or labor contracts (baseball players and club owners, employers and employees) often involve on-going economic relationships where specific investments have been made. Hence the costs of unresolved disputes may be dramatic (strikes, lower productivity, loss of trust, etc.). To avoid such costs, several dispute resolution methods exist. Alternative dispute resolution (ADR) refers to any mode of dispute resolution that does not utilize the court system, such as arbitration, neutral assessment, conciliation, and mediation. ADR share the feature that a third party is involved who offers an opinion about the dispute to the disputants. In recent decades, many countries have adopted rules requiring parties to go through some form of ADR before resorting to trial [1]. This paper will contribute to the current debate concerning court-mandated ADR, but whereas the debate concentrates on the parties incentives, we will focus on the third party incentives.

Court annexed arbitration is one such procedure. Disputants whose cases fit some criteria established by court must participate in arbitration in a prerequisite to trial. It is a hybrid of mediation and arbitration. In arbitration the disputing parties present their case to a third party intermediary (or a panel of arbitrators) who examines all the evidence and then makes a decision for the parties. Arbitration may be binding or non-binding. In binding arbitration the arbitrator's decision is final and cannot be appealed. In non-binding arbitration the arbitrator's decision is not final and may be rejected by the parties. The award made is merely an advisory opinion. The role of an arbitrator is thus similar to that of a mediator. [2] Non-binding arbitration is a process which is conducted as if it were a conventional arbitration, [3] except that the award issued by the tribunal

1. Court-annexed ADR programs were set up throughout the U.S. Moreover, under the amended Civil Rule 16 (effective March 1, 2008) every civil case is subject to compulsory ADR. In France, parties to a divorce dispute are required by civil procedure rules to take part in a mediation before having their case heard in court. In the same way, parties to a labor conflict are required to take part in a conciliation.

2. Although we systematically refer to arbitration and arbitrator behavior, our work also applies to any third party whose decision can be rejected and who cares about her reputation.

3. Two procedures are commonly employed when disputes are settled by arbitration. In final-offer arbitration, the contending parties simultaneously submit a proposition to an arbitrator who must select one of the two final offers; this is in contrast to conventional arbitration, in which the arbitrator may

is not binding on the parties, and they retain their rights to bring a claim before the courts or other arbitration court (See Bennett (2006) for an introduction to non-binding arbitration). [4]

Non-binding arbitrator behavior has received much less attention than binding arbitrators. This article attempts to fill that gap. In non-binding arbitration, as in conventional arbitration, the arbitrator fashions an award based on an analysis of her external judgment of what would constitute a fair award. But, in non-binding arbitration (contrary to binding arbitration), the contending parties can appeal to a court if the award does not suit. The incentives created by this institutional structure would be for the arbitrators to make decisions that are acceptable to the parties. A key point of our analysis is that arbitrators do not like their proposals to be rejected by the parties. Empirical research (Farber and Bazerman 1986) finds that the perceived quality of arbitrators plays a part in their selection by parties. Their prestige and reputation increase their prospects of being chosen to arbitrate future disputes. Arbitrators want to maintain a favorable reputation for two reasons : first they may have a human concern about their prestige ; second their reputation may influence their career. The arbitrator knows that his acceptability to the parties in the future depends on the quality of her decision. In other words, arbitrators take into consideration how their decisions affect the probability that the award will be accepted by both parties. We formalize the effect of reputation-seeking behavior on arbitration awards. We show that arbitrators act strategically, and may move some distance away from their preferred outcomes in order to retain cases in the arbitration court. Arbitrators might want to make decisions that differ from their preferred awards when their preferred awards would be rejected. Precisely, we show that arbitrator bias is determined by the legal background (expected judgment and trial costs). Arbitrators seek to pre-empt

select any figure above, below, or between the parties' offers.

4. In Florida several statutes "authorize or require non-binding arbitration as a method of resolving certain disputes relating to condominiums, cooperatives, homeowners associations, mobile home park lot tenancies, medical malpractice and sign owners." (in Daniel Morman and Jonathan Whitcomb "The Nonbinding Arbitration Minefield In Florida", *Florida Bar Journal*, May 2007. See also Edmund D. Edelman and Daniel J.B. Mitchell, "Binding-Nonbinding Arbitration : A New Process to Resolve Interest Disputes", *California Public Employee Relations Journal*, Issue 164, February 2004.

rejections and try to make decisions that will prevent cases going to court. However if the bias becomes too costly then arbitrators will not skew their decision and will choose their preferred recommendation. When arbitrators mimic judges, parties who take their cases to arbitration receive a decision close to the justice they would have received had their cases gone to court. This result calls into question a criticism often made of arbitration, namely that arbitration is a form of private justice which may be inconsistent with public justice and unfair. In non-binding arbitration, arbitrators may have an incentive to design their award so as to mimic the public judgment if the disutility suffered from bias is not too high. The decision bias may be welfare enhancing since the third party decision is closed to the judge decision and the bias allows the litigants to save the trial costs and to continue their relation.

The remainder of this paper is organized as follows. After reviewing the related literature in section 2, we present the basic model (section 3). In section 4 we set out the results. Section 5 examines the effect of the main parameters of the model on the arbitrator's decision. In section 6, three extensions are considered, one involving uncertainty about a litigants' characteristics, the other considering a link between the arbitrator's award and the judgment made by the court and the last introducing an "offer of settlement" rule. Section 7 briefly concludes.

# 2 Related Literature

Obviously, our work is closely related to the vast literature on alternative dispute resolution, in particular arbitration. This literature identifies how litigants and arbitrators behave in binding arbitration (among others : Farber and Bazerman 1986, Farber 1980, Ashenfelter and Bloom 1984 and more recently Armstrong and Hurley 2002). Very little work seems to have been done on non-binding arbitration and non-binding arbitrators. Shavell (1995) examines the social costs and benefits of binding and non-binding alternative dispute resolution. Spurr (2000) analyzes pre-trial mediation and then tests a model in which each party takes into account the possibility of paying or being paid a penalty that is imposed on a party who unreasonably rejects the mediation award. The model

predicts (1) the probability that each party will accept or reject the mediation award, and (2) if the award is rejected and the case is later settled, how the settlement payment compares to the mediation award. We are not aware of a paper about arbitrators' behavior in non-binding arbitration. Yet, the fact that arbitration is nonbinding will affect the arbitrator's behavior since the parties remain free to go to court to litigate their dispute. We will show how this freedom will affect the arbitrator's behavior. Especially we will show that the arbitrator will arbitrate in the shadow of the law as parties bargain in the shadow of the law in the pre-trial negotiations.

Our paper is also related to the literature on careerist judges. Landes and Posner (1976) conjecture that judges follow precedents to avoid the disutility of being overruled. Miceli and Cosgel (1994) and Whitman (2000) assume that judges suffer a utility loss when their rulings are overturned by others and gain utility when they are cited. Levy (2005) analyzes how careerist judges formulate their decisions using information they uncover during deliberations as well as relevant information from previous decisions. He shows that a careerist judge tends to be creative, that is, she tends to contradict previous decisions more than an efficient judge would do. Iossa and Palumbo (2007) study the monitoring role given to decision-makers by endogenous appeals, i.e., appeals triggered by the parties involved in the dispute. Their main conclusion is that the incentive of the parties to monitor decision-makers through appeals is enhanced when information provision is delegated to the parties themselves rather than to an independent investigator. In Shavell (2006) the ability of litigants to appeal decisions of judges to a higher authority may lead to the making of better decisions because the appeals process constitutes a threat to adjudicators whose decisions deviate too much from socially desirable ones. Our model is close to that of Shavell (2006) since there exists a mechanism that constitute a threat to adjudicators. In his model, it is the appeal process, in our model it is the decision from the parties to accept or reject the proposal. We depart from it in considering careerist arbitrators (In shavell's model, a reversal penalty is imposed on the adjudicator if his decision is not equal to the socially optimal decision). Arbitrators do not like their proposal to be rejected by the parties since refusals mean they have not succeeded in resolving the dispute. Arbitrators also suffer disutility from deviating from their preferred

award. Contrary to Shavell, the disutility is not a constant but an increasing function of the deviation.

# 3 The Basic Model

Consider a one-shot game involving an arbitrator, A, and two individuals, a plaintiff (P) and a defendant (D). The parties are engaged in a contractual relationship (union and management, franchisor and franchisee, firm A and firm B) which generates an initial surplus $W$. The parameter $\alpha \in [0, 1]$ formalizes the share of the surplus that the plaintiff can appropriate. A dispute arises between P and D that is first arbitrated. [5] Once the arbitrator has given her verdict, the disputants face a binary choice : they can accept the arbitration award or go to court. If they both accept, the case is resolved, and the defendant pays the plaintiff the amount of the award. If either party rejects, the case proceeds on toward trial. We suppose that the choices are made simultaneously. [6] The arbitration procedure is considered a peaceful mechanism for resolving disputes, whereas trial is considered a formal and conflicting procedure. We assume that if both parties accept the arbitrator's award, they cooperate fully and the surplus remains $W$. If trial occurs, we assume that the relationship is broken and $W = 0$. [7]

We assume that first the arbitrator has a somewhat exogenously derived notion of an "appropriate award", $z$. The notion of an "appropriate award" is investigated empirically by Farber and Bazerman (1986). They argue that the "appropriate award" is independent of the particular type of arbitration. The "appropriate award"' is argued to be a function of the facts of a given situation. The facts are considered to be exogenous to the bargaining process. The "appropriate award" reflects the arbitrator's personal view of how the case should be decided, based, for example, on her background, on her information about the

5. Either the parties have previously included an arbitration clause in their contract, or the parties decide to go to arbitration to save the costs of a trial or to ensure the privacy and confidentiality of the procedure.

6. In mediation systems and court-annexed arbitration, programs each party must submit its response to the mediation award without knowing the response of the other party.

7. This value has no impact on our results.

case,[8] on her private interest in the decision,[9] on her sense of justice[10]... The rules that govern arbitrators provide for flexible proceeding and do not require the strict application of legal rules. The "appropriate award", $z$, is then used to determine the arbitration award, $a$. In conventional arbitration, the arbitration award is a weighted average of the arbitrator's notion of an "appropriate award" and the average of the parties' offers (Farber and Bazerman 1989). In final-offer arbitration, the arbitrator decides in favor of the party whose offer is closest to her "appropriate award" (Farber, 1980 ; Ashenfelter and Bloom, 1984 ; Farber and Bazerman, 1986). In our model (non-binding arbitration), the arbitration award may correspond to or differ from the "appropriate award" depending on the possibility that the award be rejected by the parties. Each time that the arbitrator makes an award $a$ that differs from her "appropriate award", she incurs a disutility of $d(a-z)$, where $d(0)=0$ and $d(x)>0$ for $x><0$. We assume that the cost function $d$ is convex : $d''>0$. In our model, the arbitrator fails to decide a case in accordance with her preferred award in order to reduce the loss of utility from the award being rejected by the parties and the case taken before a court.[11] The arbitrator derives a positive constant utility $X$, measured in terms of social status, reputation and prestige, whenever her award is accepted.[12] Note that the arbitrator's utility does not depend on the value of the judgment.[13] Furthermore, the arbitrator receives an amount $F$ in fees whatever

8. $z$ may be the result of an updated belief about the case according to a received signal about the state of the world. For example as in Iossa 2007.

9. An arbitrator might have a private interest in a decision, such as because she has been bribed or threatened.

10. Several economists, following Bankston (1976), have supposed that arbitrators formulate the "appropriate award" using the following process : $z=\lambda\left[(1-\theta)f(I)+\theta X\right]$ where $f(I)$ denotes arbitration-acceptable information ; $X$ denotes information that is external ; and $\theta$ and $\lambda$ represent possible sources of bias.

11. In Miceli and Cosgel (1994) the judge suffers a utility loss when her privately optimal outcome for the case does not coincide with her actual decision in the case. Miceli and Cosgel make the assumption that the utility loss is a constant and does not depend on the "distance" between the judge's private optimum and the judge's actual decision.

12. In arbitration procedures, the parties select the arbitrator. Consequently their reputation affects their future earnings.

13. In conventional arbitration, and specially in non-binding arbitration, an arbitrator has large flexibility to recommend an award based on her own perception of fairness and may depart from the judgment.

her decision is. In summary, we can represent the utility level of the arbitrator as follows. If the award $a$ is accepted by both parties, then the arbitrator receives utility equal to

$$U_A = F + X - d\,(a - z) \tag{1}$$

If the award is rejected, then the arbitrator receives utility equal to

$$U_A = F \tag{2}$$

Of course, the arbitrator's decision affects litigants' utilities. The preferences over possible awards are given by $U_P(a) = a$ for the plaintiff and by $U_D(a) = -a$ for the defendant. The arbitrator's fees, $F$, are shared between the litigants : the plaintiff bears a part $\gamma$ of the fees, and the defendant bears $(1 - \gamma)$.
Let $c_P$ denote the plaintiff's expected costs and $c_D$ the defendant's expected costs if either party opts for the court.

Let $j$ be the judgment made by the court. Litigants are assumed to know $j$, $c_P$ and $c_D$. The judgment, $j$, may be seen as the optimal adjudication given the facts of the case and legal principles. In other words, $j$ represents the judgment made by an independent court without bias and without error. In other words, $j$ is the decision society considers to be fair given the facts of the case. It could be assumed that $j$ is the adjudication which maximizes a social welfare function (as in Shavell (2006)). We focus on the case where $z$ is unequal to $j$ as is the case of interest. [14]

The expected utility of each litigant is represented in the following game matrix.

The structure of the game is as follows :

*Period 0.* Mandatory non binding arbitration occurs. $z$ is exogenously determined.

*Period 1.* The arbitrator chooses $a$.

*Period 2.* Either litigant may accept or refuse the arbitrator's decision.

"This is important because arbitration, unlike courts, is not subsidized by the government ; arbitrators' fees must be defrayed by the disputants. The public subsidy of adjudication places arbitration at a cost disadvantage vis-à-vis the courts. One way to overcome this disadvantage is to offer a distinctive service and not just copy the judge's decision", Posner (2005). We relax this assumption in section 6.

14. We focus on mandatory arbitration. But if we consider voluntary arbitration, parties would choose arbitration in order to save trial costs ($c_P$ and $c_D$) and to maintain their contractual relationship ($W$).

TABLE 1 – Game matrix

| P \ D | *accept* | *refuse* |
|---|---|---|
| *accept* | $a + \alpha W - \gamma F$ , $-a + (1-\alpha) W - (1-\gamma) F$ | $j - c_P - \gamma F$ , $-j - c_D - (1-\gamma) F$ |
| *refuse* | $j - c_P - \gamma F$ , $c_D - (1-\gamma) F$ | $j - c_P - \gamma F$ , $-j - c_D - (1-\gamma) F$ |

*Period 3.* If the arbitrator's decision is accepted by both parties, the game ends. If the arbitrator's decision is refused, a trial occurs.

# 4 The Results

We can determine the equilibria of the game using backward induction.

## 4.1 The parties' decisions

We focus on the situation in which both parties accept the arbitration award. We are assuming that if the litigants are indifferent, they will accept the award. The equilibrium $(accept, accept)$ appears if :

$$a \geq j - \alpha W - c_P \tag{3}$$

which is the condition for which the plaintiff accepts the arbitration award. And if

$$a \leq j + (1-\alpha) W + c_D \tag{4}$$

which is the condition for which the defendant accepts the arbitration award. We denote by $\underline{z}$ the award $a$ that makes the plaintiff just indifferent between the award and the expected benefit of going to court (condition 3 is satisfied with equality). And similarly we denote by $\bar{z}$ the award $a$ that makes the defendant just indifferent between accepting the award and going to court (condition 4 is satisfied with equality). Thus, an arbitration award $a$ is unopposed by both parties if and only if it falls within the interval $[\underline{z}, \bar{z}]$, which corresponds to the domain where both parties value the arbitration at least as much as

the trial. As long as $\bar{z}$ is larger than $\underline{z}$ there will be mutually agreeable arbitration awards. Thus the range of potential arbitration agreements is given by :

$$\Delta = \bar{z} - \underline{z} = W + c_P + c_D. \tag{5}$$

**Lemma 1** *The range of potential arbitration agreements $\Delta$ is defined by the loss of the surplus in the event of trial, $W$, and by the total legal costs, $c_P + c_D$.*

The loss of surplus in the event of trial is enough for the range of potential arbitration agreements to exist (even if there are no legal costs). Notice that the range of potential arbitration agreements is determined neither by the arbitrator's fees, nor by the judgment. [15]

## 4.2 The arbitrator's decision

It was assumed that arbitrators can perfectly anticipate whether the parties will accept or not the arbitral award or will prefer instead to go to court. They know the range of potential arbitration agreements, as they know the litigants' parameters : the parties' cost of going to court, and how the surplus is shared. [16]

If the arbitrator's "appropriate award" lies within the range of potential arbitration agreements, $z \in [\underline{z}, \bar{z}]$, then the arbitrator chooses her preferred award. The award is accepted by both parties and the game ends. The arbitrator does not bias her decision and selects her preferred award. There is no trial.

If the arbitrator's preferred award lies outside the range of potential arbitration agreements, $z \notin [\underline{z}, \bar{z}]$, the arbitrator faces a choice between making a different award which satisfies both parties but does not correspond to her ideal and accepting refusal by the dissatisfied party. The arbitrator is indifferent between the case in which her award is refused by the plaintiff (defendant) and the case in which her award is accepted if and only if :

$$X - d\,(a - z) = 0. \tag{6}$$

15. When litigants have different prior, the range of potential arbitration agreements depends on the difference between the prior. See for example, Daughety and Reinganum (2008).

16. In practice, arbitrators may be unlikely to perfectly know the litigants' parameters. We relax this assumption in section 6.

Given the properties of the function $d$, two critical values $\hat{a}$ exist : $\hat{a}_1$ and $\hat{a}_2$, with $\hat{a}_1 < \hat{a}_2$ (See the Appendix 8.1 for graphic illustrations).

There are two possible cases to study : Case i. $z < \underline{z}$ and Case ii. $z > \bar{z}$. Case i corresponds to the case in which the arbitrator prefers to award less than the plaintiff is willing to accept (condition 3 is not satisfied). Case ii corresponds to the case in which the arbitrator prefers to award more than the defendant is willing to pay (condition 4 is not satisfied).

**Case i :** $z < \underline{z}$

When the award that makes the arbitrator indifferent between recommending her preferred award and compromising is greater than the minimum amount the plaintiff is willing to accept ($\hat{a}_2 > \underline{z}$) the arbitrator compromises at $a = \underline{z}$ since the cost to compromise $d(\underline{z} - z)$ is smaller than the cost that makes her indifferent $d(\hat{a}_2 - z)$. The award is accepted by both parties. There is no trial. See sub-case 1 in the Appendix 8.1.

When the award that makes the arbitrator indifferent between recommending her preferred award and compromising is smaller than the minimum the plaintiff is willing to accept ($\hat{a}_2 < \underline{z}$), it is too costly for the arbitrator to bias her decision because she would have to opt for an award that is too distant from her preferred award ($\underline{z}$ is too far from her preferred award $z$ : $d(\underline{z} - z) > d(\hat{a}_2 - z)$). The arbitrator chooses $a = z$ which is refused by the plaintiff and the case goes to trial. See sub-case 2 in the Appendix 8.1.

**Case ii :** $z > \bar{z}$

When the award that makes the arbitrator indifferent between recommending her preferred award and compromising is greater than the maximum amount the defendant is willing to pay : $\hat{a}_1 > \bar{z}$, it is too costly for the arbitrator to bias her decision and she chooses $a = z$ to minimize her disutility from a recommendation below her preferred choice. The award $z$ is refused by the defendant and the case goes to trial. See sub-case 3 in the Appendix 8.1.

When the award that makes the arbitrator indifferent between recommending her preferred award and compromising is smaller than the maximum amount the defendant is willing to pay : $\hat{a}_1 < \bar{z}$, the arbitrator compromises and opts for $a = \bar{z}$. In this case, the cost of compromising $d(\bar{z} - z)$ is smaller than the cost that makes her indifferent

$d(\hat{a}_1 - z)$. This award, $\bar{z}$, is accepted by both parties. There is no trial. See sub-case 4 in the Appendix 8.1.

To summarize these findings, we state

**Proposition 1** *a) If $z \in [\underline{z}, \bar{z}]$, the arbitrator chooses her preferred award since both parties would be content with such an award and there will be no trial.*

*b) When $z < \hat{a}_2 < \underline{z}$ and when $z > \hat{a}_1 > \bar{z}$, it is too costly for the arbitrator to bias her decision because she would have to opt for an award that is too distant from her preferred award. The matter goes to trial.*

*c) If $z < \underline{z} < \hat{a}_2$ or if $\hat{a}_1 < \bar{z} < z$, then the arbitrator bends her decision in favor of the plaintiff ($a = \underline{z}$) in the first case and in favor of the defendant ($a = \bar{z}$) in the second case. Here, the gain from reputation is greater than the loss from bias. There is no trial.*

The arbitrator's decision always differs from the judgment $j$ (since we consider $z \neq j$). Situations a) and b) correspond to the states in which the arbitrator's behavior remains unaltered by considerations of reputation. Situation b) corresponds to the case in which the arbitrator does not bias and chooses her appropriate award. The arbitrator's decision differs from $j$ and a trial occurs. Situation c) corresponds to the case in which the non-binding arbitrator alters her decision in favor of one party so that her decision will be accepted and there will be no trial. The arbitrator's decision is closer to $j$ since we have $z < \underline{z} < j$ or $j < \bar{z} < z$.

To analyze the impact of bias in terms of welfare, we retain two criteria : the percentage of conflicts that end up with an arbitration award and the "appropriateness" of the arbitrator's award. [17] The first criterion is based on the idea of minimizing dispute resolution costs. In case of failure of arbitration, parties have to go to court which is costly as each litigant must pay the trial costs ($c_P + c_D$). Moreover, arbitration is better for the future relations between the disputing parties than trial ($W$ is lost in case of trial). Thus a welfare criteria will be the percentage of conflicts that end up with an arbitration award. The higher this percentage is, the higher the welfare is (parties save $c_P + c_D + W$). [18] The

17. See Olszewski (2011) for the use of two similar criteria to compare conventional and final-offer arbitration.

18. This criteria is based on the usual criterion in the existing literature on arbitration. In this literature,

second criterion includes the "appropriateness" of the arbitrator's award. This criterion reflects the fact that the parties may wish to depart as little as possible from the judge's decision. [19] About these two criteria we can establish the following proposition :

**Proposition 2** *The decision bias (case c) is such that*
*- the arbitrator's decision is closer to $j$ than if the arbitrator does not bias, and*
*- there is no trial (both parties accept the arbitration award).*

The first point of the previous proposition can be put in relation to the empirical study of Chew (2011). He shows that arbitrators' decision-making processes mirror judges' decision-making processes, referencing legal principles and precedents and following the same interpretational norms. In another context, Shavell (2006) showed that the appeals process leads adjudicators to make decisions that more closely resemble the socially optimal decision. In our model, the threat to see her decision refused by the parties leads the third party to make a decision closer to $j$. Furthermore, there is no trial.

# 5 Changes in legal parameters

Here we set out to answer the question how the legal parameters (the court's judgment, $j$, which may be affected by rules of damages control : caps on damages, punitive damages for example, and the costs of going to court, $c_P$ and $c_D$) affect the arbitrator's behavior through the thresholds $\underline{z}$ and $\bar{z}$. We show that by serving as an alternative for the parties, the court induces the arbitrator to adjust his verdict.

An increase in the judgment $j$ is favorable to the plaintiff. This change has the effect of raising both the lower and the higher thresholds. In other terms, the change moves the contract zone to the right. Different effects may occur depending on the initial situation and the magnitude of the increases.

the criterion is the percentage of conflicts that ends up with a settlement. Indeed arbitration is a costly procedure, whereas settlement is free (or less costly). Hence arbitration has to make parties to reach a settlement without using the system. In our model, arbitration is mandatory and non-binding : trial plays the part of arbitration and arbitration acts as settlement.

19. This criteria is based on the view that arbitration is a form of private justice which may be inconsistent with public justice.

Consider the case in which initially the arbitrator's preferred award lies within the range of potential arbitration agreements, $z \in [\underline{z}, \bar{z}]$. If the increase in the minimum the plaintiff is willing to accept, $\underline{z}$, is comparatively small, the preferred award continues to lie within the interval $[\underline{z}, \bar{z}]$, and, therefore, the arbitrator continues to choose it. If the increase is large enough and the minimum the plaintiff is willing to accept exceeds the arbitrator's preferred award, $\underline{z} > z$, so that $z$ is no longer in the contract zone, then, the selected award will depend on whether or not $\underline{z}$ increases enough to exceed the arbitrator's threshold $\hat{a}_2$. If it does, the award remains unchanged. If it does not, the arbitrator alters her decision and chooses $a = \underline{z}$, that is, the award changes and becomes the new value of $\underline{z}$. The increase in $\bar{z}$ has no effect. In the case in which initially the arbitrator's preferred award is less than the minimum the plaintiff is willing to accept, $z < \underline{z}$, then, the selected award will depend on whether or not $\underline{z}$ increases enough to exceed the arbitrator's threshold $\hat{a}_2$. If it does, the arbitrator alters her decision and chooses $a = z$, that is, the award changes and becomes the arbitrator's preferred award (We move from sub-case 1 to sub-case 2 in the Appendix 8.1). If it does not, the award remains unchanged. Finally, consider the case in which initially the arbitrator's preferred award is greater than the maximum the defendant is willing to pay, $z > \bar{z}$. If the increase is large enough and $\bar{z}$ oversteps $z$, so that $z \in [\underline{z}, \bar{z}]$, then the arbitrator does not bias her decision. If the increase is comparatively small, the results are the same as in case ii. It depends on whether the maximum the defendant is willing to pay $\bar{z}$ exceeds the arbitrator's threshold or not.

In a nutshell, an increase in the judgment may induce the arbitrator to bias her decision depending on the comparative impact on the bounds of the contract zone. The change in the arbitrator's award is in favor of the plaintiff except if the plaintiff's and defendant's thresholds ($\underline{z}$ and $\bar{z}$), move too far to be worth deviating from the preferred award. In these cases, the arbitrator will choose her preferred award whereas before the change she skewed her decision. This change may thus be harmful to the party.

The analysis of the effects of a decrease in the judgment is quite similar. It is straightforward that a decrease in the judgment may induce the arbitrator to bias her decision depending on the comparative impact on the bounds of the range of potential arbitration

agreements. The change in the arbitrator's award is in favor of the defendant unless the defendant's and plaintiff's thresholds, ($\underline{z}$ and $\bar{z}$) move too far to be worth deviating from the preferred award.

Let us now examine the effects of a change in the trial costs. An increase in the trial costs, $c_P$ and $c_D$, decreases the minimum the plaintiff is willing to accept, $\underline{z}$ and increases the maximum amount the defendant is willing to pay, $\overline{z}$, so the range of potential arbitration agreements will expand. If initially $z \in [\underline{z}, \bar{z}]$, there is no change. The arbitrator's preferred award remains in the range of potential arbitration agreements and is chosen by the arbitrator and accepted by both parties. Consider the case in which initially the arbitrator's preferred award is less than the minimum the plaintiff is willing to accept, $z < \underline{z}$. The result depends on the relative position of $\hat{a}_2$. If initially $\hat{a}_2 > \underline{z}$, then the arbitrator continues to choose $\underline{z}$ but the value has decreased. If initially $\hat{a}_2 < \underline{z}$, it depends on whether the decrease in $\underline{z}$ is large enough, so that $\hat{a}_2$ oversteps $\underline{z}$. If it does, then the arbitrator biases her decision to $\underline{z}$, whereas before she opted for $z$ (in the Appendix 8.1, we move from sub-case 2 to sub-case 1). If the decrease in $\underline{z}$ is large enough, so that $z \in [\underline{z}, \bar{z}]$, then the arbitrator does not bias her decision. The increase in $\bar{z}$ has no impact. In the case in which initially the arbitrator's preferred award is greater than the maximum the defendant is willing to pay, $z > \bar{z}$ we have the following results. If the increase in $\bar{z}$ is large enough, so that $z \in [\underline{z}, \bar{z}]$, then the arbitrator does not bias her decision. If the increase in $\bar{z}$ is such that $\bar{z}$ stays below $\hat{a}_1$, the arbitrator's decision does not change. If $\bar{z}$ oversteps $a_1$, then the arbitrator chooses $\bar{z}$ whereas she does not bias before (we move from sub-case 3 to sub-case 4 in the Appendix 8.1).

The analysis of the effects of a decrease in the trial costs, $c_P$ and $c_D$, is similar.

To summarize these findings, we state

**Proposition 3** *A change in the legal system that makes it more (less) attractive for the plaintiff (an increase (decrease) in $j$ or a decrease (increase) in $c_P$) may induce the arbitrator to opt for an award that is more (less) favorable to the plaintiff. However, if the change in the thresholds ($\underline{z}$ and $\bar{z}$) is such that to bias becomes too costly, then the arbitrator decides to choose her preferred award.*

*Similar results hold for any change in the legal system concerning the defendant.*

As parties bargain in the shadow of the law, arbitrators arbitrate in the shadow of the law. However, if the changes of legal parameters are too favorable to a party, the party becomes too demanding and that becomes too costly for the arbitrator to deviate from her preferred award. This result comes from the fact that in our model the disutility suffered from biasing is an increasing function of the deviation (contrary to the model of Shavell (2006) where the disutility is a constant penalty).

Raising the level of prestige, $X$, obtained in case of acceptation or decreasing the disutility incurred from differing from the "appropriate award" induces the arbitrator to bias her decision (the bias decision is closer to $j$), and makes refusals less likely. The number of trials is diminished, thus trial costs are saved and the relation between the disputing parties can continue.

# 6 Extensions of the Basic Model

Let us now consider three extensions of the basic model.

## 6.1 Uncertainty about the rejection of the arbitrator's decision

In the basic model, the arbitrator is able to predict when her decision will be reversed. Here, we assume that the arbitrator has imperfect information about the litigants, more specifically she does not know how the surplus is shared, $\alpha$.[20] Arbitrators are uncertain when their decisions will be rejected. So, the arbitrator may bias her decision because she may misgauge whether or not a litigant will refuse her decision. To investigate this issue, suppose that litigants differ in the value of $\alpha$.[21] Let $g(\alpha)$ be the probability density of $\alpha$ and $G(\alpha)$ be the cumulative distribution function. For simplicity, we assume that $c_P = c_D = 0$.

20. To investigate these issues it could be assumed that litigants differ in the legal costs and that arbitrators know only their distribution. See Shavell (2006) for a similar analysis in the context of appeal.

21. Plaintiff and defendant know the value of $\alpha$.

The plaintiff will accept the arbitration award if and only if

$$\alpha > \frac{j-a}{W}. \tag{7}$$

Hence, the plaintiff will accept the award if and only if his type $\alpha$ is higher than $\underline{\alpha} = \frac{j-a}{W}$. Note that if $a \geq j$, then the plaintiff will always accept the arbitration award. If $a < j$, the probability that the award will be accepted by the plaintiff is $1 - G(\underline{\alpha})$.

The defendant will accept the arbitration award if and only if

$$\alpha < 1 + \frac{j-a}{W}. \tag{8}$$

Hence, the defendant will accept the award if and only if his type $\alpha$ is less than $\bar{\alpha} = 1 + \frac{j-a}{W}$. Note that if $a \leq j$, then the defendant will always accept the arbitration award ($\bar{\alpha} \geq 1$). If $a > j$, then the probability that the award will be accepted by the defendant is $G(\bar{\alpha})$.

The arbitrator knows that if she makes an award $a < j$, the probability that both litigants accept is $1 - G(\underline{\alpha})$. If she makes an award $a > j$, the probability that both litigants accept is $G(\bar{\alpha})$. If $a = j$, both litigants will always accept the award.

Hence, the expected utility of an arbitrator is

$$EU_A = \begin{cases} F + G(\bar{\alpha})\left[X - d\left(a - z\right)\right] & \text{for } a > j \\ F + \left[X - d\left(j - z\right)\right] & \text{for } a = j \\ F + (1 - G(\underline{\alpha}))\left[X - d\left(a - z\right)\right] & \text{for } a < j \end{cases} \tag{9}$$

If $z < j$, then the arbitrator's decision $a$ is such that $z < a < j$.[22] Hence, we can restrict attention to $a$ in $[z, j]$. For such $a$, equation 9 is

$$F + (1 - G(\underline{\alpha}))\left[X - d\left(a - z\right)\right], \tag{10}$$

the derivative with respect to $a$ of which is

$$\frac{dEU_A}{da} = -(1 - G(\underline{\alpha}))d'\left(a - z\right) - g(\bar{\alpha})\frac{-1}{W}\left[X - d\left(a - z\right)\right]. \tag{11}$$

The intuition behind expression 11 is as follows. The first term is the expected marginal disutility from bias. The second term is the marginal effect to the arbitrator from the

22. $a$ cannot exceed $j$ : if $a = j$, $EU_A = F + [X - d(j - z)]$, whereas if $a > j$, $EU_A = F + G(\bar{\alpha})\left[X - d\left(a - z\right)\right]$ which is inferior since $d\left(j - z\right) < d\left(a - z\right)$. Moreover $a$ cannot lie below $z$. $EU_A$ is higher at $z$ : $EU_A = F + (1 - G(\underline{\alpha}))X$, than at a lower $a$.

marginal impact on the probability that the award be accepted, $g(\bar{\alpha})\frac{-1}{W}$ times the net benefit from being accepted, $X - d\,(a - z)$.

If $z > j$, then the arbitrator's decision $a$ is such that $j < a < z$.[23] Hence, we can restrict attention to $a$ in $[j, z]$. For such $a$, equation 9 is

$$F + G(\bar{\alpha}))\,[X - d\,(a - z)]\,, \tag{12}$$

the derivative with respect to $a$ of which is

$$\frac{dEU_A}{da} = G(\bar{\alpha})d'\,(a - z) + g(\bar{\alpha})\frac{-1}{W}\,[X - d\,(a - z)]\,. \tag{13}$$

**Proposition 4** *Assume that the arbitrator does not know perfectly whether her decision would be refused by the parties, she chooses $a^* \in [z, j]$ if $z < j$ and $a^* \in [j, z]$ if $z > j$. The probability of rejection is positive.*

Shavell (2006) has a similar result in a context of appeals process. We differ from him by considering an increasing function of the deviation $d(a - z)$.

## 6.2 Judge's behavior

In the basic model we assume no link between the arbitrator's decision and the judgment made by the court. Here we assume that arbitration reputation is affected by the degree to which court judgments match their recommendations. An arbitrator with whom the courts agree would advertise this fact to potential litigants, as an encouragement both to select them for the role and to accept the arbitrator's award.

We define $q$ as the arbitrator's and litigant's assessment of the probability that the judge will depart from the arbitrator's decision. We assume there is no link between the judgment and the arbitration award. Indeed in many jurisdictions, the arbitrator's decision is not admissible at trial. Furthermore, a main feature of arbitration is confidentiality. Arbitration is held in private settings, providing parties with an alternative to the openness of courtroom proceedings. Consequently, $q$ is not a function of the arbitration award. The

23. $a$ cannot exceed $z$ : $EU_A$ is higher at $a = z$ : $EU_A = F + G(\bar{\alpha}))X$, than at a greater $a$. $a$ cannot lie below $j$ : if $a = j$, $EU_A = F + (1 - G(\underline{\alpha}))\,[X - d\,(j - z)]$, whereas if $a < j$, $EU_A = F + (1 - G(\underline{\alpha}))\,[X - d\,(a - z)]$ which is inferior since $d\,(j - z) < d\,(a - z)$.

plaintiff accepts the arbitrator's award if $a > j - \frac{\alpha W}{q} - \frac{c_P}{q}$ which defines $\underline{z}^J$. The defendant accepts the arbitration award if $a < j + \frac{\alpha W}{q} + \frac{c_D}{q}$ which defines $\bar{z}^J$. The range of potential arbitration agreements is given by $\frac{c_P+c_D+W}{q}$ which is greater than $\Delta$. The number of acceptable arbitration agreements is greater than in the basic model. If it is certain that the judge will choose the same award as the arbitrator ($q = 0$), then the arbitrator's award is always accepted by both parties. Also let $u$ be the utility the arbitrator receives if the judge follows her decision and $v$ be the disutility if the judge departs from her decision. [24] Also the arbitrator's expected reputational utility is given by $(1-q)u - qv = u - q(u+v)$. But recall that this utility can be received only if the arbitrator's award is rejected and the case goes to trial. Combining this with equation 2 yields the arbitrator's overall expected utility :

$$U_A = F + u - q(u + v). \tag{14}$$

This new feature has an impact on the values for which the arbitrator is indifferent between the case in which her award is refused by the plaintiff (defendant) and the case in which her award is accepted. These values satisfy the following condition :

$$X - d\,(a - z) = u - q(u + v). \tag{15}$$

Proposition 1 still holds (with the values of $\underline{z}^J$, $\bar{z}^J$ and $\hat{a}^J$ as defined above). This extension modifies the arbitrator's incentives to bias and the bias value.

**Proposition 5** *If the expected utility received if the judge follows her decision ($u(1-q)$) is greater than the expected disutility of the decision not being followed ($qv$), then the arbitrator biases more often. Otherwise, the arbitrator biases less often. In this extension, the bias is farther from $j$ than in the basic model since we have $\underline{z}^J < \underline{z} < j < \bar{z} < \bar{z}^J$.*

## 6.3 Offer of settlement rule

In many US courts, the party who rejects the arbitration award but does not do better at trial then has to pay the trial costs of the other party as well as her own. This rule

24. Note that the disutility does not depend on the difference between the judgment and the award. We have avoided adding this complication because it would not bring new insights : the imitation effect would just be reinforced.

is similar to an "offer of settlement" rule which may shift legal costs against parties that reject a settlement proposed by one of the parties and then fail to improve on that proposal at trial.[25] Let $\beta$ denote the fraction of the other party's costs that the party who rejects the award would have to pay in the event of cost-shifting.

To investigate this issue, suppose that litigants and arbitrators know only the distribution of $j$. Let $H(x)$ represent the probability that the judgment will be less than or equal to $x$. Assume, without loss of generality, that there exists a maximum possible judgment $J$. Then we obtain (i) $0 \leq H(j) \leq 1$, for all $j$ in the support $[0, J]$, (ii) $H(j) = 1$ for all $j \geq J$, and (iii) $H(j)$ is (weakly) increasing in $j$. Let $\bar{j}$ denote the expected value of the judgment, that is, $\bar{j} = \int j dH(j)$.

The expected utility of each litigant is represented in the following game matrix.

TABLE 2 – Game matrix

| P \ D | *accept* | $refuse$ |
|---|---|---|
| *accept* | $a + \alpha W - \gamma F$ , $-a + (1-\alpha) W - (1-\gamma) F$ | $\bar{j} - \gamma F - H(a) c_P - [1 - H(a)] (1 - \beta) c_P$ , $-\bar{j} - (1-\gamma) F - c_D - [1 - H(a)] \beta c_P$ |
| $refuse$ | $\bar{j} - \gamma F - H(a) \beta c_D - c_P$ , $-\bar{j} - (1-\gamma) F - H(a)(1-\beta) c_D - [1 - H(a)] c_D$ | $\bar{j} - \gamma F - H(a)(\beta c_D + c_P) - [1 - H(a)] (1 - \beta) c_P$ , $-\bar{j} - (1-\gamma) F - H(a)(1-\beta) c_D - [1 - H(a)] (\beta c_P + c_D)$ |

The equilibrium (accept, accept) appears if :

$$a > \bar{j} - \alpha W - H(a) \beta c_D - c_P, \tag{16}$$

and

$$a < \bar{j} + (1-\alpha) W + c_D + [1 - H(a)] \beta c_P. \tag{17}$$

We note $\bar{z}^R = \bar{j} - \alpha W - H(a) \beta c_D - c_P$ and $\underline{z}^R = \bar{j} + (1-\alpha) W + c_D + [1 - H(a)] \beta c_P$.

25. The most famous of such rules is the Rule 68. See among others : Spier 1994, Bebchuk and Chang 1999.

The range of potential arbitration agreements is now given by $\Delta^R = \bar{z}^R - \underline{z}^R = W + c_P + c_D + \beta\left[(1 - H(a))c_P + H(a)c_D\right]$ which is greater than $\Delta$. The arbitrator biases less often.

Proposition 1 still holds with $\bar{z}^R$ and $\underline{z}^R$. The arbitrator who biases her decision (case c)) opts for $\bar{z}^R$ or $\underline{z}^R$ which are farther from $j$ than were $\bar{z}$ and $\underline{z}$ if we assume that the perception of judgment is unbiased on average. Here we have : $\underline{z}^R < \underline{z} < j$ and $\bar{z}^R > \bar{z} < j$.

**Proposition 6** *If an offer of settlement rule exists, arbitrators bias less often and choose an award farther from $j$ when they bias.*

# 7 Conclusion

In this paper, we analyze compulsory non-binding arbitrator behavior as an illustration of compulsory non-binding ADR. We answer the question why an independent third party may be induced not to opt for her preferred award, that is, the award she thinks appropriate. Considerations of reputation or prestige combined with the possibility that the award will be rejected may explain this behavior. We have focused on situations in which the third party has reputation considerations and she also suffers disutility from deviating from her preferred award. We show that the third party may be induced to bias her decision to ensure her award is accepted, when the compromise is not too costly to her compared with her gain of reputation derived from the award being accepted. Moreover, we show that the legal system's parameters (the expected judgment and the trial costs) exert an influence on the third party because they act as a benchmark for the litigants and hence for the third party who is concerned about her reputation. When one party gains some power (for example the expected judgment moves in their favor), then the third party's bias will move in that direction unless it moves too far to be worth deviating from the preferred outcome. Hence third parties adjudicate in the shadow of the law. The threat of refusals leads them to take decisions that more closely resemble the judge's decision (that is assumed embodies the social interest). Given these results, compulsory nonbinding ADR should be encouraged, since on the one hand disputants avoid the trial (disputants accept the proposal of the third party) and on the other hand the proposal of the third party is influenced by the judgment which would be given by the judge and who

is regarded as fair. Nevertheless, this recommendation is worth only if the third parties have career concerns and thus value the acceptance of their proposal. If the third parties are very attached to the choice of their "appropriate award" (the disutility is large in case of bias), bias will not occur and the results do not hold any more : third party will not bias her decision and the matter goes to trial.

Let us provide an assessment of the role of the main assumptions on which our results rest :

First we assume that the judgment is socially desirable and is known to the parties and the third party. The assumption that the judgment that the judge will decide is common knowledge is not critical. If parties and arbitrators know the probability distribution of the judgment, the threat of trial will be imperfectly informative but the behavior of the parties will be the same except that they consider the expected judgment and not the judgment. It will influence the range of potential arbitration agreements : $[\bar{j}-\alpha W-c_P, \bar{j}+(1-\alpha W)+c_D]$. Proposition 1 still holds. The arbitrator chooses her preferred award if it is in the range of potential agreements. Otherwise she biases when it is not too costly. The problem is that the arbitrator is influenced not by the true judgment but by an imperfect perception of it. Thus the bias may not be welfare enhancing. If the perception of judgment is unbiased on average, $\bar{j} = j$, then the arbitrator makes the same decision that she would if the information is perfect.

Second we do not consider the possibility of settlement. Settlement negotiations may be conducted before arbitration or after the arbitration award has been rejected and before the trial. In the first case, given our assumption of perfect and symmetric information, we would find that arbitration would not occur since settlement avoids arbitration costs $F$. The litigants will make a settlement agreement which divides in some way the costs avoided by settling. The settlement will be for an amount depending on the bargaining power of each litigant. In the latter case (settlement before trial), the availability of such negotiated agreements will affect the values of $\underline{z}$ and $\bar{z}$. Since settlements are more favorable than trials, both litigants will accept the arbitration award less often. If settlement is allowed, arbitrators will behave as described in proposition 1, except that the value of $\underline{z}$ and $\bar{z}$ are given by the expected settlement which is to say that the contract zone is smal-

ler than if settlement is not allowed. Here we refer to the simplest bargaining theory. We do not consider that the arbitrator's recommendations may affect settlement. In a more elaborate analysis, a recommendation may serve as a focal point through its influence on bargainer beliefs. That is, bargainers' expectations of judgment may converge upon the recommendation, which can help eliminate bargainer optimism that is likely to otherwise increase dispute rates. Furthermore, a recommendation may reduce uncertainty surrounding the likely outcome from litigation, thereby increasing dispute rates for risk-averse bargainers. For a recent analysis of these two effects, see Dickinson and Hunnicut, 2009.

# Références


[1] Armstrong, Michael and W. J. Hurley. 2002. "Arbitration using the Closest Offer Principle of Arbitrator Behavior," 43 *Mathematical Social Sciences* 19-26.

[2] Ashenfelter, Orley, 1987. "Arbitrator Behavior," 77 *American Economic Review* 342-346.

[3] Ashenfelter, Orley and David Bloom, 1984. "Models of Arbitrator Behavior : Theory and Evidence," 74 *American Economic Review* 111-125.

[4] Bebchuk Lucian and Howard F. Chang 1999. "The Effect of Offer-of-Settlement Rules on the Terms of Settlement," 28 *The Journal of Legal Studies* 489-513.

[5] Bennett, Steven. 2006. "Non-binding Arbitration : an Introduction," 1 *Dispute Resolution Journal.*

[6] Bloom, David and Christopher Cavanagh. 1986. "An Analysis of the Selection of Arbitrators," 76 *American Economic Review* 408-422.

[7] Chew, Pat. 2011. Arbitral and Judicial Proceedings : Indistinguishable Justice or Justice Denied ? 46 *Wake Forest Law Review* 185-208.

[8] Daughety, Andrew and Jennifer Reinganum. 2008. "Settlement" in The Encyclopedia of Law and Economics (Second Ed.) - Vol. 10 - Procedural Law and Economics, Ed. by Chris William Sanchirico. Published by Edward Elgar.

[9] Dickinson David L. and Lynn Hunnicutt. 2009. "Nonbinding Suggestions : the Relative Rffects of Focal Points Versus Uncertainty Reduction on Bargaining Outcomes," 69 *Theory and Decision* 615-634.

[10] Farber, Henry. 1980. "An Analysis of Final-Offer Arbitration," 24 *Journal of Conflict Resolution* 683-705.

[11] Farber, Henry and Max Bazerman. 1986. "The General Basis of Arbitrator Behavior : An Empirical Analysis of Conventional and Final-Offer Arbitration," 54 *Econometrica* 1503-1528.

[12] Iossa, Elisabetha, and Palumbo, Giuliana. 2007. "Information Provision and Monitoring of the Decision-maker in the Presence of an Appeal Process," 163 *The Journal of Institutional and Theoretical Economics* 657-682.

[13] Levy, Gilat. 2005. "Careerist Judges and the Appeals Process," 36 *The RAND Journal of Economics* 275-297.

[14] Micel, T.J. and Cosgel, Metin. 1994. "Reputation and Judicial Decision-Making," 23 *Journal of Economic Behavior and Organization* 31-51.

[15] Olszewski, Wojciech. 2011. "A welfare analysis of arbitration", 3 *American Economic Journal : Microeconomics* 174-213.

[16] Shavell, Steven. 1995. "Alternative Dispute Resolution : an Economic Analysis," 24 *Journal of Legal Studies* 1-28.

[17] Shavell, Steven. 2006. "The Appeals Process and Adjudicator Incentives," 35 *Journal of Legal Studies* 1-29.

[18] Spier, Kathryn 1994. "Pretrial Bargaining and the Design of Fee-shifting Rules," 25 *RAND Journal of Economics* 197-214.

[19] Spurr, Stephen J. 2000. "The role of Nonbinding Alternative Dispute Resolution in Litigation," 42 *Journal of Economic Behavior and Organization* 75-96.

[20] Whitman, D .G. 2000. "Evolution of the Common Law and the Emergence of Compromise," 29 *Journal of Legal Studies* 753-781.

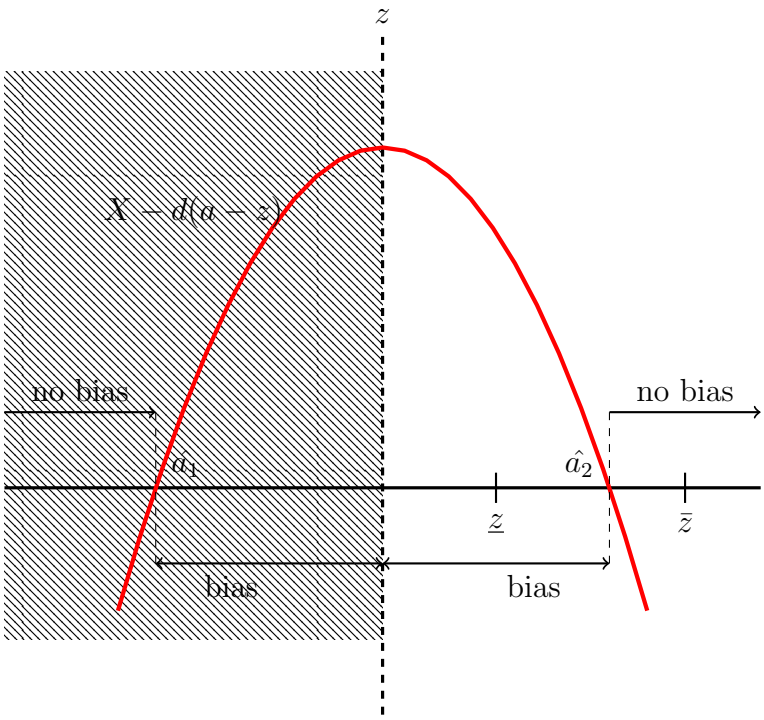
# 8 Appendix

## 8.1 Graphic illustration of proposition 1

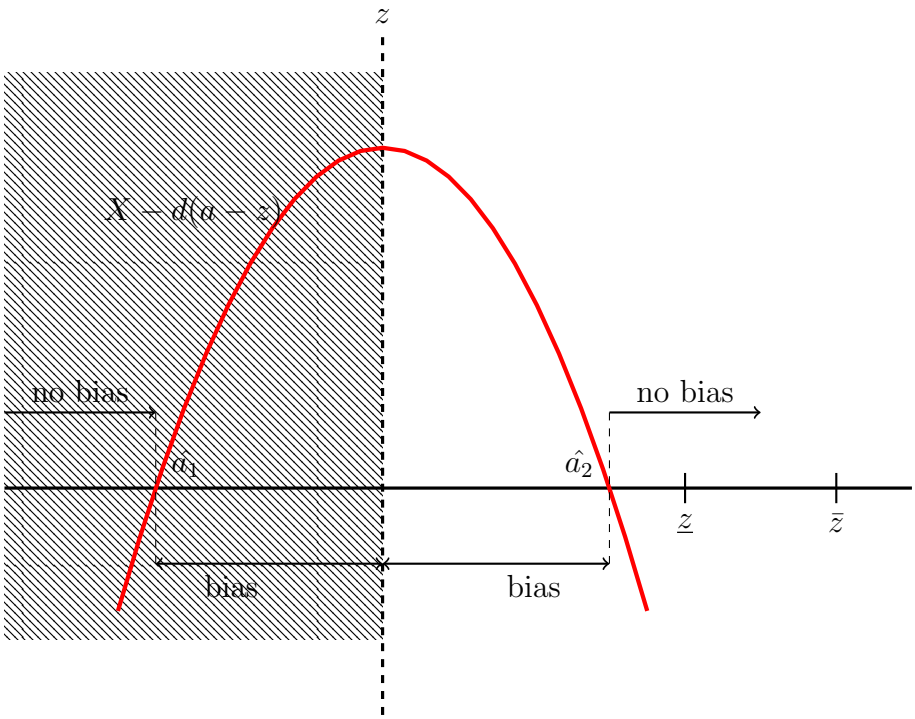
We use $d(a-z) = (a-z)^2$

**Case i :** $z < \underline{z}$

**Sub-case 1 :** $\underline{z} < \hat{a}_2$

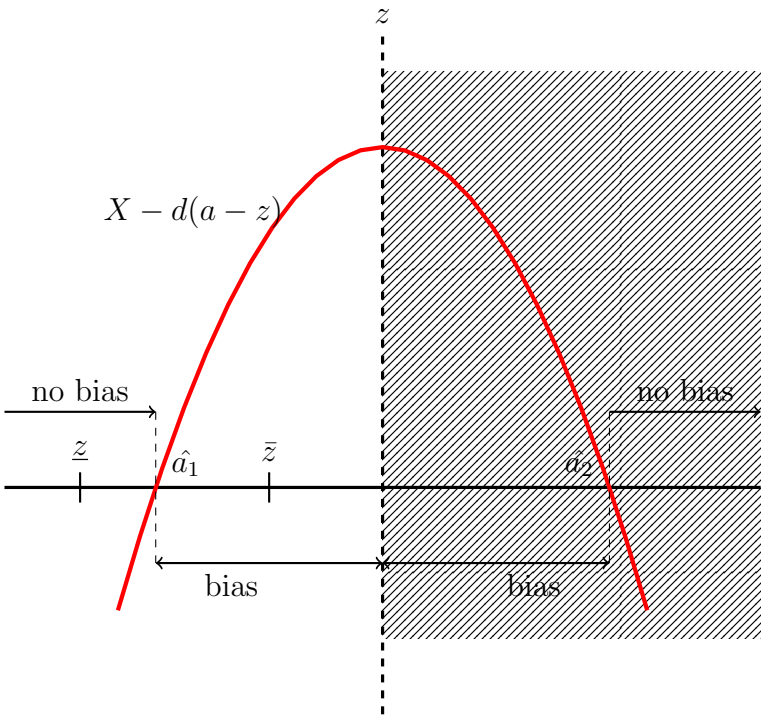



**Sub-case 2 :** $\underline{z} > \hat{a}_2$

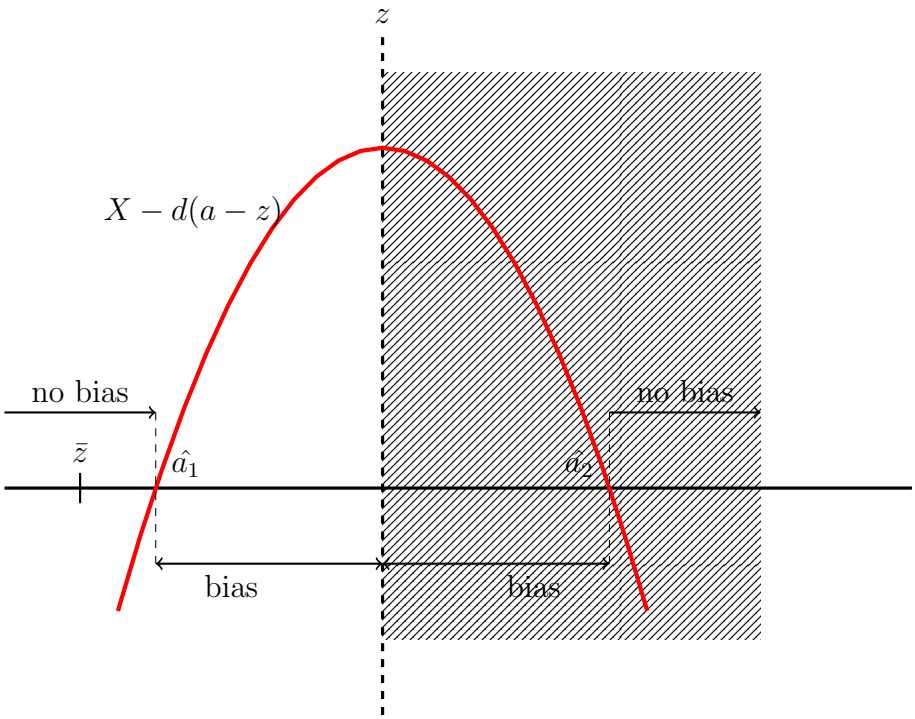



**Case ii :** $z > \bar{z}$

**Sub-case 3 :** $\bar{z} < \hat{a}_1$




**Sub-case 4 :** $\bar{z} > \hat{a}_1$


$z$
$X - d(a - z)$
no bias
no bias
$\bar{z}$
$\hat{a}_1$
$\hat{a}_2$
bias
bias