CENTRE DE RECHERCHE SUR LES STRATÉGIES ÉCONOMIQUES

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June 2020

Working paper No. 2020 – 04

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June 18, 2020

Abstract

We analyze how environmental taxes should be optimally levied when the regulators and firms face costs uncertainties in a Stackelberg-Cournot game. We allow linear-quadratic payoffs functions coupled with an affine information structure encompassing common and private information with noisy signals. In the first period, the regulator chooses the intensity of emissions taxes in order to reduce externalities. In the second period, facing industry-related and firm-specific shocks, firms compete in the marketplace as Cournot rivals and choose outputs. We show that, given costs uncertainties with non-uniform quality of signals across firms, the regulator sets differentiated tax policy. We also examined the social value of information under ex-ante calibrated emissions taxes. We argue that the magnitude of the associated social benefits and costs of more precise private signals hinge largely and fundamentally on the value of the ratio of the slopes of the marginal damage and the marginal consumer surplus. The lack of accurate data clouds the regulatory process by preventing the necessary fine-tuning of the tax rules towards specific environmental circumstances. Finally, we investigate information sharing between polluters and its impacts on welfare. We stress that, when there are threats of severe environmental damages under deep uncertainties, collusion is welfare reducing and may jeopardize the regulatory process. Numerical simulations illustrate the results that the model delivers.

JEL Classification Numbers: D43, D83, H23, L13, Q58.

Keywords: Differentiated Taxes, Costs Uncertainties, Signaling, Precision, Information Sharing, Collusion, Energy Markets.

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Introduction

Environmental economists have considered several broad classes of instruments to correct negative externalities. Pricing pollution is typically the most economically efficient way to drive down emissions and has some economic advantages over purely legal-based regulatory actions. To do so, public authorities may put in place a price-based and/or quantity-based regulatory policy, the main purpose being to give the appropriate incentives to polluters in managing their externalities. By explicitly pricing the pollution externality, such instruments provide clear and strong incentives to polluters to seek out and exploit the lowest cost ways of reducing emissions.[1]

Environmental taxation is a market-based approach designed to regulate emissions and protect environmental quality. Environmental taxes directly set a price on pollution. Following decades of large-scale implementation and experimentation, it is widely accepted today that environmental taxes represent a cost-effective regulatory mechanism to steer agents' behavior toward a greener economy. In general, polluters have no sufficient incentives to manage their emissions effectively if they don't face the social costs of their actions. Well designed, environmental taxes alter relative prices, lead to an adjustment in polluter's behavior, and tackle pollution issues.[2]

Not surprisingly, under idealized conditions, the tax-setting task can be straightforward optimally achieved. If public authorities could observe perfectly all sources of information at the time the policy is set, then they could easily impose well-structured direct taxes on emissions that maximize social welfare. Such taxes bring private costs into line with social costs and thereby achieve the efficient levels of production and emissions. Unfortunately, the perfect information framework ignores some important features of real-world policy settings and many existing tax policies appear to fall short of the theoretical ideal. Such externality-correcting taxes are clearly not plausible in a real world characterized by large informational distortions and uncertainties.[3]

[1]Our paper does not fit within the prices vs. quantities dichotomy. Our focus is not on ranking the most efficient policy choice to put a price on pollution under cost uncertainties. There is actually a fair-sized literature on the optimal policy tools to tackle pollution problems in the presence of cost uncertainties. Since Weitzman's seminal work (1974), many environmental economists analyzed the typical instrument choice, between the use of prices, quantities, or a combination of both. See, e.g., Aldy and Stavins (2012), Goulder and Parry (2008), Goulder and Schein (2013), Muller and Mendelsohn (2009), Nordhaus (2007), Pizer (2002), Pizer and Prest (2020), Stavins (2020), and the many further references cited therein.

[2]There is a vast array of economic analysis that investigates emissions taxes to achieve environmental objectives in the most efficient way. The stragegy in this literature is to determine the least-cost approaches to achieve a given objective. See, e.g., Aldy (2017), Carattini et al. (2018), Metcalf and Weisbach (2009), Newell and Stavins (2003), Nordhaus (2007), Stiglitz et al. (2017), and the many further references cited therein. Some references explored policy design elements without going through modeling exercises or the formal analysis of this paper.

[3]While pollution taxes can deliver socially efficient environmental protection and substantial associated

Even if environmental taxes are well understood and experienced today, processes of setting are shrouded with uncertainties and informational asymmetries. Environmental regulators do not know as much about regulated firms' abatement and production costs as do the firms themselves. Firms typically know their marginal costs with greater precision than the regulator. In addition, they may not have incentives to reveal their true marginal costs to the regulator and to their rivals. This informational asymmetry is one of the great difficulties of policy making because firms have both the opportunity and the incentive to exploit their informational advantages to undermine the intended goals of a well-meaning regulator. Further, at the firm level, the production process itself involves considerable uncertainties, which indirectly affects the efficiency of emissions taxes. Firms are buffeted by various kinds of shocks to costs, and their ex-ante estimates of production and abatement costs differ, for many reasons, from the ex-post observations.[4] Hence, firms' production adjustment decisions are dependent on receiving information about costs shocks and the associated estimates.

Motivated by these facts and to account for real-world complications, this paper seeks to overcome some identified weaknesses in the design of existing emissions taxes, and to identify ways that help environmental regulators to set more effective taxes. Our purpose is to refine our understanding of pollution taxes, their performance and implementation by confining our attention to informational uncertainties about costs. We look at answering the following questions. Facing industry-wide and private shocks, what are the properties of the optimal tax policy in uncertain polluting industries? What role does precision of privately-held information play in pollution problems? How does the precision of information affect social welfare? What is the social value of information in regulating emissions through taxes?

To this end, our work builds on a growing literature that characterizes efficient emissions taxes under costs uncertainties. We develop a two-period Stackelberg-Cournot game with uncertainties about the state of the world, which allows us to analyze pollution behavior and the properties of firms' production strategies in informationally complex markets. In the first period the regulator has no information at the time polluters make their decisions, nor can collect the information that is dispersed among them. Before observing the actual

health benefits, it is often administratively impossible, technologically too costly, or politically infeasible to price actions according to the externalities that they generate. Although these issues and other non-purely-economic matters are important in any tax-setting process, they are beyond the scope of this paper. For a balanced overall discussion of theoretical and practical issues concerning taxes, see notably Metcalf and Weisbach (2009) and the review paper of Goulder and Schein (2013) and the further references that they cite.

[4] In an empirical study where 28 different regulations in the United States are reviewed, Harrington et al. (2000) showed that generally, the difference between ex-ante and ex-post cost estimates is large. They highlight that ex-ante unit costs of production and abatement are inaccurate if technological innovation is not accounted for or cost information is out-of-date. Further, ex-ante costs of a complete abatement program can be over or underestimated for the same reasons (Elofsson, 2007).

industry outputs, it can only commit to tax schedules that are contingent on signals that will be available to firms in the second period. The intensity of emissions taxes is designed to maximize the expected social welfare and to reduce negative externalities. Then, in the second period, facing industry-related shocks (common signal) and firm-specific shocks (private signals), firms compete in the marketplace as Cournot rivals and choose outputs. This setting allows us to focus on the regulator's information deficit and on the strategic behavior of rival firms when unraveling information from signals and to analyze the influence of common and privately-held signals on the efficiency of the regulatory instrument. In addition, we can also consider to what extent the signals are informative and how varying the noise in signals impacts welfare.

A large literature spanning the field of environmental economics has investigated emissions taxes under different informational structures (See for example, Christiansen and Smith, 2015; Elofsson, 2007; Espínola-Arredondo and Muñoz-García, 2015; Fikru and Gautier, 2016; Goulder and Parry, 2008; Heuson, 2010; Ikefuji et al., 2016; Mideksa and Weitzman, 2019; Pindyck, 2007; Pizer, 2002; Stavins, 1996, 2020; Weitzman, 1974; Zhao, 2003).[5] Here we set-up a model based on signaling games as in Elnaboulsi et al. (2018). We adopt a linear-quadratic framework. We have relied on assuming linear demand and marginal costs functions to keep our model tractable and to avoid mathematical complications. In addition, we put a high premium on statistical inferences in setting the tax policy to handle uncertainties by considering linear conditional expectations.

Firms in polluting industries are buffered by shocks to costs and may learn about through common and private observations. Here, we consider that firms receive private signals about the private-value shocks and industry-wide signals about shocks that have a common value nature. We assume that the expectations of the true state of the world conditional on the signals is linear in the signals. The linearity of conditional expectations is substantive. When a firm receives shocks on costs, it can form a conditional expectation about its rivals' costs. In fact, based on its own expected marginal costs, each firm in the polluting industry can form expectations about others' costs, which can be achieved, for instance, through released data on previous years production in markets, e.g. energy, subject to government-required information disclosure, or on emissions of facilities covered by mandatory public programs.

While the previous literature usually considers symmetric and normally distributed signals and shocks, in this paper we provide a richer information structure. First, we step away from Gaussian setting by allowing signals to have some interesting arbitrary probability dis-

[5] Emissions taxes have been explored in depth conceptually. Given the huge number of published papers on environmental taxation under costs uncertainties, we have only included some references that we have subjectively judge to be most relevant to this paper. Some references contain formal arguments in favor of differentiated taxes that parallel the argument developed here.

tributions. Then, we introduce asymmetric signals in a Cournot polluting industry subject to emissions taxes. Thus, private-cost shocks to a given firm have a private value nature, which means that polluters may, to some extent, determine their information structure. In this way, we allow for essentially different quality precision among private signals involved in the model. Finally, in the vein of the literature focusing on information sharing, we extend our model by considering the case where polluting firms subject to environmental regulation collude under cost-sharing agreements (See for instance, Amir et al., 2010; Currarini and Feri, 2015; Evans and Weninger, 2014; Gal-Or, 1986; Ganuza and Jansen, 2013; Li, 1985; Shapiro; 1986; Vives, 1988, 1990). Here, we try to answer a simple question whether or not firms have incentives to pool their information about costs. We also investigate firms' incentives to share their information. To this end, we assume that the full vector of costs is shared perfectly by every one and can be accessed through different channels.

The equilibrium notion we use is that of subgame-perfect Nash equilibrium. We proceed in the usual way by solving the second-period production issue first, and then solve the first-period tax setting policy. In oligopolistic industries, firms must account for the strategic behavior of rival firms when unraveling information from signals, and they must account for how their own action influence the expectations of signals that rivals receive, and hence their inferences and output choices. Thus, this strategic interaction between players in the industry may yield positive information externalities. Given linear conjectures about a rival's production set at the second period, we solve a firm's optimization problem to obtain consistent linear outputs, and then we determine how emissions taxes should be optimally levied in the first period of the game.

Our model deliver some interesting positive properties in the design of emission taxes. Consistent with earlier works, the first contribution is to show that there exists a unique linear Bayesian Nash equilibrium.[6] We argue that, to overcome some of environmental challenges, instead of setting the same tax rate on regulated emissions, public authorities may calibrate differentiated rates to reflect costs uncertainties and the resulting source-specific damages (Elnaboulsi et al., 2018; Fowlie, 2010; Fowlie and Muller, 2019; Muller and Mendelsohn, 2009). When ex-post realized costs manifest differently than expected, implementing a differentiated tax policy may achieve the socially efficient outcome. Intuitively, this is because differentiated taxes are calibrated to reflect source-specific costs and the associated damages. Since biases are not uniform across firms within an industry in the real world, policy incentives should reflect differences in cost signals. Otherwise, setting the same tax

[6]See for instance, Bernhardt and Taub (2015), Currarini and Feri (2015), Gal-Or (1986), Ganuza and Penalva (2010), Hurkens (2014), Jeitschko et al. (2018), Lambert et al. (2018), Shapiro (1986), and Vives (2011, 2017).

rate can jeopardize the regulatory approach: the tax rate will be systematically too low on dirty firms and too high on clean firms. Thus, this result provides regulators an understanding of how such tax policy can be used in the presence of uncertainties about costs and the uncertain associated environmental damages.

It is worth noting that regulators' decisions are contested in court and the existing tax policies are increasingly more subject to contentious debate today. Legislators in many countries are questioning the effectiveness of the proposed tax-based instruments. The main reason is that actual taxes are failing to adequately accommodate non-uniformly expected production and abatement costs and the associated damages.[7] Being aware of the uncertainties resulting from different non-uniform shocks will help to strengthen future environmental decision-making. Regulators may rely on and take advantage of a large array of public and private industry-specific available data sets in the course of administrating differentiated emissions taxes, which can be an answer to questions about how to refine tax-based policy designs to account for non-uniform costs signals. In the presence of privacy and confidentiality concerns in some strategic industrial polluting sectors, we stress the need to review the best available information regarding industry-specific variables to support a reasonable modeling framework for analyzing costs, emissions, damages, and other impacts of regulatory decisions. For instance, the U.S. government collects high-frequency data on sulfur dioxide (SO_2) emissions from power plants. Thus, a SO_2 sector-specific emission tax could be implemented based on these monitored data, emissions inputs, or some other related measures.[8]

The second contribution of this paper is to analyze welfare properties when the regulator tailors environmental prices through taxes in uncertain industries. To this end, we examine the value of information to decision makers under ex-ante calibrated emissions taxes by using

[7] For instance, in a 2014 decision concerning air quality problems involving nitrogen oxides (NO_x) and sulfur dioxides (SO_2), the United States Supreme Court acknowledged the significant and inherent uncertainties in the estimation of pollution damages and differences that can arise between ex-ante expected versus realized production and abatement costs. The Court ruled that regulations limiting harmful emissions should proceed, notwithstanding these uncertainties surrounding pollution damages and production and abatement costs. Furthermore, the Court found that uncertainties should be considered in the design of any regulatory instrument limiting harmful emissions (The US Supreme Court on *EPA v. EME Homer City Generation, L.P.*, no. 12-1182, April 29, 2014). Clearly, under the U.S. legal system, any regulatory instrument could not ignore the relationship between sources and receptors in matters involving air quality standards (Schmalensee and Stavins, 2013).

[8] Parry and Small (2005) offer another example. They note that some externalities associated with driving light-duty vehicles are a function of fuel consumption, e.g., CO_2 emissions, while others are a function of vehicle miles traveled such as accidents or local air pollution. As a result, implementing a simple gasoline tax will not ensure that drivers bear the accurate social cost of driving. In their analyses of the externalities from driving in the United States, they found that a vehicle miles traveled tax yields about four times greater welfare gain than the optimal gasoline tax. Thus, an ex-ante differentiated tax policy will deliver clear and strong incentives to buy environment-friendly cars.

precision criteria. We investigate how signals' precision affects social welfare. We find that, under some environmental circumstances, uncertain industries are socially less harmful than deterministic ones if cost uncertainty is privately observed.

One would hope that improvements in the precision of information would increase the expected welfare. However, the magnitude of the associated benefits and costs of more precise signals depends on the severity of environmental damages. Our results hinge largely and fundamentally on the value of the ratio of the slopes of the marginal environmental damage and the marginal consumer surplus. We find that, in the context where marginal costs involve uncertainties with private information at the moment the policy is set, less precise signals are welfare enhancing. Uninformative signals yields industry-wide overproduction which entails an increase in consumer surplus and emissions. But the implied advantages of imprecise information offset the resulting environmental damages. Hence, if environmental impacts are less adverse than anticipated, i.e., damages are "acceptable", then the regulator must be lenient with such industry-wide overproduction. With cost uncertainty, policy discretion would be highly desirable.

Having said that, however, when there are threats of serious environmental damages, more precise information is welfare enhancing. This is particularly the case when the regulator is dealing with irreversible loss, and when it is uncertain about the likelihood of that loss. Our findings confirm the clear-eyed assessment according to which there is no reason for postponing any regulatory policy to correct externalities. Under deep uncertainties, environmental taxation should be precautionary: taxes must be set accordingly in order to avoid irreversible damages and diminish environmental harm, including threats to human life or health. In this case, well informed polluting firms have no incentives to increase their outputs in order to avoid the burden of the tax, which yields a reduction in emissions. Therefore, more precise information leads to higher social welfare. Numerical results show how welfare varies with signals precision under different adverse impacts of pollution. When environmental damages are expected to be high, then precise signals boost social welfare.

Our paper also contributes to the ongoing policy debate on collusion between polluters by means of truthful sharing agreements about costs value, which may cloud the regulatory process. Information sharing, even if mandated through regulations, could increase transparency to an excessive extent, and eventually set the basis for anti-competitive behavior. We asked a simple question whether or not firms have incentives to pool the information about their marginal costs. Then, we investigate the welfare consequences of concerted practices between players within an industry. Our model offers some useful insights.

First, we show that, due to the prevailing uncertainties and the regulator's information deficit about costs, the optimal tax rules is the same as in the non-sharing game, which is quite intuitive. Then, we show that, when the uncertainty is characterized by privately-held information, collusion through cost sharing agreements is a dominant strategy in highly uncertain industries.[9] Less informed firms have greater incentives to collude than well informed ones. Numerical simulations highlight the intuition of how the magnitude of marginal costs impacts the decision to adopt collusive behaviors.

Finally, the main result we draw from the welfare analysis of anti-competitive behaviors is that, in settings where firms face environmental regulation trough taxes, the policy conclusion is quite ambiguous and hinges on the nature of environmental damages. While information sharing in Cournot oligopoly markets without externalities seems to be unambiguously welfare enhancing, collusion that would arise from cost sharing agreements is better for social welfare if and only if the slope of the marginal environmental damages is less than the slope of the marginal benefits for consumers.[10] Collusion creates positive externalities and generates information gains due to information aggregation and the subsequent production rationalization. Hence, information sharing should be treated more leniently under Cournot competition with environmental externalities. However, there is a range of situations where collusive behaviors may be welfare reducing. This is the case that emerges when the slope of the marginal damages is higher than the slope of the marginal benefits for consumers. This is true when the economy faces extreme environmental circumstances, which implies that the resulting damages are large enough to shave off all efficiency gains from information-pooling. Increasingly stringent measures have to be taken in order to reduce emissions. Further, the regulator should prohibit any exchange of information between Cournot oligopolist to avoid welfare loss.

We set up an analytically-tractable model with the emphasis on clarity of exposition. Yet, the model embodies enough of real-world issues to give some useful insights on environmental regulation. The structure of our model is flexible and may characterize several polluting industries in which firms are engaged in a competition à la Cournot, at least locally, and are shrouded with perpetual uncertainties about costs. The regulator's uncertainty about the industry shocks can lead to inefficient policies when setting taxes to drive down emissions and to correct negative externalities. For instance, the model may represent an industry, such as the U.S. energy sector or the European wholesale energy market, where an homogeneous product, i.e., electricity, can be produced with different energy inputs. Some

[9] As in Shapiro (1986) for instance, the expected profits from pooling of information are larger than those of remaining independent. Hence, collusion is a dominant strategy.

[10] Here, we recover a well-known result showing that collusion is welfare improving due to large production rationalization effect of cost information exchange (Vives, 1988, 1990). Amir et al. (2010) obtain robust and clear-cut results for the incentives and welfare effects of information sharing when information is firm-specific in industries without environmental externalities.

inputs, e.g., fossil fuels sources, entail low marginal production and abatement costs and generate considerable Green-house gas (GHG) emissions. Others, like non-fossil renewable energy sources, have higher marginal production and abatement costs and implies lower GHG emissions. Polluters have no sufficient incentives to manage their emissions effectively if they don't face the social costs of their actions. To avoid more severe interferences with the climate system, it is important to set an effective regulatory policy. Before observing the actual industry production, the regulator commits to price-based policy, which is crucial to reduce carbon emissions. It sets, for example, contingency-based emissions taxes in order to correct negative externalities arising from the energy market (e.g., SO_2/CO_2 emissions). In the electricity markets, the production costs can, to some extent, be estimated from data of plants and some price indexes. Still, an operator has private information about some inputs and how a power unit is operated in details. Thus, well designed, carbon taxes alter relative prices and lead to an adjustment in agents' behavior. Unfortunately, GHG emissions are not priced adequately. About half of the emissions covered by carbon pricing initiatives are still priced almost US $10 per ton of carbon dioxide equivalent, way below the needed level to curb global emissions (Carattini et al., 2018; Carattini et al., 2019; World Bank, 2019).

The remainder of the paper proceeds as follows. The next section presents evidence of real-world costs uncertainties. Section 2 lays out the model set-up with shocks on costs that have common and private values nature. Section 3 solves the model and delivers the tax equilibrium. Section 4 presents detailed analysis of welfare and the social impacts of signals' precision. Information sharing between polluters and its effects on the regulatory policy are examined in section 5. Finally, conclusion summarizes the paper and offers directions for future research. All proofs are presented in the appendix.

1 Real-world uncertainties

In real-world polluting industries, firms face persistent and perpetual cost uncertainties, which implies potentially significant forecast errors and hence, makes them fail to deliver cost-effective decisions. Such uncertainties also cloud the regulators' task in setting efficient tax policies because they might err not only in their expectations, but also in their beliefs about firms' decisions, as the process depends on many different random variables, only some of which are observable to individual polluters.[11]

When it comes to deal with environmental issues, it is hard to address all sources of un-

[11] In the electricity sector, for instance, uncertainties include the cost of fuels, the cost to operate existing power plants, the cost to construct and operate new power plants and infrastructure, demand, and policies affecting the electric power sector. One may add uncertainty in future economic conditions that could affect fuel supplies, technology costs, and electricity demand in the electricity sector.

certainty. There is a considerable body of work focusing on the regulator's information deficit and treating environmental matters under cost-related shocks or other issues that affect uncertainty of a firm's economic context. Here, we survey briefly some sources of uncertainty that we subjectively judge relevant to this paper. Much remains unknown regarding the sources of uncertainties about costs.

For different reasons, firms may simply not know with certainty their own costs function. As highlighted by Weitzman (1974), engineers may fail to understand the magnitude of their task, which entails systematic errors in costs estimates. Uncertainties also reflect different kind of firms' expectations about production and abatement costs,[12] newly introduced products,[13] technological choice which is, after all, notoriously difficult to predict, creating errors about future costs,[14] drifts and volatility of some inputs and energy commodities (whether oil, natural gas or coal) prices implying an increase in pollution-intensive output,[15]... Cost uncertainties may further be exacerbated by political risks and swings, regulatory regimes and producers subjective probabilities of some exogenous stochastic economic shocks.

Firms routinely and purposely also overestimate or underestimate their costs for strategic reasons. For instance, firms may internally formulate different expectations over carbon prices. According to the Carbon Disclosure Project, firms have deliberately and actively employed quite varying expectations over carbon prices in their investment analysis and strategic planning: the average carbon price among U.S. firms disclosing use of an internal carbon price in their operations was around \$40 per ton of CO_2 in 2017, with a standard deviation of \$33 per ton of CO_2 (Aldy and Gianfrate, 2019).[16]

[12]For instance, carbon prices reveal occasional spikes and troughs, and do not follow what would be interpreted, even loosely, as clear price path. Under the E.U. Emissions Trading System, emission allowances were trading at €5 per ton of carbon dioxide in 2017, but jumped to more than €30 per ton in 2019. At the same time, Sweden faces a separate carbon tax greater than €90 per ton. Such volatility in prices creates significant forecast errors, and hence uncertain trajectory of inputs prices and the implied costs.

[13]For example, in the chemical industry, when a new product is released, it is uncertain whether the product will turn out to be toxic in the future. If it is found to be toxic, firms are eventually expected to pay a certain liability based on the some toxicity indicators. Though, this uncertainty enters into the cost function.

[14]For instance, given uncertainty over inputs expected prices such as the rate of interest or the price of carbon in a tradable allowance market, it is difficult to derive the necessary optimally conditions for firms to invest in irreversible long-lived abatement technologies. In the absence of a clear price path, firms may not be able to smooth their rate of investment over time due to the prevailing forcast errors.

[15]As an example, under the California Regional Clean Air Incentives Market program (RECLAIM), NO_x allowance prices increase from about US \$1,000 per ton in 1999, to more than US \$20,000 per ton in 2000, to more than US \$120,000 per ton in 2001. This dramatic run-up resulted from the electricity crises when existing generators failed to meet demand. During the crisis, California power companies relied much more often on the dirtiest generators to meet occasional peak load (Fowlie et al., 2012).

[16]This average does not include the mass of firms that implicitly use a price of zero and do not participate to the disclosure initiative. Even within the same industry and country, there is substantial difference in internal estimations depending on the company's objectives. For instance, in 2017, ExxonMobil used \$80 per ton of CO_2, ConocoPhillips used \$43 while Devon Energy used only \$24 per ton.

Further, with the onset of global warming, firms consider climate risk and the related regulatory policies into their calculations due to the prevailing adverse shocks such as severe hurricanes, heat waves, fires, and droughts and their impact on costs. Firms have to gauge the impact of regulatory changes and assess exposure to the increasing climate risk. Managing these uncertainties is similar to managing other financial risks and compliance risks, which implies frequent errors in firms' expectations about costs.

These uncertainties suggesting that firms' perceptions about costs may be substantial, do not end there. Overlapping and new environmental regulations are also a major source of uncertainties, which implies difficulties to characterize the expected benefits and costs of the regulation.[17] Under some circumstances, while new mandates might be advantageous to at least some firms in an industry to overestimate costs, others would very likely call for a cost underestimate (Harrington et al., 2000). In addition, new regulations, e.g., air quality mandates, drive firms to change their operations in order to reduce pollution. Changes such as modifying the mix of inputs altering production, impose costs on the firm. These costs may spur innovative activity and reward other firms which produce lower-emitting technologies and processes. Further, as highlighted in Goulder and Stavins (2011), implementing specific environmental provisions may cause firms to modify their investment decisions in green technologies or in new pollution control equipment to satisfy the mandate. This would affect firms' expected marginal costs to comply with other public programs.

A prominent example is the overlapping nature of energy and climate policies at the national and regional levels. For instance, the European climate change program includes, among other things, a set of policies to address GHG emissions from electricity generation. The E.U. Emissions Trading Scheme is the major policy instrument in this respect. However, it is combined with electricity taxes, support schemes for renewable energy sources and measures promoting energy efficiency. The regional policy is complemented by domestic measures (Lehmann, 2012). Many European countries employ a wide array of climate-oriented energy policies such as renewable energy subsidies or efficiency resources standardized mandates, which request firms to undertake abatement investment decisions that would sacrifice cost effectiveness, and hence reduce the necessary efforts to comply with some other public programs, e.g., CO_2-reducing mandatory programs. Such overlapping degenerates, oftenly, into a policy mess and prevent firms to deploy least-cost compliance strategies.[18]

[17] Hahn and Tetlock (2008) provide a good discussion on how regulations and laws introduced by public authorities impose costs on firms. One may add uncertainty related to a pending public policy decision, or some other issues and adverse shocks affecting a firm's economic context.

[18] Another example of overlapping State regulations is the limited cost-effectiveness of Acid Rain programs in the United States to reduce SO_2 and NO_x emissions from coal-fired and electric generating plants, which induced higher compliance investment costs in scrubbers (Frey, 2013).

2 The model set-up

2.1 Basic elements

Demand and Consumer Surplus We consider a market where two non-identical risk-neutral firms, $N = \{i, j\}$, strategically compete in quantities, *à la Cournot*.[19] There is a continuum of consumers of the same type with a quadratic utility function which takes the form:[20]

$$u(q_i, q_j, r) \equiv aQ - \frac{b}{2}Q^2 + r; a, b > 0 \tag{1}$$

where $Q = (q_i + q_j)$ is the total output supplied by the industry and $r \geq 0$ is the *numéraire* commodity produced in an exogenous market and thus can be neglected throughout the further analysis. Consumers choose (q_i, q_j) so as to maximize their preferences, which gives rise to a linear demand structure. The implied inverse demand function is given by:

$$p(q_i + q_j) = a - b(q_i + q_j), \text{ for } i, j, \text{ with } i \neq j \tag{2}$$

where p designates the market-clearing price. In the following, we assume that the demand intercept a is sufficiently high to avoid shutdown.[21] The consumption of the bundle (q_i, q_j) gives consumers a net surplus of:

$$CS = \frac{b}{2}Q^2 \tag{3}$$

Firms' Production and Pollution The production of goods results in pollution that can be reduced by investing in abatement. Firms are subject to some liability rule defining the compensation rate on emissions. Facing a per-unit of emissions pollution tax, $\tau_i > 0$, a firm i can undertake environment-friendly measures or may vary the level of its production to reduce emissions, and thus the tax burden.

We assume that firm i's net emissions are additively separable in output and abatement efforts.[22] Thus, if producer i chooses output level q_i and abatement level x_i, net emissions by firm i are defined by:

$$e_i(q_i, x_i) = q_i - x_i \tag{4}$$

[19] The basic structure and notations of the model are in line with Elnaboulsi et al. (2018). The set-up can be extended to $n-$olibopoly.

[20] This specification is oftenly used in the relevant literature to generate linear demand functions (see for instance, Angeletos and Pavan, 2007; Bayona, 2018; Myatt and Wallace, 2015).

[21] In addition, this allows us to ignore the nonnegativity constraint on prices (and quantities), and hence we entirely deal with linear Bayesian Nash equilibria (Hurkens, 2014).

[22] This formulation is frequently used in the literature for tractability. Petrakis and Xepapadeas (2003) showed that the results from an additively seperable function are robust to the more general model in which emissions are proportional to output.

The variable $x_i \in [0, q_i)$ is the firm's effective investment in abatement technologies. In the following, we simplify our formulation by considering that emissions by firm i, e_i, are proportional to firm output q_i:

$$e_i = \varphi q_i;\ \forall i \in N, 0 < \varphi < 1 \tag{5}$$

where φ is the constant marginal damage per unit of output. One would have consider that this parameter varies with i, $i \in N$. However, we have relied on simplifying the model by assuming that φ is the same across firms within an industry, to avoid mathematical complications.[23] Since investment decisions are not considered in the paper, this assumption simplifies the exposition of the results and thus we can focus on costs uncertainties and the effects of varying the informativeness of private signals on the expected optimal environmental policy. In addition, considering different φ does not change qualitatively our results.

Following standard convention, we consider that environmental damages generated by the industry are given by the following quadratic convex function:

$$D = \frac{1}{2} dE^2;\ d > 0 \tag{6}$$

where $E \equiv \sum_{i \in N} e_i$ represents the aggregate level of emissions. The positive coefficient d is an exogenous variable that captures the steepness of marginal damages of emissions among firms or equivalently the degree of convexity of the damage function. This type of damage function is commonly used in the relevant literature and assumes that a marginal increase in output entails a positive and increasing environmental damages. Note that the damage function is exogenous for consumers, i.e., they do not take into account the effect of their consumption decisions on the environment.

The regulator As we stated before, the risk-neutral social planner seeks to set an environmental tax τ_i, based on emissions, for i, j, with $i \neq j$. To do so optimally, the regulator maximizes the following expected social welfare function which includes the expected consumer surplus, $\mathbb{E}(\widetilde{CS})$, firms' expected profits, $\sum_{i \in N} \mathbb{E}(\widetilde{\pi}_i)$, and the government's total expected revenues generated by the compensation rule on the remaining emissions, $\mathbb{E}(\widetilde{R})$,

[23] φ may also represent the investment coefficient of emissions reduction, e.g., carbon reduction. Note that, the modeling approach is more appropriate for industries where technologies are similar but not necessarily identical.

minus the expected value of environmental damage due to firms' production process, $\mathbb{E}(\widetilde{D})$:[24]

$$\max_{<\tau_i,\tau_j>} \mathbb{E}_{\widetilde{c}_i,\widetilde{c}_j}\left[\widetilde{W}\left(\tau_i,\tau_j\right)\right] = \max_{<\tau_i,\tau_j>} \mathbb{E}_{\widetilde{c}_i,\widetilde{c}_j}\left[\left(\widetilde{CS}-\widetilde{D}\right)+\sum_{i\in N}\mathbb{E}(\widetilde{\pi}_i)+\ell\ \mathbb{E}(\widetilde{R})\right] \tag{7}$$

where ℓ is a positive parameter representing the marginal social benefit of public revenues.[25] It can also be seen as the marginal cost of public funds. Empirical studies found that $1 < \ell < 2$. In the following, we restrict our analysis to this empirically relevant range (See for instance, Elnaboulsi et al., 2018).

The pollution-tax game can be described as a Stackelberg game where the regulator is the *leader* and firms are the *followers*. In the first stage, before observing firms' output decision and without knowing beforehand the realization of the random variables, the regulator sets $\tau_{i,i\in N}$ in order to maximize the expected welfare. In the second stage, given $\tau_{i,i\in N}$, firms compete as Cournot rivals and decide simultaneously the level of production and the implied emissions in order to maximize the expected profits.

2.2 Information structure

We assume that the technology used by each player in the marketplace is stochastic but it exhibits constant returns to scale. This means that, for a given state of the world, the marginal production and abatement cost, c, is constant.[26] Linear marginal costs are compatible with the framework adopted in our work, and allow us to focus on the impact of information precision on the intensity of the tax rate and on welfare.

Our model features common and private information with noisy signals. Before choosing their production strategies, firms face the same prospects and have access to some common and private information values about marginal costs. Some shocks are commonly observed by all firms such as such as industry-wide shocks. Other shocks are private in nature, e.g., an idiosyncratic cost shock specific to one firm's technology affecting indirectly a rival in the industry. Hence, we consider that each firm receives unbiased noisy estimates of its uncertain

[24] $\mathbb{E}[\cdot]$ and $\mathbb{E}[\cdot|\widetilde{c}_1,\widetilde{c}_2]$ denote the expectation and conditional expectation operators. Note that random variables are denoted with a tilde while realized values lack the tilde. The mean of a random variable is denoted with bar.

[25] The indirect social benefit of environmental taxation is known as the "second dividend" according to Goulder's definition (Goulder, 1995). Pollution taxes may generate substantial revenue that can be used to meet existing fiscal needs or to lower the burden of distortionary taxes that weight on the rest of the economy, such as on labor and capital.

[26] As an example, the cost parameter c could be a unit ex-post pollution damage that is assessed on a firm, say an electricity generator, and for which the firm has common and private estimates before submitting its supply function (See for instance, Vives, 2017).

marginal cost, such that:

$$\widetilde{c}_i = \widetilde{s} + \widetilde{\varepsilon}_i, \text{ for } i, j, \text{ with } i \neq j. \quad (8)$$

The first component, $\widetilde{s}$, is a positive random variable and represents the industry-related shocks. We assume that this common cost component is the same for both firms. In addition, we suppose that $\widetilde{s}$ is distributed according to some prior density with mean μ_s and finite variance $\sigma_s^2 \in \mathbb{R}_+$, i.e., $\widetilde{s} \sim (\mu_s, \sigma_s^2)$. Let $\frac{1}{\sigma_s^2}$ denotes the precision of $\widetilde{s}$. Hence, lower value of σ_s^2 (respectively higher) means that all firms are more (less) informed about the magnitude of the true value of the common component of the marginal cost.

Industry-related shocks to production could be related to the cost of some common inputs of production, or wages for instance. The random common statistics can also be dispersed data on outputs, investments or sales volumes that are collected, aggregated and processed by industry-related systems and trade associations platforms. For example, costs shocks caused by changes in energy prices and published by a trade association are normally common shocks. In some circumstances, public interventions are also source of persistent industry-wide shocks since new environmental regulation heavily affects common components of marginal costs.

The second component of the marginal cost, $\widetilde{\varepsilon}_i$, is also a random variable but it represents firm-specific shocks. It can be viewed as the remaining costs' uncertainties or noise terms that are not correlated across players. The realization of firm i's private signal, $\widetilde{\varepsilon}_i$, is not observable by player j as well as by the regulator. In the following, we consider that firms receive asymmetric quality of signals. For i, j, with $i \neq j$, $\widetilde{\varepsilon}_i$ is distributed according to some prior density with mean μ_{ε_i} and variance $\sigma_{\varepsilon_i}^2 \in \mathbb{R}_+$, i.e., $\widetilde{\varepsilon}_i \sim \left(\mu_{\varepsilon_i}, \sigma_{\varepsilon_i}^2\right)$. We assume that $\widetilde{\varepsilon}_i$ is uncorrelated with $\widetilde{s}$ and $\widetilde{\varepsilon}_j$, with $j \neq i$. Let us define $\frac{1}{\sigma_{\varepsilon_i}^2}$ as the the precision of $\widetilde{\varepsilon}_i$: it measures the amount of information firm i is to receive.

There can be several situations where a given firm may not have complete information about its own costs, and hence production decisions are based on expected costs. Some costs components are firm-specific shocks. For instance, production decisions in some industries are taken before production contracts are signed and input costs are known, e.g., energy supply negotiated contracts. Further, some ex-post emissions damage that is assessed to a firm would be a source of private cost shock. In addition, a player in the industry may developed an environment-friendly innovative process, e.g., energy-saving R&D, and it may not be clear how this innovation is cost-reducing. Finally, there may be incomplete information about geological or meteorological conditions for firms that extract natural resources, i.e., raw resources used in the production of energy. For example, while the costs of exploration of crude oil are considered as common to the industry, the quality of new oilfields and hence the related costs are likely to be only privately known.

Our framework follows the large literature on signaling, but provides a rich asymmetric information structure in the analysis of environmental taxes in uncertain industries: it allows each firm to have a privileged view of its own sliver of the world, and successfully hide part of its marginal cost due to the nature of emissions or some other factors; and most importantly, we consider that firm-specific noise terms have different variances, $\mathrm{var}(\widetilde{\varepsilon}_i) \neq \mathrm{var}(\widetilde{\varepsilon}_j), \forall i \neq j$, which allows us for asymmetries in firms' payoff structure. Thus, we do not impose symmetry on the quality of signals since the precision of i possibly differs from the precision of j, which is crucial when firm i is better informed about costs than j, i.e., $\sigma^2_{\varepsilon_i} < \sigma^2_{\varepsilon_j}$.

2.3 Timing

Having described the information structure, the sequence of events and actions is as follows:

1. In the first stage of the game, the risk neutral regulator sets environmental taxes $\{\tau_i\}_{i\in I}$ optimally to maximize the expected welfare, $\mathbb{E}\left[\widetilde{W}\right]$. The regulatory instrument is made under informational constraint, i.e., before the realization of the random variables. Thus, the regulator's decision is made subject to prior expectations about the state of the world that will prevail later.

2. In the second stage of the game, the common component of the marginal costs $\widetilde{s}$, which represents the industry-wide shocks, is drawn randomly. This signal is observed by both players, but not the regulator.

3. In the third stage of the game, the private signal of the marginal cost $\{\widetilde{\varepsilon}_i\}_{i\in I}$ is drawn randomly. It may be considered as firm-specific shocks and is a private information: prior to taking any action, and after the realization of $\widetilde{\varepsilon}_i$, each player i privately observes its signal $\widetilde{c}_i$, but not the regulator neither the rival.

4. In the final stage, given the marginal costs and the intensity of environmental taxes chosen in the first stage, risk neutral firms are considered as Bayesian decision makers, acts simultaneously in the product market and choose their output and emission abatement levels. Then, payoffs are realized for each players.

Clearly, under the above framework decisions are made sequentially and subject to uncertainties about the eventual state of the world. Figure 1 models the finite horizon version of the game. It shows the information structure and decisions over time.

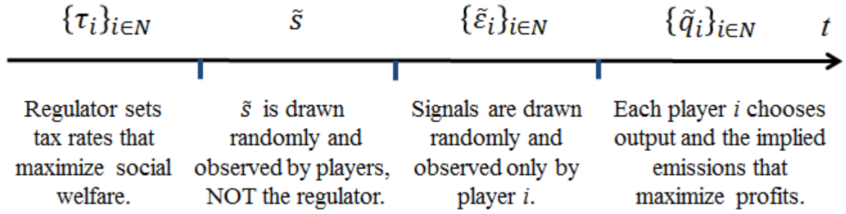

Figure 1: Timeline of events in a simple two-period tax game.

The structure of our model describes some but by no means all pollution problems, where the first period decision affects players' actions in the second production period. For instance some industrial activities are highly dependent on the relative price of inputs (e.g., gas, coal or pesticides), the business cycle, as well as on the weather. Firms learn about their costs over time, and therefore about their emissions, as uncertainties unfold. In addition, some players in the marketplace have access to different abatement technologies which are sometimes privately known. Further, the rapidly expansion of data on economic activity today creates new opportunities for players in the marketplace to conduct and evaluate strategic decisions.[27]

3 Equilibrium

3.1 Firms' Behavior

To compute the equilibrium, we proceed using backward induction: in the second period of the game, players comply with the instruments defined by the regulatory policy in place and compete *à la Cournot* in the marketplace given conjectures about rivals' costs; then we move back to the initial period where the regulator commits to a tax-based policy and choose the intensity of firm-specific emissions tax rule.

In the second period of the game, each firm makes its own decision based on unknown common and private signals that it receives. Some available and common information are relevant for production costs, such as released data on previous years production and emissions of facilities covered by public programs, weather conditions, actual fuel and carbon prices, etc. However, each player in its setting has some private information and conditions its output on the rival's private signals.

Lemma 1. *A firm can make inferences about the marginal cost of its rival based upon its private information. For every player i and each player j, with $i \neq j$, the conditional*

[27] The model may also be applied to large range of pollutants that affect a given period and then dissipate. For example, the release of some toxic chemicals might cause immediate harm and the chemical might then degrade or be absorbed in the environment.

expectation $\mathbb{E}[\tilde{c}_i|\tilde{c}_j]$ *is a linear function of* $\tilde{c}_j$ *and is given by:*

$$\mathbb{E}[\tilde{c}_i|\tilde{c}_j] = \gamma_j \tilde{c}_j + \lambda_j, \tag{9}$$

where $\gamma_j = \frac{\sigma_s^2}{\sigma_s^2 + \sigma_{\varepsilon_j}^2}$, $\lambda_j = \mu_i - \gamma_j \mu_j$, $\mu_i = \mu_s + \mu_{\varepsilon_i}$, *and* $\mu_j = \mu_s + \mu_{\varepsilon_j}$.

Proof. Using a result proved by Ericson (1969) and based on the Law of Iterated Expectations, it is easy to compute the posterior expectations $\mathbb{E}[\tilde{c}_i|\tilde{c}_j]$, i.e., firm i's expectations about firm j's signal. □

Lemma 1 implies that firm i's expected cost conditional on $\tilde{c}_j$ is linear, which yields an affine information structure. It simply states that, a firm in the marketplace can form expectations about others' costs based on its own marginal costs. This implies that signals are affiliated such that if the signal of one player increases, then it increases the probability that the competitor has a high signal relative to the probability that the competitor has a low signal. In other words, if one player has precise information, then the rival is more informed as well based on the inference effect. Thus, one firm's signal is relevant for its rival. After observing its cost signal, a given firm understands and internalizes how its action affects rivals' information set and output decisions. This creates positive externality in the industry.

Many interesting and common examples of joint distribution functions satisfy this property and give rise to an affine posterior expectation, e.g., multivariate normal distribution (see for example, Bernhardt and Taub, 2015; Ganuza and Penalva, 2010; Lambert et al., 2018; Vives, 2017). A large literature spanning multiple fields used this property of linearity in the analysis of a diverse variety of questions in economics, such as the value of macroeconomic information in financial markets (Morris and Shin, 2002), business cycles and large oligopoly games (Angeletos and Pavan, 2007), the structure of markets under price and quantities competition (Gal-Or, 1986; Li, 1985; Shapiro, 1986, Vives, 1988), monopolistic competitive firms (Hellwig, 2005), the properties of games on networks (Currarini and Feri, 2015; Myatt and Wallas, 2015, 2018), the value of public information in environmental regulation (Elnaboulsi et *al.*, 2018), among many others.

Under our setting, the linearity of conditional expectations is crucial to study the impact of signals' informativeness on welfare. However, beyond this property, we impose no restrictions on the nature of the probability distributions of firm-specific costs errors, i.e., the distribution of signals may be considered arbitrary.[28] By providing richer asymmetric

[28] Note that the linearity of conditional expectations allows us to step away from the assumption of joint normality of signals and parameters of the Cournot game with incomplete information. Further, this assumption makes it possible to circumvent negative quatities and prices issues that can arise under the Gaussian

information structure, it is possible to characterize the effect that information has on the dispersion of conditional expectations. The last expression can be written as:

$$\mathbb{E}[\tilde{c}_i|\tilde{c}_j] = \gamma_j\left(\tilde{c}_j - \mu_j\right) + \mu_i, \text{ for } i, j, \text{ with } i \neq j. \quad (10)$$

where γ_j refers to the relative precision of the private information held by firm j, keeping σ_s^2 implicitly fixed. Since $\sigma_{\varepsilon_i}^2$ ranges from 0 to ∞, γ_i ranges from 0 to 1, and represents the informativeness of the signal, i.e., the different qualities of the information sources. Thus, firm i is more informed than firm j if $\gamma_i > \gamma_j$, which implies that firm i will get better off: a firm generates positive revenues if it has access to more precise and accurate information sources.

The dispersion effect arises because the sensitivity of conditional expectations to the realized value of the signal depends on the informational content of the signal (Ganuza and Penalva, 2010). When firm j receives perfect information and observes its shock with no measurement errors $\left(\sigma_{\varepsilon_j}^2 = 0\right)$, then $\gamma_j = 1$ and the signal is said to be perfectly informative. In this case, a more accurate information structure leads to a more disperse distribution of the conditional expectation, $\mathbb{E}[\tilde{c}_i|\tilde{c}_j] = \tilde{c}_j - \mu_{\varepsilon_j} + \mu_{\varepsilon_i}$. When firm j receives no information $\left(\sigma_{\varepsilon_j}^2 \to \infty\right)$, then $\gamma_j \to 0$ and the signal is uninformative. In this case, if the informational content of j's signal is low, conditional expectations are concentrated around the ex-ante unconditional expected valuation, i.e., $\mathbb{E}[\tilde{c}_i|\tilde{c}_j] = \mu_i$. Thus, the inference effect of firm j on firm i is higher if firm j's signal is less informative. Firm i only relies on its μ_i to infer firm j's expected marginal costs. Note that, if firms are equally informed $\left(\sigma_{\varepsilon_i}^2 = \sigma_{\varepsilon_j}^2\right)$, then $\mathbb{E}[\tilde{c}_i|\tilde{c}_j] = \gamma\tilde{c}_j + \lambda$, where $\gamma = \frac{\sigma_s^2}{\sigma_s^2 + \sigma_\varepsilon^2}$ and $\lambda = (1-\gamma)\,\mu_c$. In this case, the information structure coincides exactly with the one adopted in Elnaboulsi et *al.* (2018).

At the second period of the game, conditional on the received signals and given the conjecture about the rival's production cost, firm i's problem is to choose $\widetilde{q}_i$ in order to maximize its profits:

$$\max_{<\widetilde{q}_i>} \mathbb{E}_{\widetilde{c}_j}\left[\left(\widetilde{p} - \widetilde{c}_i - \varphi\tau_i\right)\widetilde{q}_i\right], \text{ for } i, j, \text{ with } i \neq j. \quad (11)$$

Standards results can be used to establish the existence of a unique Bayesian Nash Equilibrium for the information structure given in this paper, with the equilibrium strategies are affine in the observed signals.[29] Hence, focusing on linear arbitrary strategies yields the following proposition.

specification.

[29] Note that linear strategies are not restrictive. Any equilibrium amongst those that involve strategies that are bounded above and below by linear strategies is itself linear (Dewan and Myatt, 2008).

Proposition 1. *Making use of Lemma 1 and given the payoff functions in the second period of the game, there exist coefficients* $(\theta_{i1}, \theta_{i2})$*, for* $i \in N$*, such that the Bayesian Nash equilibrium for this regulatory game is unique and characterized by a linear decision rule. More explicitly, the equilibrium production strategy is given by:*

$$\widetilde{q_i} = \theta_{i1} + \theta_{i2}\widetilde{c_i}, \text{ for } i \in N \tag{12}$$

where,

$$\theta_{i1} = \frac{a + \varphi\left(\tau_j - 2\tau_i\right)}{3b} + \frac{2\lambda_i\left(2 - \gamma_j\right) - \lambda_j\left(2 - \gamma_i\right)}{3b\left(4 - \gamma_i\gamma_j\right)}, \tag{13}$$

and,

$$\theta_{i2} = -\frac{\left(2 - \gamma_i\right)}{b\left(4 - \gamma_i\gamma_j\right)}. \tag{14}$$

Proof. See Appendix A. □

In general, Bayesian games may result in, alongside pure-strategy equilibria, mixed-strategy ones. However, given the structure of the information environment, best responses are always unique where each firm follows a linear decision rule. Lemma 1, together with linear demand and constant marginal costs assumptions, ensure that the equilibrium quantities of our Stackelberg-Cournot model can be explicitly characterized in compact closed form, and are linear in their arguments. In addition, facing industry-wide shocks and privately-held information, the classical linearity of best reaction functions is substantive, allowing us to keep results tractable. Further, linear equilibria are very useful in empirical analysis.

3.2 Regulator's problem and the optimal tax rates

Similar to the associated literature, we focus in our analysis on the interior solution. Given the best response functions for both firms, the regulator maximizes the expected welfare to set the optimal taxes in the first stage of the game:

$$\max_{<\tau_i,\tau_j>} \mathbb{E}_{\widetilde{c}_i,\widetilde{c}_j}\left[\widetilde{W}\left(\tau_i,\tau_j\right)\right] \ni \widetilde{W}\left(\tau_i,\tau_j\right) \equiv -\left(\frac{b + d\varphi^2}{2}\right)\widetilde{Q}^2 + \sum_{i\in N}\left[\left(a - \widetilde{c_i} + \varphi\,\ell'\,\tau_i\right)\widetilde{q_i}\right] \tag{15}$$

where $\ell' = \ell - 1$ and the parameter ℓ, with $1 < \ell < 2$, represents the relative weight the regulator places on the revenue from emissions taxation: a higher value of ℓ implies a higher value on the revenue component.

Under our setting, we show that there exists a unique equilibrium for the environmental regulatory game. We describe the equilibrium tax rule in the following proposition.

Proposition 2. *In uncertain polluting industries characterized by common and private information structure about costs, a risk neutral planner sets, as an environmental pricing policy, differentiated emissions taxes given by:*

$$\tau_i = \frac{(2\omega + \ell' - 1)\left[2a - (\mu_i + \mu_j)\right]}{4\varphi(\omega + \ell')} + \frac{(1 - \ell')(\mu_i - \mu_j)}{4\varphi\ell'}; \forall i, j, i \neq j \tag{16}$$

where $\mu_i = \mu_s + \mu_{\varepsilon_i}$, *for* $i \in N$, *and the parameter* $\omega \equiv \left(\frac{1}{3} + \frac{d\varphi^2}{3b}\right) \geq \max\left\{\frac{1-\ell'}{2}, \frac{1}{3}\right\}$.

Proof. Using the same steps as Elnaboulsi et al. (2018), we can deliver the intensity of emissions taxes. See Appendix B. □

Unless $\mu_i = \mu_j$, this proposition states that, while in real world strong administrative and political economy arguments in favor of uniform taxation remain, cost-based policy differentiation is unambiguously welfare maximizing and is robust to unanticipated cost realizations. In other words, a uniform emission-based policy simply can not correct efficiently a pollution externality. Therefore, environmental taxes must be designed to accommodate non-uniform expected signals that yield heterogeneous damages.[30] Thus, the intensity of emissions liabilities must be corrected by the differentiation coefficient defined by $\frac{(1-\ell')\left(\mu_{\varepsilon_i} - \mu_{\varepsilon_j}\right)}{4\varphi\ell'}$. In fact, for i and j, with $i \neq j$, we can express the tax rule as follows:

$$\tau_i = \frac{(2\omega + \ell' - 1)\left[2(a - \mu_s) - \left(\mu_{\varepsilon_i} + \mu_{\varepsilon_j}\right)\right]}{4\varphi(\omega + \ell')} + \frac{(1 - \ell')\left(\mu_{\varepsilon_i} - \mu_{\varepsilon_j}\right)}{4\varphi\ell'} \tag{17}$$

If $\mu_i = \mu_j$, i.e., $\mu_{\varepsilon_i} = \mu_{\varepsilon_j} = \mu_\varepsilon$, the second term in the right hand side of the last equation is equal to zero. Therefore, the implied tax rule is

$$\tau_i = \frac{(2\omega + \ell' - 1)\left[2(a - \mu_s - \mu_\varepsilon)\right]}{4\varphi(\omega + \ell')} \tag{18}$$

Hence, the main difference between the last two equations is $\frac{(1-\ell')\left(\mu_{\varepsilon_i} - \mu_{\varepsilon_j}\right)}{4\varphi\ell'}$ that can be used to set differentiated tax policy to regulate emissions, which is more efficient than many existing policies under which emissions taxes do not reflect the realization of expected cost signals, and the associated environmental damages.

For example, it is known that a unit of electricity corresponds to different amount of GHG emissions according to what type of power input is used, e.g., coal versus renewables.

[30] For instance, some pollutants, e.g., nitrogen oxides (NO_x) and sulfur dioxide (SO_2), have historically been subject to undifferentiated market-based regulation in the United States. Fowlie and Muller (2019) showed that, when ex-post realized abatement costs manifest differently than expected, it is possible to achieve the socially efficient outcome under differentiated taxes.

If, based on some averaged observations, the regulator sets some arbitrary uniform emissions taxes that correct for the externality produced by the activity, then such energy-tax approach is inefficient and fails to incentivize abatement. This implies that the associated energy-efficiency policy also fails to achieve climate targets. The average-tax policy that aims to penalizes GHG emissions at the same tax rate, entails under-taxing dirty activities and over-taxing green activities, which may be considered as a subject to contentious debate.

One may verify that, under our setting, the regulator's objective function is concave, so the optimal tax rates can be analytically derived by considering the first-order conditions of the objective function. Note that, since we consider constant marginal costs, the optimal tax rule does not depend upon the quality of information as given by γ_i, for $i \in N$.

The parameter ω may represent the severity of environmental damages due to emissions. To see this, let's define $\eta \equiv \frac{d\varphi^2}{b}$ as the ratio of the slopes of the marginal environmental damage ($d\varphi^2$) and the marginal consumer surplus (b). We can write ω as a function of η:

$$\omega = \frac{(1+\eta)}{3} \tag{19}$$

If the slope of the marginal environmental damage is higher than the slope of the marginal benefits for consumers, i.e., $\eta \geq 1$, then $\omega \geq \frac{2}{3}$. This means that environmental damages are significantly higher than consumer surplus (CS), which represents a serious pollution case. In this case, the regulator needs to strengthen regulation such that the damage from the externality is completely controlled. Another possible interpretation is that ω represents the regulator's environmental valuation in terms of its vulnerability to environmental externalities. A regulator who is highly concerned with the market failure arising from negative externalities tailors higher emissions taxes in order to prevent more welfare loss. Here we place constraints on the value of ω to ensure that, in equilibrium, quantities and prices are strictly positive, and hence firms pay a non-negative emissions tax. We therefore require that $\omega \geq \max\left\{\frac{1-\ell'}{2}, \frac{1}{3}\right\}$.

Proposition 3. *Given the optimal tax rates, the regulator expects the equilibrium output for each firm i, for $i \in N$, industry output, and market-clearing price as follows:*

$$\mathbb{E}\left[\widetilde{q}_i\right] = \frac{(1+\ell')\left[2a - \left(\mu_i + \mu_j\right)\right]}{12b(\omega+\ell')} + \frac{(1+\ell')\left(\mu_j - \mu_i\right)}{4\ b\ell'} \tag{20}$$

$$\mathbb{E}\left[\widetilde{Q}\right] = \frac{(1+\ell')\left[2\alpha - \left(\mu_i + \mu_j\right)\right]}{6b(\omega+\ell')} \tag{21}$$

$$\mathbb{E}\left[\widetilde{p}\right] = \frac{2a\left(3\omega + 2\ell' - 1\right) + (1+\ell')\ \left(\mu_i + \mu_j\right)}{6(\omega+\ell')}. \tag{22}$$

Proof. The determination of these expressions is straightforward using Proposition 2. □

The concavity of the regulator's objective function leads, in equilibrium, to positive quantities and price. Further, since we assumed constant returns to scale, the equilibrium strategies do not depend on the precision of the information given by γ_i, for $i \in N$.

Substituting $\mu_i = \mu_s + \mu_{\varepsilon_i}$, for $i \in N$, into the expression of the expected output for each polluting firm, we obtain

$$\mathbb{E}\left[\widetilde{q}_i\right] = \frac{(1+\ell')\left[2\left(a-\mu_s\right)-\left(\mu_{\varepsilon_i}+\mu_{\varepsilon_j}\right)\right]}{12b(\omega+\ell')} + \frac{(1+\ell')\left(\mu_{\varepsilon_j}-\mu_{\varepsilon_i}\right)}{4\ b\ell'} \tag{23}$$

and

$$\mathbb{E}\left[\widetilde{Q}\right] = \frac{(1+\ell')\left[2\left(\alpha-\mu_s\right)-\left(\mu_{\varepsilon_i}+\mu_{\varepsilon_j}\right)\right]}{6b(\omega+\ell')} \tag{24}$$

If firm i observes, let's say, a lower cost signal, it has strong incentives to expand output. The reverse holds for the observation of a high cost signal, which is economically intuitive. In addition, lower (respectively higher) cost signals entails an increase (respectively decrease) in the industry output. This result is intuitive under Cournot competition.

4 Welfare analysis

Under environmental regulation, how the expected private cost affects welfare? What are the social welfare effects of information precision? Who benefits from having more precise prior information about marginal costs? The purpose of this section is to answer these questions. To this end, recall that the expected welfare function is given by:

$$\mathbb{E}_{\widetilde{c}_i,\widetilde{c}_j}\left[\widetilde{W}\left(\tau_i,\tau_j\right)\right] = \overline{W} = -\frac{(b+d\varphi^2)}{2}\mathbb{E}_{\widetilde{c}_i,\widetilde{c}_j}\left[\widetilde{Q}\right] + \sum_{i\in N}\mathbb{E}_{\widetilde{c}_i,\widetilde{c}_j}\left[\left(a-\widetilde{c}_i+\varphi\ell'\tau_i\right)\widetilde{q}_i\right] \tag{25}$$

Using the above results in equilibrium and considering that, for $i \in N$, $\sigma_i^2 = \sigma_s^2 + \sigma_{\varepsilon_i}^2$, i.e., $\gamma_i = \frac{\sigma_s^2}{\sigma_i^2}$, yield the following welfare expression:

$$\begin{aligned}\overline{W} \ = \ & -\frac{(b+d\varphi^2)}{2}\left[\frac{\left(2-\gamma_i\right)^2\sigma_i^2}{b^2\left(4-\gamma_i\gamma_j\right)^2} + \frac{\left(2-\gamma_j\right)^2\sigma_j^2}{b^2\left(4-\gamma_i\gamma_j\right)^2} + \frac{2\left(2-\gamma_i\right)\left(2-\gamma_j\right)\sigma_s^2}{b^2\left(4-\gamma_i\gamma_j\right)^2}\right] + \frac{\left(2-\gamma_i\right)\sigma_i^2}{b\left(4-\gamma_i\gamma_j\right)} \\ & + \frac{\left(2-\gamma_j\right)\sigma_j^2}{b\left(4-\gamma_i\gamma_j\right)} + \frac{(1+\ell')^2\left[2a-\left(\mu_i+\mu_j\right)\right]^2}{24b\left(\omega+\ell'\right)} + \frac{(1+\ell')^2\left(\mu_i-\mu_j\right)^2}{8b\ell'}\end{aligned} \tag{26}$$

Proposition 4. *Given the structure of the information environment adopted in this paper, and assuming a is large enough to avoid shutdown, the expected welfare is decreasing with respect to the unconditional expected marginal cost, $\mu_i = \mu_s + \mu_{\varepsilon_i}$, if $\mu_i \leq \mu_j$. However, an increase in the unconditional expected marginal cost is welfare enhancing, i.e. $\frac{\partial \overline{W}}{\partial \mu_i} > 0$, if and only if*

$$\left(\mu_i - \mu_j\right)\left(4\ell' + 3\omega\right) > 2\ell'\left(a - \mu_i\right) \tag{27}$$

or equivalently,

$$\frac{\mu_{\varepsilon_i} - \mu_{\varepsilon_j}}{a - \mu_s - \mu_{\varepsilon_j}} > \frac{2\ell'}{2\ell' + \omega} \tag{28}$$

Proof. See Appendix C. □

This proposition states that, if $\mu_i \leq \mu_j$ $\left(\text{similarly } \mu_{\varepsilon_i} \leq \mu_{\varepsilon_j}\right)$, then any increase in firm i's expected marginal costs entails welfare loss. This means that if the more productively efficient firm i sees an increase in its expected costs, then the social welfare decreases. This conclusion is economically intuitive. Firm i may adopt environmentally friendly technology to avoid the burden of the tax, which yields higher marginal costs. This implies a decrease in the overall industry output (and emissions), and an increase in prices. Due to costs' inference effect, this provides incentives for firm j to increase its production. In fact, facing under-production and higher prices, the less productively efficient rival j behaves strategically and substantially increases its own production to offset the reduction in output. As a reaction, firm i aggressively increases its production, and competition in the product market between producers is exacerbated. Such overproduction entails an increase in emissions and induces welfare loss.

However, if firm i's unconditional expected marginal cost, μ_i, is large relative to μ_j, or equivalently $\mu_{\varepsilon_i} > \mu_{\varepsilon_j}$, then $\frac{\partial \overline{W}}{\partial \mu_i} > 0$. An increase of firm i's expected marginal costs is welfare enhancing. This is the case when firm i goes more and more green. This implies a positive gap between the rival j's marginal cost and its own. To avoid the burden of higher environmental taxes, firm j's may have incentive to adopt cleaner technologies and hence reduce its emissions. The environment overall impact is hence positive, which entails welfare increase. Thus, going green in the product marketplace creates positive externalities by inducing rivals to adopt environmental friendly technologies.

Proposition 5. *When the regulator sets up emissions taxes to correct pollution under costs uncertainties, an increase in the relative precision of information, i.e. the signal becomes more informative, induces a decrease in the expected welfare as long as $\omega \leq \frac{4}{5}$, where $\left(\gamma_i, \gamma_j\right)$ being defined on the support $\mathcal{A} =]0,1] \times]0,1]$. However, if $\omega > \frac{4}{5}$, then an increase in* *the relative precision of information may enhance the expected welfare, which is true under severe environmental damages. Thus,*

- *If* $\omega \in \left[\frac{1}{3}, \frac{4}{5}\right]$ *, then,*

$$\frac{\partial \overline{W}}{\partial \gamma_i}\left(\gamma_i, \gamma_j\right) < 0, \frac{\partial \overline{W}}{\partial \gamma_j}\left(\gamma_i, \gamma_j\right) < 0 \tag{29}$$

- *If* $\omega > \frac{4}{5}$*, then,*

$$\begin{cases} \frac{\partial \overline{W}}{\partial \gamma_i}\left(\gamma_i, \gamma_j\right) = \frac{\partial \overline{W}}{\partial \gamma_j}\left(\gamma_i, \gamma_j\right) > 0 \Leftrightarrow \left(\gamma_i, \gamma_j\right) \in A_-^\rho \\ \frac{\partial \overline{W}}{\partial \gamma_i}\left(\gamma_i, \gamma_j\right) = \frac{\partial \overline{W}}{\partial \gamma_j}\left(\gamma_i, \gamma_j\right) = 0 \Leftrightarrow \left(\gamma_i, \gamma_j\right) \in A_0^\rho \\ \frac{\partial \overline{W}}{\partial \gamma_i}\left(\gamma_i, \gamma_j\right) = \frac{\partial \overline{W}}{\partial \gamma_j}\left(\gamma_i, \gamma_j\right) < 0 \Leftrightarrow \left(\gamma_i, \gamma_j\right) \in A_+^\rho \end{cases} \tag{30}$$

  where the family of the non-empty sets $A_-^\rho, A_0^\rho, A_+^\rho$*, defines a partition of* $\mathcal{A}$ *and* $\rho \equiv \frac{5\omega-4}{\omega}$.

Proof. See Appendix D. □

In this proposition, we used precision criteria to analyze the value of information to a decision maker. Proposition 5 shows the conditions under which uncertain industries with private information about costs are socially less harmful than deterministic ones. We obtain two major results:

1. In uncertain markets with environmental externalities in which there is incomplete information about costs, more precise signals yield welfare loss as long as $\omega \leq \frac{4}{5}$, or equivalently the ratio of the slopes of the marginal environmental damage ($d\varphi^2$) and the marginal consumers' surplus (b), $\eta \leq \frac{7}{5}$. Viewed another way, if this condition is binding, then less precise prior information leads to greater welfare. Under Cournot competition, a firm may benefit from having less precise information than its competitor and hence has greater incentive to increase its production (in order to improve its less precise prior information). But, due to costs' inference effects, imprecise information provides a mechanism that enables the rival to expand its production also. Therefore, competition between players in the marketplace is exacerbated. This yields an increase in the industry output. Such overproduction enhance consumers' surplus but entails an increase in emissions. However, the social cost of environmental damages due to the advantages of imprecise information continues to be "acceptable" as long as the slope of the marginal damages relative to the marginal benefits for consumers $\eta \leq \frac{7}{5}$. Therefore, the regulator must be lenient with the industry-wide overproduction as long as consumers are taking advantage of imprecise information. This conclusion

is consistent with the literature on information precision under Cournot competition without environmental externalities (See for instance Gal-Or, 1988).

2. Conversely, more precise signals are welfare enhancing when $\omega > \frac{4}{5}$ or equivalently $\eta > \frac{7}{5}$, i.e., there are threats of serious environmental damage. Under such circumstances, well informed firms have no incentives to produce further in order to avoid the burden of the tax, which yields a reduction in emissions, and hence improves social welfare. When the social cost of environmental damages is so important, there is no reason for postponing any regulatory policy to correct externalities. This is particularly the case when the regulator is dealing with irreversible loss, and when it is uncertain about the likelihood of that loss. The regulator has to be extremely cautious and must treat the question very carefully by setting the intensity of the policy instrument accordingly in order to avoid or diminish environmental harm, including threats to human life or health.

The theoretical analysis and its social implications are illustrated with the following numerical simulations based on admissible parameters values. Our purpose is to provide more intuition of how social welfare moves with the precision of signals for different values of ω. Figure 2 shows results from simulations performed in the following case parameter grid: $\gamma_i \in [0,1]$, $\gamma_j \in [0,1]$, and $\omega \in \left]\frac{1}{3}, 1.3\right]$.

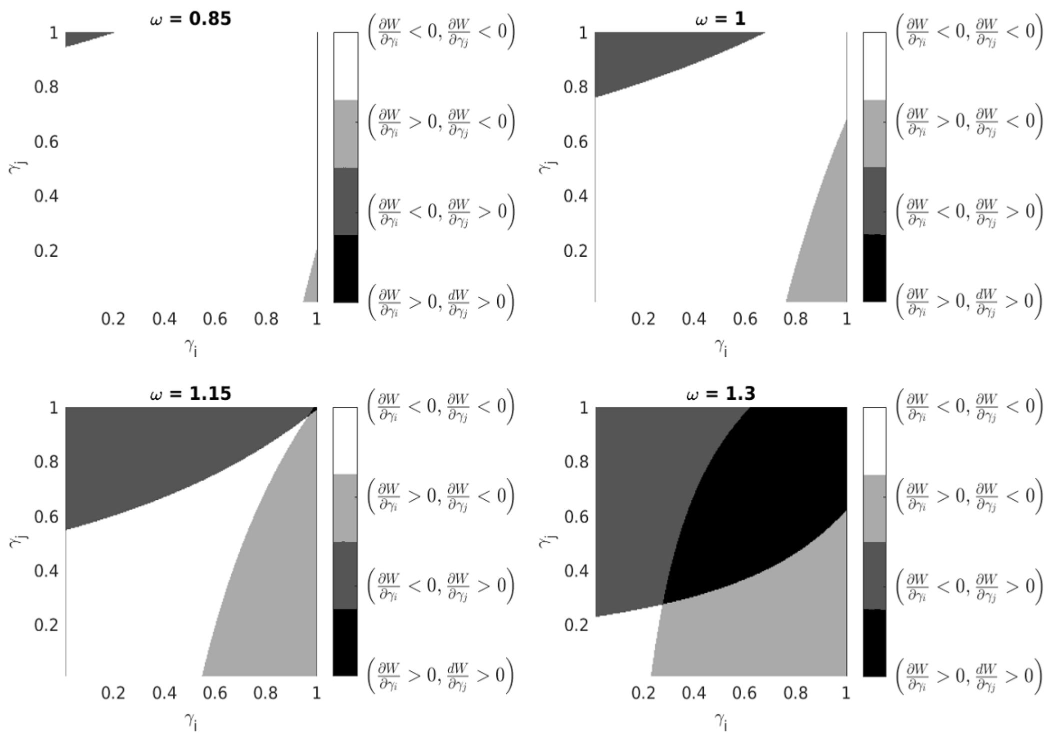

Figure 2: Social welfare and the precision of signals for admissible value of ω.

The upper left panel in Figure 2 illustrates the consequences of signals' precision on welfare when $\omega = 0.85$ or $\eta = 1.55$. It shows that when both signals $\left(\gamma_i, \gamma_j\right)$ become more informative, then social welfare decreases (white zone). However, if, let's say, firm i becomes well informed (higher γ_i) while firm j is uninformed (lower γ_j), then $\frac{\partial \overline{W}}{\partial \gamma_i} \geq 0$ (light gray tiny area). This means that any improvement in i's signal is welfare enhancing if j's signal is weak (and vice versa). Numerical results also show that when $\omega = 1.3$ or $\eta = 2.90$ (right lower panel in Figure 2), then any improvement in both signals $\left(\gamma_i, \gamma_j\right)$ entails an increase in social welfare (black area).

In Table 1, we summarize the likelihood of welfare improvement with small changes of signals' precision.

	$\omega \geq \max\left\{\frac{1-\ell'}{2}, \frac{1}{3}\right\}$				
	0.80	0.85	1.00	1.15	1.30
$\frac{\partial \overline{W}}{\partial \gamma_i} > 0$ or $\frac{\partial \overline{W}}{\partial \gamma_j} > 0$	0.0000	0.0065	0.0912	0.2746	0.6452
$\frac{\partial \overline{W}}{\partial \gamma_i} > 0$ and $\frac{\partial \overline{W}}{\partial \gamma_j} > 0$	0.0000	0.0000	0.0000	0.0004	0.3479

Table 1: Simulation results and the likelihood that precise information is welfare enhancing.

Entries indicate the likelihood that signals precision is welfare enhancing for different values of ω.[31] It is easy to verify that, for any $\left(\gamma_i, \gamma_j\right)$, with $i \neq j$, the probability that an improvement in the precision entails an increase in social welfare is zero provided that $\omega \leq \frac{4}{5}$. However, this probability increases for higher values of ω: more precise information is welfare enhancing.

To end this section, notice that social welfare is increasing in ℓ' representing the marginal social benefit of environmental taxation. Differentiating the welfare function with respect to ℓ' and rearranging the expression yield:

$$\frac{\partial \overline{W}}{\partial \ell'} = \frac{(2\omega + \ell' - 1)(1 + \ell')\left[2a - \left(\mu_i + \mu_j\right)\right]^2}{24b(\omega + \ell')} - \frac{(1 - \ell')(1 + \ell')\left(\mu_i - \mu_j\right)^2}{\ell'^2} \tag{31}$$

This partial derivative is positive if and only if,

$$\frac{(2\omega + \ell' - 1)\left[2a - \left(\mu_i + \mu_j\right)\right]^2}{24b(\omega + \ell')} \geq \frac{(1 - \ell')\left(\mu_i - \mu_j\right)^2}{\ell'^2} \tag{32}$$

or,

$$\frac{\ell'^2 \, (2\omega + \ell' - 1)\left[2a - \left(\mu_i + \mu_j\right)\right]^2}{24b(\omega + \ell')(1 - \ell')\left(\mu_i - \mu_j\right)^2} \geq 0 \tag{33}$$

[31] It represents, for each value of ω, the proportion of the surface (in Figure 2) where the welfare function is increasing with respect to γ_i and γ_j, total surface area being equal to 1.

which is always true under our assumptions. This observation implies that, an increase in the weight on revenues, ℓ', is welfare enhancing. In addition, improving the environmental quality through taxes generates extra revenues that can be recycled and hence may have positive impacts on the rest of the economy by reducing distortive taxes. For instance, extra revenues from a widely applied energy tax (or Carbon tax) might be used to diminish the burden of distortive taxes on labor or heavy regulated markets.

5 Information sharing

5.1 Incentive to Share Information

We consider the possibility that firms adopt an anti-competitive behavior and collude by means of truthful sharing agreements about costs value,[32] which can be ensured through different channels such as using information sharing platforms. This means that firm i is allowed to observe firm j's signals if and only if it reveals its own signals to j.[33]

The information sharing part of the model has many applications in different fields of environmental economics analysis. For instance, increased risks in exploration and high volatility of crude oil prices are considered as important reasons for information sharing activities (Banal-Estañol, 2007). Another real-world example experiencing serious and unfortunate distortions of competition is the energy market. Under different E.U. and U.S. transparency-enhancing, resilience and security regulations, increased effort and focus has been put on information sharing in the energy sector.[34] Several platforms were performed to disclose energy-related information to market participants. This gives players free access to indicators on capacities, data on scheduled availability of units, day-ahead generation forecasts, etc. and provides them with focal points around which to align their decisions. It is true that such platforms allowing information sharing among energy sector stakeholders

[32] Note that if a mechanism of verification does not exist, a firm may have incentive to report untruthfully the real value of its costs. We ignore this possibility in this paper since we focus on the consequences of information pooling on welfare under environmental regulation through emissions taxes. Having said that, Li (1985) and Shapiro (1986), among others, have shown that in a Cournot market with uncertainties about private costs, firms completely reveal information in equilibrium.

[33] While we do not attempt to analyze the exchange of information process between players, one may assume that an outside agency such as an energy or environmental agency conducts the transmission of the information held by market participants: the agency collects and publicizes information, dependent upon the type of the competition in the industry and the source of uncertainty in the market, i.e., the cost parameter that is different for each player.

[34] According to the European Commission, at least 80% of European companies have experienced at least one cybersecurity incident in 2015, and the number of security incident across all industries worldwide rose by 38% the same year. See the European Union Agency for Network and Information Security, Report on Cyber Security Information Sharing in the Energy Sector, Final, November 2016. Available at https://www.enisa.europa.eu, accessed 15 March 2020.

are essential to better address resilience and security issues such as risks, vulnerabilities and threats, even if the quality of the shared information is not always at the required level. However, increased transparency would only be helpful up to a point. By providing good opportunities to freely and legally disseminate industry-related information, these platforms also facilitate cooperation and ease actions coordination between players in the industry. Hence, increased transparency gives firms incentives to adopt collusive behavior that undermines the environmental policy.

Suppose that firms adjust their information about costs, internalize their payoff interdependencies, and coordinate their actions, while the regulator remains uninformed about firms' costs. In this case, information sharing through concerted practices may generate various inefficiencies and hence clouds the regulatory process. In this section we examine if firms have incentives to share their information.[35] The following propositions summarize our findings.

Proposition 6. *Suppose that a sharing agreement (S) is reached between firms and ensured using an information sharing platform or any other channel. Suppose also that firms truthfully signal their private information and receive perfectly the full vector of rivals' costs. In this case, since the regulator can neither foresee nor control the uncertainties at the time it sets the environmental policy which remains in force for an extended period of time, then the optimal tax rules is the same as in the non-sharing information game (NS).*

Proof. The proof follows similarly to the proof of Proposition 2, and it is therefore skipped (See Appendix B). □

The interpretation of Proposition 6 should be relatively clear and economically intuitive. Since the regulator's information set remains the same, information sharing does not affect the expected welfare maximization problem, which gives the same tax rules.

Proposition 7. *When a regulator uses taxes as a pricing policy to correct harmful externalities in uncertain industries, firm i has incentives to collude with firm j and vice versa if and only if*

$$\left(c_j - \mu_j\right) \geq \left(c_i - \mu_i\right) z\left(\gamma_i, \gamma_j\right) \tag{34}$$

where the function $z\left(\gamma_i, \gamma_j\right) = \left(\frac{2-2\gamma_i\gamma_j+3\gamma_i}{4-\gamma_i\gamma_j}\right) > 0$, is defined on the support $\mathcal{D} = [0,1]^2$, $\forall \gamma_i, \gamma_j \in [0,1]$, reaching its maximum value, $z_{\max}\left(\gamma_i, \gamma_j\right) = 1.25$, for $\left(\gamma_i = 1, \gamma_j = 0\right)$, and its minimum value, $z_{\min}\left(\gamma_i, \gamma_j\right) = 0.5$, when $\gamma_i = 0$, $\forall \gamma_j \in [0,1]$. Further, collusion by sharing

[35] This paper concentrates on a firm's incentives to share its private information with competing firms, but it does not consider merging issues.

information is a dominant strategy in highly uncertain industries than under deterministic ones if the uncertainty is characterized by privately hold information.

Proof. See Appendix E. □

Here, equilibrium outcomes can be solved given the information revealed. If firms share perfectly the full vector of costs, a decision of whether to exchange information is based on profit comparisons, and there is no strategic interaction via observation of prices or quantities. Given the linear specification of our model, and since profits are positively correlated to production, comparing firms' output levels in each informational regime is sufficient to prove the condition under which there is a net gain from information sharing. Proposition 7 states that information sharing is a dominant strategy for firms provided that the last relation holds.[36] It is mutually beneficial to firms to cooperatively exchange their costs information and collude. This is true because firms' profits under cost sharing agreements exceed their expected profits under competition à la Cournot at the second period of the game. Further, the incentives to collude in the industry are increased only if the cost uncertainties are large enough, i.e., lower values of information precision (γ_i, γ_j).

Figure 3 shows numerical illustrations of our findings for reasonably realistic values of (c_i, c_j) and for $\gamma_i, \gamma_j \in [0, 1]$. Numerical simulations provide the intuition of how the magnitude of marginal costs impacts the decision to collude through information-pooling agreements in uncertain markets.

It can be easily verified that information sharing occurs if both firms, i and j, have relatively high marginal costs (c_i, c_j), i.e., firms are environmentally friendly. In addition, higher uncertainties (less precise information) increase the incentive to collude (white area). The uncertainty with private information generates extra profits for colluding firms, i.e., $\nabla q_i \equiv q_i^S - q_i^{NS} > 0$, and hence there are greater incentives to collude in more uncertain industries. The regulator cannot ignore cost sharing agreements between players in the industry because changes in output level entails an increase in emissions, which jeopardizes the regulatory approach. Conversely, not to share information is a stable configuration in uncertain industries if firms have low marginal costs (c_i, c_j), i.e., firms are productively efficient but not environment friendly (black area).

[36] For a given precision, this result is consistent with Gal-Or (1986), Shapiro (1986), and Ganuza and Jensen (2013).

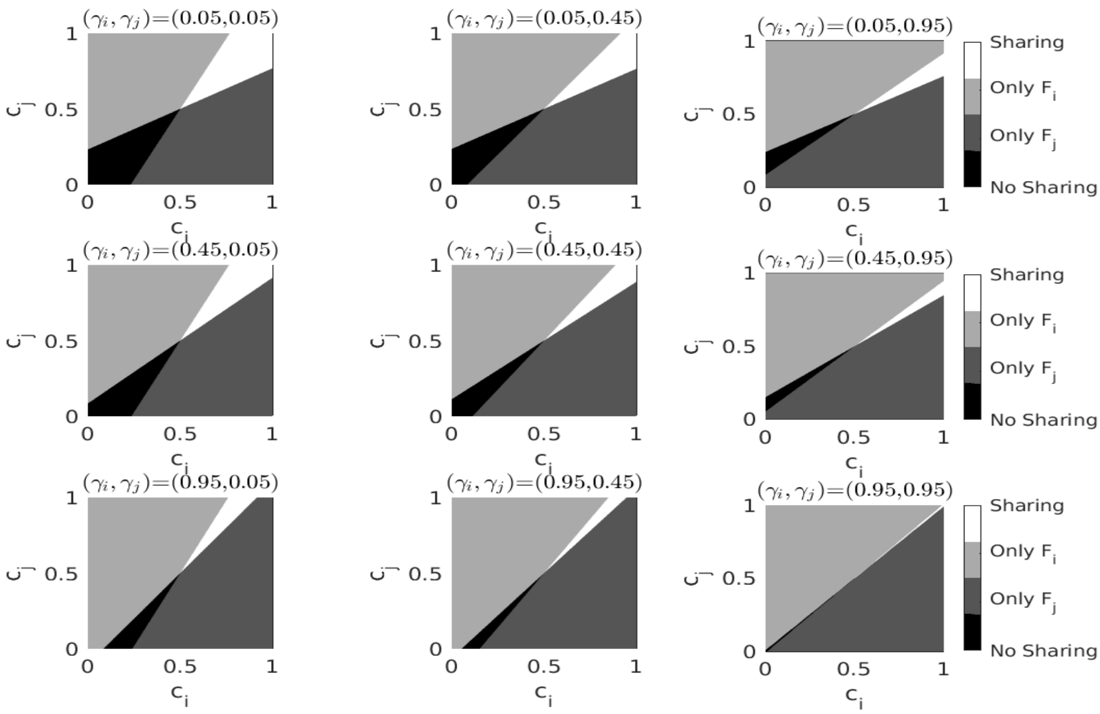

Figure 3: Simulations results, incentives to share information for admissible values of $(c_i, c_j)\,,\, \left(\gamma_i, \gamma_j\right) \in [0,1]^2$.

Further, if firm i gets access to more reliable source of information and the cost signal is less noisy, then firm i is not willing to share any information with firm j who may have unilateral incentives to share information. In particular, if $\sigma^2_{\varepsilon_i} \rightarrow 0$, i.e., $\gamma_i \longrightarrow 1$, the signaling distortions completely vanish (approaching the full information case), then i is reluctant to collude with j: the expected profits from remaining independent are larger than those from pooling of information. By contrast, when $\sigma^2_{\varepsilon_i} \rightarrow +\infty$, i.e., $\gamma_i \longrightarrow 0$, the signal is infinitely noisy so that the informational content is worthless, and hence the signaling distortions increase firms' willing to share information (light gray area). The same analysis holds for j (dark gray area). Note that, if firm i's signal is sufficiently strong while firm j's signal is weak, then j is willing to share information with its rival, while firm i is not. In this case, non sharing information is always a stable issue, i.e., an equilibrium.

Finally, since changes in firms' profits in equilibrium can be captured by the variation of firms' outputs under sharing and non-sharing regimes, we can analyze how information precision affects the output gap arising from a more efficient distribution of production across firms, i.e., changes in emissions under both cases. Let's define the output gap as $\nabla q_i \equiv q_i^S - q_i^{NS}$, for $i \in N$. From appendix E, we write

$$\nabla q_i = \frac{\left(c_j - \mu_j\right) - 2\left(c_i - \mu_i\right)}{3b} + \frac{\left(2 - \gamma_i\right)\left(c_i - \mu_i\right)}{b\left(4 - \gamma_i \gamma_j\right)}, \text{ with } i \neq j \tag{35}$$

Differentiating this relation with respect to the relative precision of i's signal, γ_i, and rearranging the terms we obtain the following simplified expression:

$$\frac{\partial}{\partial \gamma_i} \nabla q_i \left(\gamma_i, \gamma_j\right) = -\frac{2\left(c_i - \mu_i\right)\left(2 - \gamma_j\right)}{b\left(4 - \gamma_i \gamma_j\right)^2} \tag{36}$$

Since the term $\frac{\left(2-\gamma_j\right)}{b\left(4-\gamma_i\gamma_j\right)^2}$ is strictly positive for $\gamma_i, \gamma_j \in [0, 1]$, we are entitled to write

$$sgn\left(\frac{\partial \nabla q_i}{\partial \gamma_i}\right) = -sgn\left(c_i - \mu_i\right) \tag{37}$$

Thus, the sign of the first derivative of ∇q_i with respect to the relative precision of i's signal depends on the sign of $\left(c_i - \mu_i\right)$. What does that mean? To display the intuition behind our findings, we subsequently consider the following two cases:[37]

1. If $c_i < \mu_i$, i.e., i's marginal cost turns out to be lower than expected, then $\frac{\partial \nabla q_i}{\partial \gamma_i} > 0$. In the case where i's true marginal cost is lower compared to its mean, a better quality signal widen the production gap between sharing and non-sharing information regimes. Higher precision of the signal entails an increase of q_i^S because more precise information signals that firm i is productively efficient in the marketplace. Hence, a more efficient distribution of the industry output across firms yields an increase of firm i's production level under information-pooling agreements. Expanding its production under deterministic markets makes firm i better off because it realizes extra profits from the resulting output gap. Having said that, firm i is not completely free in its maneuver in the marketplace because it is not environmentally friendly (lower value of marginal cost): q_i^S heavily depends on the regulator's environmental consciousness and can be taxed accordingly.

2. Conversely, if $c_i > \mu_i$ then $\frac{\partial \nabla q_i}{\partial \gamma_i} < 0$. The intuition behind this result is that, any improvement of i's signal lowers the production gap between the sharing and non-sharing cases. When the signal becomes more informative, and if the true value of the marginal cost is higher than its average (which signals that the firm is environmentally friendly but productively inefficient), firm i must behave strategically and reduces its production under cost sharing agreements $\left(q_i^S\right)$. Hence, well informed, firm i has more

[37] A similar analysis can be undertaken with respect to j's relative precision of information. We can show that, the first derivative of ∇q_i with respect to γ_j yields: $\frac{\partial \nabla q_i}{\partial \gamma_j} = \frac{\gamma_i(c_i-\mu_i)(2-\gamma_i)}{b\left(4-\gamma_i\gamma_j\right)^2} \Rightarrow sgn\left(\frac{\partial \nabla q_i}{\partial \gamma_j}\right) = sgn\left(c_i - \mu_i\right)$. If $c_i > \mu_i$, then $\frac{\partial \nabla q_i}{\partial \gamma_j}$ increases in response to a higher precision of j's signal. If $c_i < \mu_i$, then a higher quality of j's information entails a decrease in ∇q_i.

incentives to reduce its output level. As a result, the production gap decreases.

5.2 Collusion and Welfare

In this section, we try to look at the issue of information sharing in industries subject to environmental pricing policy through taxes, seeking to answer a simple question: Should a Cournot oligopolist be allowed to share information about their costs in uncertain polluting industries? To this end, we investigate how collusion through cost sharing agreements affects social welfare. Note that, antitrust authorities do not forbid the exchange of information *per se* as long as it is not used with the purpose to collude or to deter entry. However, suspicions remain.

An extensive literature has examined the incentives of a Cournot oligopolist to share information about private costs without environmental externalities (See for instance, Amir et al., 2010; Banal-Estañol, 2007; Gal-Or, 1986; Li, 1985; Shapiro, 1986; Vives, 1988, 1990). Industry-wide information sharing increases expected profits, reduces expected consumer surplus, and enhances social welfare defined as the expected sum of industry profit and consumer surplus. The welfare analysis of information-pooling agreements about costs is difficult to resolve and is quite complex in uncertain industries with environmental externalities.[38] Thus, in order to translate our findings to regulatory policy, we state verbally our findings and relegate the formal expressions to the appendix. The following proposition fully characterizes the impact of collusion between firms on welfare.

Proposition 8. *Under the informational framework adopted in this paper, i.e., common and private information, when taxes are used to regulate an uncertain and asymmetric polluting Cournot industry, collusion that would arise through cost sharing agreements is unambiguously better for welfare as long as the ratio of the slopes of the marginal environmental damage* ($d\varphi^2$) *and the marginal consumers' surplus* (b), η, *is less than one. However, if the ratio is higher than one, i.e.,* $\eta > 1$, *then collusion may have large negative impact on social welfare. This is the case under extremely and severe environmental irreversibilities.*

Proof. See Appendix E. □

To deliver the insight behind this proposition, we write the welfare function in terms of equilibrium outputs since collusion through cost sharing agreements is feasible as long as $q_i^S \geq q_i^{NS}, \forall i \in N$. We have relied on simplifying the welfare function, but justified, to avoid

[38] In the classical literature on information exchange, aggregating industry profits and consumer surplus yields the expression for social welfare. When it comes to uncertain polluting industries, the expected welfare function is quite more complicated and includes further environmental damages and tax revenues, which makes our task more complex.

mathematical complications. Let's examine more closely the expression of the welfare total differential:

$$\Delta W = b\,(2 - 3\omega)\,Q\,(\Delta q_i + \Delta q_j) + 2b\,(q_i \Delta q_i + q_j \Delta q_j) + \varphi\,(1 + \ell')\,(\tau_i \Delta q_i + \tau_j \Delta q_j) \quad (38)$$

ΔW measures the change in welfare that would arise following an increase in output levels, i.e., higher expected profits from information-pooling and possible collusion. The sign of ΔW is indeterminate: the second and third terms on the right hand side of this expression are unambiguously strictly positive; however, the sign of the first term on the right hand side of the last expression depends on the sign of $(2 - 3\omega)$. Therefore, the overall social effect of collusion depends on the value of ω.

Recall that $\omega = \frac{b+d\varphi^2}{3b}$ where d characterizes the convexity of environmental damages,[39] and b captures the concavity of the benefits for consumers. We consider the following two subsequent cases:

1. If $\omega \leq \frac{2}{3}$, i.e., the ratio $\eta \leq 1$, then $\Delta W \geq 0$. In words, when the slope of the marginal environmental damages $(d\varphi^2)$ is less than the slope of the marginal benefits for consumers (b), then, for an exogenously given level of information precision, collusion between players is welfare improving. This observation implies that firms would have an incentive to share information cooperatively since information sharing increases the industry profits. Further, collusive behavior is welfare improving. In uncertain polluting industries, collusion generates information gains due to the information aggregation and eventually the subsequent production rationalization. Total output may be produced in a more cost-efficient way since information-pooling allows firms to better adjust to costs shocks. Consequently, social welfare increases even if collusion decreases the consumer surplus.

2. However, there is a range of situations where anti-competitive behaviors through cost sharing agreements may be welfare reducing. In fact, if $\omega > \frac{2}{3}$, then ΔW may be negative. This is true when the slope of the marginal damages $(d\varphi^2)$ relative to the slope of the marginal benefits for consumers (b) is higher than one, i.e., $\eta > 1$. This is particularly the case under severe and extreme environmental circumstances such as serious environmental irreversibilities. The resulting environmental damages are large enough to shave off all efficiency gains from information-pooling agreements. Therefore, the potential impact of information sharing on environmental damages must be assessed, and a safe environmental policy must be urgently instituted to reduce

[39] Note that the parameter d also captures the regulator's environmental conscious.

collusive risk.

In terms of policy implications, collusion between firms through cost sharing agreements may emerge in a wide range of uncertain polluting Cournot markets, e.g., energy markets, because firms benefit from positive externalities generated by such agreements. Anti-competitive behaviors lead to large efficiency gains and are welfare improving as long as the marginal environmental damage is less than the marginal benefit for consumers. This result is consistent with the literature on collusion through information sharing under Cournot competition with uncertainties about costs. It also provides a rational for antitrust authorities to be lenient with information sharing even if it reduces expected consumer surplus.

Not surprisingly, however, if the slope of the marginal environmental damage is higher relative to the slope of the marginal consumer surplus, collusion may be welfare reducing. This is true under severe environmental damages typically associated with some Cournot industries. Hence, where there are threats of serious damages, there is no reason for postponing any carbon pricing policy such as emissions taxes, and regulatory restrictions on information exchange must be adopted. Environmental regulators and antitrust authorities have to work closely and information sharing should be treated significantly more vigorously in order to avoid further damages and welfare loss. This observation sheds light on previously unknown room for policy improvement in terms of setting environmental taxes, which highly depends on the political will and the regulator's environmental conscious. Facing controversial and scientifically debatable risks due to emissions, and by the precautionary principle, if the regulator is uncertain about the timing and likelihood of social losses, it should set the policy instrument accordingly in order to avoid or diminish environmental harm, including threats to human life or health. Pricing actions through taxes, thus, aim to avoid huge future direct and social irreversible losses.

Concluding Remarks

Over the last few decades, despite tremendous progress in the field, environmental taxes have come under increasing criticism from both outside and inside the environmental economics profession due to the large number of empirical anomalies and failures to deal with some externalities, e.g., GHG emissions or high-risk toxic pollutants. Regulators in different countries around the world are lagging on taking strong actions on emissions and delivering an effective tax-based policy. It is crystal clear that, to reduce emissions, sharply and effectively, a great surge has to be done in decision makers' attitude toward policy settings. To succeed in such a difficult task, the time has come for designers of future environmental taxes to stop behaving like group therapists, complaining and confessing, and vigorously act to curb emissions.

This paper investigates emissions taxes under costs uncertainties and market power. We considered affine information structure with common and private signals. We determined the Bayesian Nash equilibrium of the game in which the regulatory instrument is made under informational constraints, and examined the consequences of varying the informativeness of signals on welfare. Although the model is built on some restrictive assumptions in order to keep results tractable, this paper provides elements to the debate of how information about costs should be used in regulatory decision making. Our findings are broadening decision makers' repertoire of the antidote for most of the environment' ills, and provide a window, hopefully, to ramp up the promised reforms around the world to reduce emissions.

We showed that, facing industry-wide and firm-specific signals, the regulator is able to set differentiated emissions taxes. Under some conditions, the proposed policy enables the fine-tuning of the intensity of the tax rates towards specific environmental circumstances. Further, the social impact of more precise signals hinge fundamentally on the value of the ratio of the slopes of the marginal damage and the marginal consumer surplus. If the severity of environmental damages increases nonlinearly for instance, then the regulator must act ambitiously enough to avoid irreversible processes that would amplify damages. This simply means that taxes must be set accordingly. To understand the potential magnitude of information precision, we turned to numerical simulations calibrated to different level of environmental damages which are inherently difficult to estimate. The final outcome depends on threats of serious damages, e.g., irreversible loss.

Then, we considered the possibility that firms within an uncertain polluting industry may cooperatively be able to share information about costs. We examined the conditions under which collusion may emerge and its social impacts. We argue that, under severe environmental damages, the implied anti-competitive behaviors is welfare reducing. Numerical simulations show how results hinge on the ratio of the slopes of the marginal damage and the marginal consumer benefit.

Clearly, persistent uncertainties and information asymmetry are crucial to policy design and decision making. Lot of deep uncertainties remain unresolved, specifically in the science of climate change which remains quite uncertain. Stripping off the cover of darkness and making information visible is not an easy task. However, current efforts to enhance informational access may offer important lessons for environmental regulation moving forward. Facing industry-wide and firm-specific shocks, there are enormous opportunities to make the best use of available set of data to enhance the quality of the environment. Such information may be used to overcome a serious lack of information on polluted activities, and could have impact on firms' behavior and levels of pollution. Furthermore, where there are threats of serious environmental damage, there is no reason for postponing any externalities pricing policy. This is the case when damages yield irreversible direct and social losses.

Our study has several limitations. Further research should address, for example, investment in clean technology issues to extend the analysis, which could yield rich policy implications. In addition, environmental damages caused by some non-uniformly mixed pollutants, e.g., NO_x emissions or many water pollutants, depend significantly on the location of the source and the spacial distribution of emissions. Hence, it is necessary to extend the model to horizontal differentiation.

Appendix

A Proof of Proposition 1

Recall that firms' information is structured such that $\widetilde{c}_i = \widetilde{s} + \widetilde{\varepsilon}_i$, for $i \in N$. First, we define each firm's profit maximization problem:

$$\max_{<\widetilde{q}_i>} \mathbb{E}_{\widetilde{c}_j} \left[\left(\widetilde{p} - \widetilde{c}_i - \varphi\tau_i \right) \widetilde{q}_i \right], \text{ for } i, j, \text{ with } i \neq j. \tag{A.1}$$

where $\widetilde{p} = a - b\left(\widetilde{q}_i + \widetilde{q}_j\right)$ and $\mathbb{E}_{\widetilde{c}_j}$ denotes the common expectation operator taken over $\widetilde{c}_j$. Differentiating equation (A.1) with respect to $\widetilde{q}_i$, setting the expression equals to zero and rearranging, the first order conditions (FOCs) for i, j, with $i \neq j$, yields the following best reaction functions:

$$\frac{\partial \mathbb{E}\widetilde{\pi}_i\left(\widetilde{q}_i, \widetilde{q}_j\right)}{\partial \widetilde{q}_i} = 0 \Rightarrow \widetilde{q}_i = \frac{a - b\mathbb{E}[\widetilde{q}_j \mid \widetilde{c}_i] - \varphi\tau_i - \widetilde{c}_i}{2b}. \tag{A.2}$$

The second order conditions (SOCs), which implies $\frac{\partial^2 \mathbb{E}\widetilde{\pi}_i(q_i, q_j)}{\partial(\widetilde{q}_i)^2} = -2b$, are satisfied since $b > 0$. Making use of the FOCs, we then can solve for the linear equilibrium in the usual way by identifying coefficients with the candidate linear strategy:

$$\widetilde{q}_i = \theta_{i1} + \theta_{i2}\widetilde{c}_i; \ \forall i, j, \text{ with } i \neq j. \tag{A.3}$$

Substituting into the expression of the FOCs, we obtain:

$$\widetilde{q}_i = \frac{a - b\mathbb{E}[\theta_{j1} + \theta_{j2}\widetilde{c}_j \mid \widetilde{c}_i] - \varphi\tau_i - \widetilde{c}_i}{2b}; \ \forall i, j, \text{ with } i \neq j. \tag{A.4}$$

Under Lemma 1, we have, for every player i, j, with $i \neq j$, $\mathbb{E}[\tilde{c}_i|\tilde{c}_j] = \gamma_j\tilde{c}_j + \lambda_j$, where

$\gamma_j = \frac{\sigma_s^2}{\sigma_s^2+\sigma_{\varepsilon_j}^2}$, $\lambda_j = \mu_i - \gamma_j \mu_j$, $\mu_i = \mu_s + \mu_{\varepsilon i}$, and $\mu_j = \mu_s + \mu_{\varepsilon_j}$. Solving for θs yields the following parameter values for i, j, with $i \neq j$

$$\theta_{i1} = \frac{a + \varphi\left(\tau_j - 2\tau_i\right)}{3b} + \frac{2\lambda_i\left(2-\gamma_j\right) - \lambda_j\left(2-\gamma_i\right)}{3b\left(4-\gamma_i\gamma_j\right)}, \tag{A.5}$$

and,

$$\theta_{i2} = -\frac{\left(2-\gamma_i\right)}{b\left(4-\gamma_i\gamma_j\right)}. \tag{A.6}$$

Finally, we obtain

$$\widetilde{q}_i = \frac{a + \varphi\left(\tau_j - 2\tau_i\right)}{3b} + \frac{2\lambda_i\left(2-\gamma_j\right) - \lambda_j\left(2-\gamma_i\right)}{3b\left(4-\gamma_i\gamma_j\right)} - \frac{\left(2-\gamma_i\right)\widetilde{c}_i}{b\left(4-\gamma_i\gamma_j\right)};\ \forall i, j, i \neq j. \tag{A.7}$$

B Proof of Proposition 2

To set the optimal tax rule, the risk neutral regulator maximizes the expected welfare defined by:

$$\max_{<\tau_i,\tau_j>} \mathbb{E}_{\widetilde{c}_i,\widetilde{c}_j}\left[\widetilde{W}\left(\tau_i,\tau_j\right)\right] \ni \widetilde{W}\left(\tau_i,\tau_j\right) \equiv -\left(\frac{b+d\varphi^2}{2}\right)\widetilde{Q}^2 + \sum_{i\in N}\left[\left(a - \widetilde{c}_i + \varphi\ \ell'\ \tau_i\right)\widetilde{q}_i\right] \tag{B.1}$$

where $\widetilde{Q} = \widetilde{q}_i + \widetilde{q}_j$ denotes the industry output. Elnaboulsi et *al.* (2018) consider marginal costs' uncertainties under publicly disclosed and private information structure in a similar Stackelberg-Cournot game to ours. The derivation here closely follows their derivation except we consider common and private information with different qualities of the information sources. Hence, the proof of this proposition can be done similarly to their proof. Making use of Lemma 1, maximizing the expected welfare with respect to τ_i and τ_j, and rearranging the FOCs lead to

$$\left(\tau_j - \tau_i\right) = \frac{\left(\mu_j - \mu_i\right) + \ell'\left[\left(\lambda_j + \mu_i\right)k_i - \left(\lambda_i + \mu_j\right)k_j\right]}{2\varphi\ell'} \tag{B.2}$$

$$\begin{aligned}\left(\tau_j + \tau_i\right) \quad = \quad & \frac{4a\omega + \left(2\omega + \ell'\right) + 2a\left(\ell' - 1\right) + \left(\mu_i + \mu_j\right)}{2\varphi\left(\omega + \ell'\right)} + \\ & \frac{\left(2\omega + \ell'\right)\left(\lambda_i k_j + \lambda_j k_i\right) - \left(6\omega + 3\ell'\right)\left(\mu_i k_i + \mu_j k_j\right)}{2\varphi\left(\omega + \ell'\right)}\end{aligned} \tag{B.3}$$

where $k_i = \frac{(2-\gamma_i)}{(4-\gamma_i\gamma_j)}$ and $k_j = \frac{(2-\gamma_j)}{(4-\gamma_i\gamma_j)}$. Using Proposition 1 and solving for τ_i and τ_j give the following tax rates:

$$\tau_i = \frac{(2\omega+\ell'-1)\left[2a-(\mu_i+\mu_j)\right]}{4\varphi(\omega+\ell')} + \frac{(1-\ell')(\mu_i-\mu_j)}{4\varphi\ell'} \tag{B.4}$$

and,

$$\tau_j = \frac{(2\omega+\ell'-1)\left[2a-(\mu_i+\mu_j)\right]}{4\varphi(\omega+\ell')} + \frac{(1-\ell')(\mu_j-\mu_i)}{4\varphi\ell'} \tag{B.5}$$

where $\omega \equiv \left(\frac{1}{3}+\frac{d\varphi^2}{3b}\right)$ and $\ell' = (\ell-1)$. Since we considered that $1 < \ell < 2$ then $0 < \ell' < 1$. To check conditions under which the regulator's objective function is concave, we compute the Hessian matrix of $\mathbb{E}[\widetilde{W}(\tau_i,\tau_j)]$:

$$H_{\widetilde{W}} = \begin{pmatrix} H_{\widetilde{W}}(i,i) = \frac{\partial^2\mathbb{E}[\widetilde{W}]}{\partial\tau_i^2} & H_{\widetilde{W}}(i,j) = \frac{\partial^2\mathbb{E}[\widetilde{W}]}{\partial\tau_j\tau_i} \\ H_{\widetilde{W}}(j,i) = \frac{\partial^2\mathbb{E}[\widetilde{W}]}{\partial\tau_i\tau_j} & H_{\widetilde{W}}(j,j) = \frac{\partial^2\mathbb{E}[\widetilde{W}]}{\partial\tau_j^2} \end{pmatrix} = \left(\frac{-\varphi^2}{3b}\right)\begin{pmatrix} (\omega+4\ell') & (\omega-2\ell') \\ (\omega-2\ell') & (\omega+4\ell') \end{pmatrix} \tag{B.6}$$

$\mathbb{E}[\widetilde{W}(\tau_i,\tau_j)]$, is concave if and only if $H_{\widetilde{W}}$ is negatively definite, namely, the following two conditions hold:

1. the first leading principal minor is negative

$$(\omega+4\ell')\left(\frac{-\varphi^2}{3b}\right) < 0 \tag{B.7}$$

   which is always true because $(\omega+4\ell') > 0$ under our assumptions;

2. and the second leading minor determinant is positive, i.e., $\det H'_{\widetilde{W}} > 0$:

$$\left|H_{\widetilde{W}}\right| = (\omega+4\ell')^2 - (\omega-2\ell')^2 > 0 \Rightarrow (2\omega+2\ell')\,(6\ell') > 0 \tag{B.8}$$

   The second condition holds as long as $\ell' > 0$ which means that $\ell > 1$. Since both conditions are satisfied under our assumptions, the tax rule maximizes the expected welfare.

C Proof of Proposition 4

Collecting all terms not involving μ_i and μ_j into Ψ, we obtain the following expected welfare function:

$$\overline{W}\left(\mu_i, \mu_j\right) = \frac{(1+\ell')^2\left[2a - \left(\mu_i + \mu_j\right)\right]^2}{24b\left(\omega + \ell'\right)} + \frac{(1+\ell')^2\left(\mu_i - \mu_j\right)^2}{8b\ell'} + \Psi \tag{C.1}$$

Differentiating equation (C.1) with respect to μ_i and rearranging the expression yield:

$$\frac{\partial \overline{W}}{\partial \mu_i} = -\frac{(1+\ell')^2\left[2a - \left(\mu_i + \mu_j\right)\right]}{12b\left(\omega + \ell'\right)} + \frac{(1+\ell')^2\left(\mu_i - \mu_j\right)}{4b\ell'} \tag{C.2}$$

Let's assume a large enough to avoid any possibility of shutdown.

1. If $\mu_i \leq \mu_j$, then the right hand side of equation (C.2) is negative. This implies that the social welfare is decreasing with respect to the unconditional expected marginal cost, i.e., $\frac{\partial \overline{W}}{\partial \mu_i} \leq 0$.

2. If $\mu_i > \mu_j$, the sign of $\frac{\partial \overline{W}}{\partial \mu_i}$ is indeterminate as the first term on the right hand side of equation (C.2) is negative, while the second term is positive. It is easy to verify that the social welfare is increasing with respect to the unconditional expected marginal cost, i.e., $\frac{\partial \overline{W}}{\partial \mu_i} > 0$, if and only if:

$$\mu_i > \frac{2a\ell' + \mu_j\left(3\omega + 2\ell'\right)}{4\ell' + 3\omega} \tag{C.3}$$

or equivalently,

$$\left(\mu_i - \mu_j\right)\left(4\ell' + 3\omega\right) > 2\ell'\left(a - \mu_j\right) \tag{C.4}$$

We can write this condition after substituting $\mu_i = \mu_s + \mu_{\varepsilon_i}$ in the last equation, for i, j, with $i \neq j$. This yields

$$\frac{\mu_{\varepsilon_i} - \mu_{\varepsilon_j}}{a - \mu_s - \mu_{\varepsilon_j}} > \frac{2\ell'}{2\ell' + \omega} \tag{C.5}$$

which is true as long as a is large enough and provided that $\mu_{\varepsilon_i} > \mu_{\varepsilon_j}$.

D Proof of Proposition 5

Collecting all terms not involving γ_i and γ_j into Φ, we obtain the following expected welfare function:

$$\overline{W} = -\frac{(b+d\varphi^2)}{2}\left[\frac{(2-\gamma_i)^2\sigma_i^2}{b^2\left(4-\gamma_i\gamma_j\right)^2}+\frac{\left(2-\gamma_j\right)^2\sigma_j^2}{b^2\left(4-\gamma_i\gamma_j\right)^2}+\frac{2\left(2-\gamma_i\right)\left(2-\gamma_j\right)\sigma_s^2}{b^2\left(4-\gamma_i\gamma_j\right)^2}\right] +\frac{(2-\gamma_i)\sigma_i^2}{b\left(4-\gamma_i\gamma_j\right)}++\frac{\left(2-\gamma_j\right)\sigma_j^2}{b\left(4-\gamma_i\gamma_j\right)}+\Phi \tag{D.1}$$

Recall that, $\forall i \in N$, $\sigma_i^2 = \sigma_s^2 + \sigma_{\varepsilon_i}^2$, i.e., $\gamma_i = \frac{\sigma_s^2}{\sigma_i^2}$. Equation (D.1) can be simplified:

$$\overline{W} = \frac{\sigma_s^2(2-\gamma_i)\left[8-6\omega+\gamma_i\left(3\omega-2\gamma_j\right)\right]}{2\gamma_i b\left(4-\gamma_i\gamma_j\right)^2}+\frac{\sigma_s^2\left(2-\gamma_j\right)\left[8-6\omega+\gamma_j\left(3\omega-2\gamma_i\right)\right]}{2\gamma_j b\left(4-\gamma_i\gamma_j\right)^2} -\frac{6\omega\sigma_s^2(2-\gamma_i)\left(2-\gamma_j\right)}{2b\left(4-\gamma_i\gamma_j\right)^2}+\Phi \tag{D.2}$$

To evaluate the sign of the derivative of $\overline{W}$ with respect to γ_i and γ_j, we consider the following function:

$$\begin{aligned}\Omega\left(\gamma_i,\gamma_j\right) &= \frac{2}{3\omega}\frac{b}{\sigma_s^2}\overline{W}\\ &= \left(\frac{4}{3\omega}\right)\frac{\gamma_i+\gamma_j-\gamma_i\gamma_j}{\gamma_i\gamma_j\left(4-\gamma_i\gamma_j\right)}-\frac{4\left(\gamma_i+\gamma_j\right)+\gamma_i\gamma_j\left(2\gamma_i\gamma_j-3\left(\gamma_i+\gamma_j\right)\right)}{\gamma_i\gamma_j\left(4-\gamma_i\gamma_j\right)^2}\\ &\quad+\frac{2}{3\omega}\frac{b}{\sigma_s^2}\Phi\end{aligned} \tag{D.3}$$

Considering the only elements in the expression of $\Omega\left(\gamma_i,\gamma_j\right)$ which depend upon information use and quality, and differentiating equation (D.3) with respect to γ_i and γ_i yield

$$\frac{\partial\Omega}{\partial\gamma_i}\left(\gamma_i,\gamma_j\right) = h\left(\gamma_i,\gamma_j\right)\left(g\left(\gamma_i,\gamma_j\right)-\vartheta\right);\ \forall i,j,i\neq j. \tag{D.4}$$

where $\vartheta = \frac{4}{3\omega}$. $g\left(\gamma_i,\gamma_j\right) = \frac{k\left(\gamma_i,\gamma_j\right)}{h\left(\gamma_i,\gamma_j\right)}$ such that

$$h\left(\gamma_i,\gamma_j\right) = \frac{(2-\gamma_i)\left(2+\gamma_i-\gamma_i\gamma_j\right)}{\gamma_i^2\left(4-\gamma_i\gamma_j\right)^2} \tag{D.5}$$

and

$$k\left(\gamma_i,\gamma_j\right)=\frac{4\gamma_i^2-12\gamma_i\gamma_j-8\gamma_i^2\gamma_j+3\gamma_i^3\gamma_j+6\gamma_i^2\gamma_j^2-2\gamma_i^3\gamma_j^2+16}{\gamma_i^2\left(4-\gamma_i\gamma_j\right)^3} \tag{D.6}$$

It follows that, $\forall\gamma_i,\ \gamma_j\in\left]0,1\right]$,

$$h\left(\gamma_i,\gamma_j\right)>0, \tag{D.7}$$

$$g\left(\gamma_i,\gamma_j\right)-1=\frac{\gamma_i^2\left(2-\gamma_j\right)\left(4+\gamma_i\gamma_j-4\gamma_j\right)}{\left(2+\gamma_i-\gamma_i\gamma_j\right)\left(4-\gamma_i\gamma_j\right)\left(2-\gamma_i\right)}>0, \tag{D.8}$$

and

$$g\left(\gamma_i,\gamma_j\right)-\frac{5}{3}=-\frac{1}{3}f\left(\gamma_i,\gamma_j\right)<0 \tag{D.9}$$

where,

$$f\left(\gamma_i,\gamma_j\right)=\frac{\left(4\left(1-\gamma_i\right)+\gamma_i\gamma_j\right)\left(8\left(1+\gamma_i-\gamma_i\gamma_j\right)+\gamma_i^2\gamma_j\right)}{\left(2+\gamma_i-\gamma_i\gamma_j\right)\left(4-\gamma_i\gamma_j\right)\left(2-\gamma_i\right)}>0 \tag{D.10}$$

Thus, we show that $1<g\left(\gamma_i,\gamma_j\right)<\frac{5}{3},\forall\gamma_i,\ \gamma_j\in\left]0,1\right]$. Then, it can be checked that $\forall\gamma_i,\gamma_j\in[0,1]$, the minimum (respectively the maximum) of the continuous and monotone function $g\left(\gamma_i,\gamma_j\right)$ is reached for $\gamma_i=0$ (respectively $\left(\gamma_i,\gamma_j\right)=(1,0)$). We can summarize our results as follows:

Corollary 1. *For $\frac{5}{3}\leq\vartheta\leq 4$, i.e., $\frac{1}{3}\leq\omega\leq\frac{4}{5}$, the expected welfare function is decreasing with respect to γ_i, and γ_j on $\mathcal{A}=\left]0,1\right]\times\left]0,1\right]$:*

$$\frac{\partial\overline{W}}{\partial\gamma_i}<0\ \textit{and}\ \frac{\partial\overline{W}}{\partial\gamma_j}<0;\forall i,j,i\neq j. \tag{D.11}$$

Corollary 2. *For $0<\vartheta<\frac{5}{3}$, i.e., $\omega>\frac{4}{5}$, we define $\rho=5-3\vartheta$ such that $0<\rho<5$. Let's denote*

$$\begin{cases} A_-^\rho=\left\{\left(\gamma_i,\gamma_j\right)\in\mathcal{A}:f\left(\gamma_i,\gamma_j\right)<\rho\right\}\\ A_0^\rho=\left\{\left(\gamma_i,\gamma_j\right)\in\mathcal{A}:f\left(\gamma_i,\gamma_j\right)=\rho\right\}\\ A_+^\rho=\left\{\left(\gamma_i,\gamma_j\right)\in\mathcal{A}:f\left(\gamma_i,\gamma_j\right)>\rho\right\}\end{cases} \tag{D.12}$$

The family of the non-empty sets $A_-^\rho, A_0^\rho, A_+^\rho$, defines a partition[40] of $\mathcal{A}$, such that

$$\begin{cases} \frac{\partial\overline{W}}{\partial\gamma_i}\left(\gamma_i,\gamma_j\right)=\frac{\partial\overline{W}}{\partial\gamma_j}\left(\gamma_i,\gamma_j\right)>0\Leftrightarrow\left(\gamma_i,\gamma_j\right)\in A_-^\rho\\ \frac{\partial\overline{W}}{\partial\gamma_i}\left(\gamma_i,\gamma_j\right)=\frac{\partial\overline{W}}{\partial\gamma_j}\left(\gamma_i,\gamma_j\right)=0\Leftrightarrow\left(\gamma_i,\gamma_j\right)\in A_0^\rho\\ \frac{\partial\overline{W}}{\partial\gamma_i}\left(\gamma_i,\gamma_j\right)=\frac{\partial\overline{W}}{\partial\gamma_j}\left(\gamma_i,\gamma_j\right)<0\Leftrightarrow\left(\gamma_i,\gamma_j\right)\in A_+^\rho\end{cases} \tag{D.13}$$

[40] A partition, $\mathcal{A}$, divides a support $[0,1]$ into disjoints subsets, $\mathcal{A}=\{A_1,\ldots,A_t\}$, i.e., $\cup_{z=1}^t A_z=[0,1]$ and $A_u\cap A_z=\varnothing$ for all $u,z=1,\ldots,t$ with $u\neq z$.

E Proof of Propositions 7 and 8

Let $\left(q_i^S, q_i^{NS}\right)$, $i \in N$, denote the optimal quantities under sharing (S) and non-sharing (NS) information regimes. Under non-sharing information, the optimal quantities are given by:

$$q_i^{NS} = \frac{a + \varphi\left(\tau_j^{NS} - 2\tau_i^{NS}\right)}{3b} + \frac{\left(\mu_j - 2\mu_i\right)}{3b} - \frac{\left(2-\gamma_i\right)\left(c_i - \mu_i\right)}{b\left(4 - \gamma_i\gamma_j\right)}; \forall i \in N, i \neq j. \tag{E.1}$$

In equilibrium, it is easy to show that, for any i, j, with $i \neq j$, $\mathbb{E}_{c_j}\left[\pi_i^{NS} \middle| c_i\right] = b\left(q_i^{NS}\right)^2$. In the sharing information case, the optimal quantities are given by:

$$q_i^S = \frac{a + \varphi\left(\tau_j^S - 2\tau_i^S\right)}{3b} + \frac{c_j - 2c_i}{3b}; \forall i \in N, i \neq j, \tag{E.2}$$

and the profits are:

$$\pi_i^S = b\left(q_i^S\right)^2; \forall i \in N, i \neq j. \tag{E.3}$$

To analyze whether or not firm i is willing to share information with firm j, it is sufficient then to compare q_i^S and q_i^{NS} for $i \in N$. We define $\nabla q_i \equiv q_i^S - q_i^{NS}$ which can be written as

$$\nabla q_i = \frac{\left(c_j - \mu_j\right) - 2\left(c_i - \mu_i\right)}{3b} + \frac{\left(2-\gamma_i\right)\left(c_i - \mu_i\right)}{b\left(4 - \gamma_i\gamma_j\right)}; \forall i \in N, i \neq j. \tag{E.4}$$

Thus, $\nabla q_i \geq 0$ if and only if $q_i^S \geq q_i^{NS}$. Unless $c_i = \mu_i$ and $c_j = \mu_j$, rearranging and simplifying $\nabla q_i \geq 0$, we obtain

$$\left(c_j - \mu_j\right) \geq \left(c_i - \mu_i\right) z\left(\gamma_i, \gamma_j\right) \tag{E.5}$$

or equivalently,

$$\left(\mu_i - c_i\right) \geq \frac{\left(\mu_j - c_j\right)}{z\left(\gamma_i, \gamma_j\right)} \tag{E.6}$$

where $z\left(\gamma_i, \gamma_j\right) = \left(\frac{2 - 2\gamma_i\gamma_j + 3\gamma_i}{4 - \gamma_i\gamma_j}\right)$. It can be easily checked that, $\forall \gamma_i, \gamma_j \in [0,1]$, $z\left(\gamma_i, \gamma_j\right) > 0$ on $\mathcal{D} = [0,1] \times [0,1]$. Further, we can show that $z\left(\gamma_i, \gamma_j\right)$ reaches its maximum value for $\left(\gamma_i = 1, \gamma_j = 0\right)$, $z_{\max}\left(\gamma_i, \gamma_j\right) = 1.25$, and its minimum value when $\gamma_i = 0$, $\forall \gamma_j \in [0,1]$, $z_{\min}\left(\gamma_i, \gamma_j\right) = 0.5$. Note that, $\left(\mu_i - c_i\right)$ captures the output gap arising from a more efficient distribution of industry output.

We have so far examined firms' incentives to collude. We now need to investigate the welfare implications of such anti-competitive behavior. In particular, we want to know, in the case of information taxation sharing, how possible collusion impacts social welfare. To this end, we write the welfare function in terms of equilibrium outputs. This simplification is purely motivated by its convenience for the calculations and ease of presentation. We can express $W(q_i, q_j)$ as

$$W(q_i, q_j) = \frac{bQ^2}{2} - \frac{d\varphi^2 Q^2}{2} + b\left(q_i^2 + q_j^2\right) + \varphi\left(1 + \ell'\right)\left(\tau_i q_i + \tau_j q_j\right) \tag{E.7}$$

Simplifying and rearranging this expression yield,

$$W(q_i, q_j) = \left(\frac{3b - d\varphi^2}{2}\right)\left(q_i + q_j\right)^2 - 2bq_iq_j + \varphi\left(1 + \ell'\right)\left(\tau_i q_i + \tau_j q_j\right) \tag{E.8}$$

We can decompose the effect of collusion on social welfare by using differential calculus. Let ΔW denotes the welfare total differential:

$$\begin{aligned}\Delta W \;=\; & \left\{\left(3b - d\varphi^2\right)\left(q_i + q_j\right) - 2bq_j + \varphi\left(1 + \ell'\right)\tau_i\right\}\Delta q_i \\ & + \left\{\left(3b - d\varphi^2\right)\left(q_i + q_j\right) - 2bq_i + \varphi\left(1 + \ell'\right)\tau_j\right\}\Delta q_j\end{aligned} \tag{E.9}$$

Rearranging the expression, we can show that

$$\Delta W = b\left(2 - 3\omega\right)Q\left(\Delta q_i + \Delta q_j\right) + 2b\left(q_i \Delta q_i + q_j \Delta q_j\right) + \varphi\left(1 + \ell'\right)\left(\tau_i \Delta q_i + \tau_j \Delta q_j\right) \tag{E.10}$$

where $\omega \geq \max\left\{\frac{1-\ell'}{2}, \frac{1}{3}\right\}$. One can note that the sign of ΔW is indeterminate as the sign of the first term on the right hand side of this expression depends on the value of $(2 - 3\omega)$, while the second and third terms are strictly positive. If $\omega \leq \frac{2}{3}$ then $\Delta W \geq 0$, which means that the overall social effect of collusion is positive. However, for large value of ω, ΔW may be negative, i.e., collusion is welfare reducing.

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