A cooperative game approach to integrated healthcare

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June 2022

Working paper No. 2022 – 06

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Abstract

This article focuses on the sharing of a bundled payment for integrated healthcare. We model this problem by means of cooperative game theory. Various approaches are considered, each of which gives rise to a particular cooperative game, and make it possible to take the chronology of medical events into account. The Shapley value, a priority rule and a proportional allocation rule are used to (partially) refund the healthcare professionals on the basis of the fee paid by the patient and we establish some properties. We also show that the core of some of these aformentioned games is non-empty and can contain these allocation rules.

Keywords: Integrated Healthcare, Healthcare chain, Chronic diseases, Shapley value

1. Introduction

Nowadays, we observe a worrying increase of patients who have chronic diseases (Hackbarth et al., 2008) in different health systems from several countries. This highlights a real problem because the countries must react to treat these patients the most efficiently, within health systems which are very fragmentated (Brekke et al., 2021). Generally, we identify three types of health professionals, from ambulatory medicine (physicians and specialists) to clinics/hospitals and social centers (retirement homes and rest houses for instance) to treat the patient with different degrees of coordination and different market structures including the possibility to have insurers such as in France or Switzerland for instance or not such as in the United States. The patients who have chronic disease or disease which require different chronological interventions will meet these different health professionals to recover in a process of healthcare which is defined from the identification of the chronic disease to the recovering (or death if the disease is uncurable). There are a lot of such chronic diseases from heart, respiratory or renal failure to diabete, but also Covid-19 in certain cases

*I would like to thank Sylvain Béal, Florence Naegelen, Michel Mougeot and Francois Maréchal for fruitful suggestions and also for their advices during the CRESE scientific day. I also thank G022 and AFSE 22 members

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in recent periods. These chronic diseases reduce the life quality and can be very expensive for patients (van Dijk et al., 2014) especially within fee-for-services health systems where the healthcare professionals could have a moral hazard to overuse in order to increase their own interests. Indeed, the healthcare profesionals do not always have the best incentives to treat the patients, and this can lead to a multiplication of consultations, even useless consultations. This problem is all the more relevant as the prospects of OECD (2017) concerning the ageing population are worrying with 9% of the population above 65 years old in 1960 to approximately 25% in two thirds of the OECD countries. Since 2020, Covid-19 has decreased the trend but it is still a challenge for countries and health systems to fight against more and more patients with chronic diseases.

Therefore, in order to increase the quality and coordination of healthcare services, the implementation of a bundled payment should be a credible solution for the future (Porter and Lee, 2013). In such a bundled payment, the patient only pays a fee which has to be shared among the participating healthcare professionals. Indeed, the integrated healthcare presents a lot of advantages, like a better coordination or better incentives between the healthcare profesionals and between the latters and the patients. It can also reduce the problem of fragmentation between services by cooperation (HCAAM, 2015). The implementation of a single bundled payment which covers all healthcare provided should increase the global quality of care (Brekke et al., 2021) and should give a better experience for patients (Stokes et al., 2018). Bundled payments in integrated healthcare are surveyed by Rocks et al. (2020) which shows that "integrated care is likely to reduce cost and improve outcome" through a meta-analysis of 34 studies. The effects of this topical issue are analyzed by some experimentations in a lot of countries since a decade. We refer to HCAAM (2015) for experiments in France, Busse and Stahl (2014) for Germany and England, Struijs and Baan (2011), de Bakker et al. (2012) and Busse and Stahl (2014) for the Netherlands that shows an improvement of the coordination of the care.

In this article, we study this problem by means of cooperative game theory, which is a set of tools relevant to analyse situations where payoffs or costs are generated by a group of agents who have mutual and conflicting interests. Cooperative game theory has been extensively used in applications in recent years (see for instance Champarnaud et al., 2021, for a recent application to revenue sharing in festivals). Here we focus on the final sharing of the bundled payment among the health professionals who participate in patient recovering. Our modelling of bundled payment problem is inspired by the literature on bankruptcy games (O'Neill, 1982; Aumann and Maschler, 1985). A classical bankruptcy game is constructed from an estate which must be shared among several claimants with different claims, the total of which exceeds the available estate. In the context of a bundled payment in healthcare, the claimants are the healthcare professionals, there claims correspond to the total cost of their respective consultations and the estate is the fee paid by patient. We will create four different bankruptcy games depending on which criteria we want to highlight more in the integrated healthcare context: the price of medical events and the order of the timeline to recover. They will also depend on what an healthcare professional claims, i.e her total turnover or what remains of the fee if all other healthcare professionals are already paid.

This article contains three contributions. Firstly, the games that we propose have new structure and depart from the literature on bankruptcy games in two directions. We allow for the possibility of an healthcare professional to act several times during the process of recovering. Furthermore, we take into account the chain of medical events in the sense that the evaluation of the bargaining power of coalitions can depend on the positions of its members in this chain. To the best of our knowledge, Ansink and Weikard (2012) is the only other bankruptcy approach including sequential aspects, in the different context of water sharing along a river. Secondly, we study the main properties of our cooperative games. In particular we prove that three out of four are convex. Convex games possess numerous interesting properties. For example, the Shapley value (Shapley, 1953) yields a core allocation in a convex game. Thirdly, for the above reason, we rely on the Shapley value in order to design an allocation to our bundled payment problem. We describe several properties of the resulting allocation depending on which bankruptcy game the Shapley value is applied to. As an illustration, if the Shapley value is applied to a specific bankruptcy game, it tends to reward more the healthcare profesionals who act at the beginning of the process. For the sake of comparison, we also study the priority rule (Moulin, 2000) and a proportional rule, two other allocation rules based on very different principles, which also belong to the core in some specific games.

The rest of the article is organized as follows. Section 2 provides the definition of the model. Section 3 provides definitions of the tools and the approaches that we use. Section 4 studies the properties of the games. Section 5 and 6 are respectively the study of the aforementioned allocation rules.

2. The model

A **bundled payement problem** is described by a quadruplet $B = (N, \{p_i\}_{i\in N}, C, F)$. The finite set $N = \{1,\dots,n\}$ contains the **healthcare professionals** that a chronic patient needs either for a general appointment or a surgical intervention. The **price** of any visit to healthcare professional $i \in N$ is $p_i > 0$, which can be the consultation price for a physician or specialist and a daily fee for hospitals, for instance. These prices are likely to be very different from one healthcare professional to another. From chronic disease identification to total recovery of the patient (or death if the chronic disease is incurable), the sequence of medical events can be represented by a **chain** C which specifies the order in which the healthcare professionals provide services to the patient. A given healthcare professional can be involved in several **events** of chain C. Formally, the chain is a finite k−dimensional vector:

$$C = (c_1, ..., c_k),$$

where, for each event $q \in \{1, \ldots, k\}$, $c_q \in N$ stands for the unique healthcare professional involved in event q. Moreover, let $q(i)$ denote the first position of C involving an healthcare professional $i \in N$ Finally, instead of paying the full costs of all events in chain C, we assume that the patient only pays a **fee** $F > 0$ for all medical services provided along chain C. The problem of sharing F among the participating heath professionals makes sense under the weak assumption that:

$$\sum_{q \in \{1,\ldots,k\}} p_{c_q} > F. \tag{1}$$

Fee F is implemented to reduce the global cost supported by the patient and depends on the severity of the chronic disease treated by the healthcare professionals. The objective is to share F among the participating health professionals by taking into account the components of problem B.

Example 1. Consider a situation where $N = \{1, 2, 3\}$ in which 1 is a practician, 2 is a laboratory, 3 is a specialist doctor. These are the health professionals who take care of a patient suffering of non-severe diabete. At the beginning, the patient needs to meet her practician. Then, if the practician suspects a disease, she refers the patient to a laboratory to make some blood tests. Then, the practician and the patient receive the results of blood test and the practician decides to meet the patient again. After this appointment, a consultation with a specialist is scheduled. This specialist will give a treatment for the patient. This simple example shows that the patient needs to meet some healthcare professionals chronologically. The associated chain is $C = (c_1, c_2, c_3, c_4)$ with $c_1 = 1$, $c_2 = 2$, $c_3 = 1$ and $c_4 = 3$. It can be summarized by $C = (1, 2, 1, 3)$ and represented as follows:

$$1 \longrightarrow 2 \longrightarrow 1 \longrightarrow 3.$$

Let us consider that the practician charges a price $p_1 = \$25$, the laboratory charges a price $p_2 = \$40$ for a blood test, and the specialist charges a price $p_3 = \$50$. Without the implementation of the integrated healthcare, the patient should have paid $25 + 40 + 25 + 50 = 140$. If the integrated healthcare is implemented, the fee F which represents a unique cost for the patient to meet all the healthcare professional she needs, would be smaller than 140. Let $F = 120$ in order to satisfy (1). □

3. A Cooperative game approach

3.1. Definitions

A **cooperative game** on a player set $N = \{1, ..., n\}$ is a characteristic function v which assigns a worth $v(S)$ to each coalition $S \subseteq N$, and such that $v(\varnothing) = 0$. The worth of coalitions is the total payoff that its members can secure by themselves. In other words, it represents "the best outcome that each subset of the participants ('players') can achieve being unaided". The coalition N of all players is called the grand coalition and is considered as actually formed, the others coming from counterfactual scenarios.

An **allocation** for v is a payoff vector $x = (x_1, ..., x_n)$ which assigns a payoff $x_i \in \mathbb{R}$ to each player $i \in N$ in order to reflect her participation to game v. It is efficient if $\sum_{i \in N} x_i = v(N)$. The **core** of game v is the set $C(v)$ of all efficient allocations x such that no coalition of players S gets a total payoff $\sum_{i \in S} x_i$ smaller than its worth $v(S)$, that is to say not smaller than what it can secure for its members. Formally:

$$C(v) = \left\{ x \in \mathbb{R}^N : \sum_{i \in S} x_i \geq v(S), S \subseteq N, \sum_{i \in N} x_i = v(N) \right\}.$$

The core can be empty. If x is a core allocation, then it is in the interest of no coalition of players to split from the grand coalition. Hence the grand coalition can be considered as stable if its members are paid according to such a core allocation. Stability is sometimes incompatible with fairness considerations.

The Shapley value of a game (Shapley, 1953) is traditionally seen as a fair allocation rule. It is efficient, additive (the Shapley value in the sum of 2 games is the sum of the Shapley values in these 2 games), assigns a null payoff to any player whose marginal contributions to coalitions are null, and assigns an equal payoff to players characterized by identical marginal contributions to coalitions. More specifically, the Shapley value is uniquely characterized by these four properties or axioms, and assigns to a player i in a game v a payoff $Sh_i(v)$ which is a weighted average of all her marginal contributions. If we denote the cardinal of coalition S by $s = |S|$, then the Shapley value of game v is given by:

$$Sh_i(v) = \sum_{S \subseteq N \setminus \{i\}} \frac{s!(n-s-1)!}{n!} \big(v(S \cup \{i\}) - v(S) \big), \quad \forall i \in N.$$

A game is convex if the marginal contributions are non-decreasing with the size of the coalition, that is:

$$v(S) - v(S \setminus \{i\}) \leq v(T) - v(T \setminus \{i\}) \quad \forall i \in S \subseteq T \subseteq N. \tag{2}$$

A bankruptcy problem $(N, E, (c_i)_{i \in N})$ is a situation in which an estate E must be shared among players in N who have claims $c_i \geq 0$, $i \in N$, satisfying:

$$E > \sum_{i \in N} c_i.$$

To each bankruptcy problem, following O'Neill (1982), Aumann and Maschler (1985) and

Curiel et al. (1987), it is possible to associate a cooperative game.
If a coalition hopes to recover as much of its members' claims as the estate allows, then we get the following game:

$$v(S) = \min\left\{E; \sum_{i \in S} c_i\right\}, \quad \forall S \subseteq N,$$

and $v(\emptyset) = 0$. To the contrary, if a coalition only expects to secure what remains of the estate once the other claimants have obtained their claims, then we get a second game:

$$v(S) = \max\left\{0; E - \sum_{i \in N \setminus S} c_i\right\}, \quad \forall S \subseteq N.$$

In the first game, the coalitions are optimistic about their opportunities whereas in the second game they are rather pessimistic. We refer to Thomson (2003) for a survey on bankruptcy problems.

3.2. Integrated healthcare games

In this section, to each $B = (N, \{p_i\}_{i \in N}, C, F)$, we will associate several cooperative games. We proceed in two steps: we begin with extra definitions regarding the chain C (subsection 3.2.1), then we introduce 4 types of cooperative games (subsection 3.2.2).

3.2.1. The chain

How to define what can be the best result for a coalition $S \subseteq N$ when its members cooperate without the other players $N \setminus S$? In other words, where should the chain stop if only the healthcare professionals in S act? The maximal chain for S denoted by $C(S)$ is precisely the set of all events from the beginning of the chain to the first event involving an healthcare professional outside of S. Note that an event involving a healthcare professional $i \in S$ does not belong to $C(S)$ if this event is located after the first event involving an healthcare professional outside of S.

There is the possibility of all healthcare professionals to act more than once in the chain. Therefore, we need a correspondance $N \longrightarrow \{1, ..., k\}$ that associates to each $i \in N$ one or more positions in the chain C. This is done by the inverse function $C^{-1}(i)$ defined as:

$$C^{-1}(i) = \left\{q \in \{1, ..., k\} : c_q = i\right\}, \quad \forall i \in N.$$

Hence, the total cost of events involving i is:

$$\sum_{q \in C^{-1}(i)} p_{c_q} = |C^{-1}(i)| p_i.$$

This total cost can be interpreted as the legitimate claim of health professional i or its bargaining power when sharing F, which refers naturally to the bankruptcy approach.

For each $S \subseteq N$, let

$$C^{-1}(S) = \bigcup_{i \in S} C^{-1}(i),$$

so that, the maximal complete chain for S is :

$$C(S) = \max_{q \in \{1,\dots,k\}:\{1,\dots,q\} \subseteq C^{-1}(S)} (c_1, \dots, c_q). \tag{3}$$

The last position of the maximal complete chain for S is denoted as q_S and the remainder of the chain is called the complementary chain $C \backslash C(S)$, which can contain whoever, even some healthcare professionals in S. Thus:

$$C(S) = (c_1, \dots., c_{q_S}) \quad \text{and} \quad C \backslash C(S) = (c_{q_{S+1}}, \dots, c_k).$$

These concepts of maximal complete chain and complementary chain will be useful to describe two of the four games in the next section.

3.2.2. The different games

In order to apprehend the problem B, we need to evaluate the bargaining power of all coalitions. Four possibilities can be obtained by answering the two following natural questions:

- Shall we account for the position of the healthcare professionals within the process of recovering?
- Should a coalition look at its opportunities with an optimistic or pessimistic view?

We can answer positively to the first question because it makes sense to think that the quality of a patient's care is determined by the first interventions she receives. In particular, the diagnosis is established at the beginning of the process and conditions the subsequent treatment. For example, if the physician refers the patient to a cardiologist for chest pains while the heart is not the source of the problem, it can lead to useless consultations and wrong guidance such as laboratory blood tests, radiology or even emergency service consultation which may lead to nothing. For these reasons, it may be interesting to provide incentives for first healthcare professionals in the process to be particularly efficient. We materialize this idea by relying, for each coalition of healthcare professionals on the maximal chain associated with this coalition. To the contrary, if one does not want to take into acccount the positions of healthcare professionals in the recovering process then, the bargaining power of coalitions can be determined by the full turnover of their members and not just by the turnover induced by the maximal chain of the coalition.

The second question is classical in the literature on bankruptcy problems (O'Neill, 1982; Aumann and Maschler, 1985). The vision of a coalition is optimistic if the coalition expects to get back the portion of its turnover covered by the fee without taking into account the claims of the other healthcare professionals. The vision of a coalition is pessimistic if it expects to obtain only what is left of the fee after each other healthcare professional is refunded the amount of her turnover, if possible. The four approaches are summarized in the following table:

	Optimistic vision	Pessimistic vision
Chain	$w_B^C(S) = \min\left\{F; \sum_{c_q \in C(S)} p_{c_q}\right\}$	$v_B^C(S) = \max\left\{0; F - \sum_{c_q \in C \setminus C(S)} p_{c_q}\right\}$
Not Chain	$u_B(S) = \min\left\{F; \sum_{i \in S} p_i \lvert C^{-1}(i)\rvert\right\}$	$z_B(S) = \max\left\{0; F - \sum_{i \in N \setminus S} p_i \lvert C^{-1}(i)\rvert\right\}$

Games u_B and z_B can be considered as bankruptcy games in which the estate is the fee F and in which the claims are the turnover of healthcare professionals. Games w_B^C and v_B^C are not bankruptcy games because the total claim of a coalition is not equal to the sum of the individual claims of its members.

Example 2. Consider a patient who have a lungs cancer. The problem $B = (N, (p_i)_{i \in N}, C, F)$ to treat this patient involves three healthcare professionals, i.e. $N = \{1, 2, 3\}$. They are respectively the practician, the specialist and the hospital, and we set $p_1 = \$25$, $p_2 = \$50$ and $p_3 = \$80$. The patient needs the following chronological treatment C:

$$1 \longrightarrow 2 \longrightarrow 1 \longrightarrow 3 \longrightarrow 3 \longrightarrow 3 \longrightarrow 1.$$

The total cost of the chain for the patient is $25 + 50 + 25 + 80 + 80 + 80 + 25 = 365$. Finally let $F = \$345$.

The maximal complete chain, its last element and the complementary chain, for each non-empty coalition are presented in the following table:

S	$C(S)$	$C \setminus C(S)$	q_S
$\{1\}$	1	$2 \to 1 \to 3 \to 3 \to 3 \to 1$	1
$\{2\}$	$\varnothing$	$1 \to 2 \to 1 \to 3 \to 3 \to 3 \to 1$	0
$\{3\}$	$\varnothing$	$1 \to 2 \to 1 \to 3 \to 3 \to 3 \to 1$	0
$\{1,2\}$	$1 \to 2 \to 1$	$3 \to 3 \to 3 \to 1$	3
$\{1,3\}$	1	$2 \to 1 \to 3 \to 3 \to 3 \to 1$	1
$\{2,3\}$	$\varnothing$	$1 \to 2 \to 1 \to 3 \to 3 \to 3 \to 1$	0
$\{1,2,3\}$	$1 \to 2 \to 1 \to 3 \to 3 \to 1$	$\varnothing$	7

Consider the coalition $\{1, 2\}$. The maximal complete chain for $\{1, 2\}$ is composed of the first three events (without discontinuity) on the chain because $1 \in S$ and $2 \in S$. The fourth event is assigned to the healthcare professional $3 \notin S$. Therefore, the cardinal of $C(S)$ is the number of elements present in $C(S)$ or the position of the last element of $C(S) : q_S = 3$. The complementary chain is $C \backslash C(S) = 3 \to 3 \to 3 \to 1$, which contains all remaining events. The four games that we propose in the context of this example are described below.

S	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$v_B^C(S)$	5	0	0	80	5	0	345
$w_B^C(S)$	25	0	0	100	25	0	345
$u_B(S)$	75	50	240	125	315	290	345
$z_B(S)$	55	30	220	105	295	270	345

Remember that the worth of each coalition allows to determine the subjective evaluation of their barganinig power in the games. This translate the power relations between the different coalitions. □

4. On the convexity of integrated healthcare games

It is known from Curiel et al. (1987) that game z_B is convex and that z_B and u_B are connected by the duality relation (Aumann and Maschler, 1985). The dual of a game v is the game v^D such that for each $S \subseteq N, v^D(S) = v(N) - v(N \backslash S)$. Since the dual of a convex game is a concave game (Bilbao, 2000), u_B is not a convex game. In this section, we show that both v_B^C and w_B^C are convex which implies that they are not the dual of each other.

Proposition 1. *For any integrated healthcare problem B, (i) the game v_B^C is convex and (ii) the game w_B^C is convex.*

The proof of proposition 1 relies on a lemma. First we need some definitions. Let A be the difference between F and the total cost of chain C:

$$A = F - \sum_{c_q \in C} p_{c_q} < 0.$$

Define a function $\psi : 2^N \longmapsto \mathbb{R}$ which assigns to each $S \subseteq N$, the real number

$$\psi(S) = \sum_{c_q \in C(S)} p_{c_q}.$$

Then, for any $S \subseteq N$, $v_B^C(S)$ and $w_B^C(S)$ can be rewritten as:

$$v_B^C(S) = \max\left\{0; A + \psi(S)\right\}, \quad \text{and} \quad w_B^C(S) = \min\left\{F; \psi(S)\right\}.$$

Lemma 1. *For any integrated healthcare problem B, ψ is a convex game.*

Proof. Recall that for each $R \subseteq N$, q_R is the last position in $C(R)$, and that $q(i)$ stands for the first position held by i in the chain C.
We want to prove (2). So choose $S, T \subseteq N$ and $i \in N$ such that $S \subseteq T \subseteq N \backslash \{i\}$. We consider two cases:

• Suppose $q(i) > q_S$. Then q_T can be smaller or greater than q_i in the following possible chains:

$$q_1 \longrightarrow q_2 \longrightarrow .... \longrightarrow q_S \longrightarrow .... \longrightarrow q(i) \longrightarrow .... \longrightarrow q_T \longrightarrow ..... \longrightarrow q_N,$$

$$q_1 \longrightarrow q_2 \longrightarrow .... \longrightarrow q_S \longrightarrow .... \longrightarrow q_T \longrightarrow .... \longrightarrow q(i) \longrightarrow ..... \longrightarrow q_N.$$

In each case, the marginal contribution of an healthcare professional i to coalition S is null. Since ψ is obviously monotonic, we obtain:

$$\psi(S) - \psi(S \backslash \{i\}) = 0 \leq \psi(T) - \psi(T \backslash \{i\}),$$

• Suppose $q(i) \leq q_S$. Then the chain looks like:

$$q_1 \longrightarrow q_2 \longrightarrow .... \longrightarrow q(i) \longrightarrow .... \longrightarrow q_S \longrightarrow .... \longrightarrow q_T \longrightarrow ..... \longrightarrow q_N,$$

where the case $q_S = q_T$ is possible. The marginal contribution of i to S is the sum of all prices charged between $q(i)$ and q_S along the chain and the marginal contribution of i to T where $S \subseteq T$ is the sum of all prices charged between $q(i)$ and $q(T)$:

$$\psi(S) - \psi(S \backslash \{i\}) = \sum_{q=1}^{q_S} p_{c_q} - \sum_{q=1}^{q(i)-1} p_{c_q} = \sum_{q=q(i)}^{q_S} p_{c_q},$$

$$\psi(T) - \psi(T \backslash \{i\}) = \sum_{q=1}^{q_T} p_{c_q} - \sum_{q=1}^{q(i)-1} p_{c_q} = \sum_{q=q(i)}^{q_T} p_{c_q}.$$

Furthermore, it holds that $q_S \leq q_T$ because $S \subseteq T$. This implies that:

$$\sum_{q=q(i)}^{q_S} p_{c_q} \leq \sum_{q=q(i)}^{q_T} p_{c_q} \Leftrightarrow \psi(S) - \psi(S \backslash \{i\}) \leq \psi(T) - \psi(T \backslash \{i\}),$$

as desired. ∎

Proof. **(Proposition 1) Part** (i): To demonstrate that the game v_B^C is convex, we adopt the proof technique in Curiel et al. (1987). For each $S \subseteq N$, we show that:

$$v_B^C(S \cup \{i\}) - v_B^C(S) \leq v_B^C(T \cup \{i\}) - v_B^C(T)$$

$$\Leftrightarrow v_B^C(S \cup \{i\}) + v_B^C(T) \leq v_B^C(T \cup \{i\}) + v_B^C(S),$$

which is equivalent to

$$\max\left\{0; F - \sum_{c_q \in C \setminus C(S \cup \{i\})} p_{c_q}\right\} + \max\left\{0; F - \sum_{c_q \in C \setminus C(T)} p_{c_q}\right\}$$
$$\leq \max\left\{0; F - \sum_{c_q \in C \setminus C(T \cup \{i\})} p_{c_q}\right\} + \max\left\{0; F - \sum_{c_q \in C \setminus C(S)} p_{c_q}\right\}.$$

Rearranging:

$$\max\left\{0; F - \sum_{c_q \in C \setminus C(T)} p_{c_q}; F - \sum_{c_q \in C \setminus C(S \cup \{i\})} p_{c_q}; 2F - \sum_{c_q \in C \setminus C(S \cup \{i\})} p_{c_q} - \sum_{c_q \in C \setminus C(T)} p_{c_q}\right\}$$
$$\leq \max\left\{0; F - \sum_{c_q \in C \setminus C(S)} p_{c_q}; F - \sum_{c_q \in C \setminus C(T \cup \{i\})} p_{c_q}; 2F - \sum_{c_q \in C \setminus C(T \cup \{i\})} p_{c_q} - \sum_{c_q \in C \setminus C(S)} p_{c_q}\right\}.$$

Using A and ψ, we can rewrite the above inequation as:

$$\max\left\{0; A + \psi(T); A + \psi(S \cup \{i\}); 2A + \psi(S \cup \{i\}) + \psi(T)\right\}$$
$$\leq \max\left\{0; A + \psi(S); A + \psi(T \cup \{i\}); 2A + \psi(T \cup \{i\}) + \psi(S)\right\}.$$

Observe that:

- $A + \psi(T) \leq A + \psi(T \cup \{i\})$ by monotonicity of ψ,
- $A + \psi(S \cup \{i\}) \leq A + \psi(T \cup \{i\})$ by monotonicity of ψ,
- $2A + \psi(S \cup \{i\}) + \psi(T) \leq 2A + \psi(T \cup \{i\}) + \psi(S)$ by Lemma 1.

Since all terms are non-negative, the proof is complete.

Part (ii): We proceed with two cases to prove that:

$$w_B^C(S) - w_B^C(S \setminus \{i\}) \leq w_B^C(T) - w_B^C(T \setminus \{i\}), \quad i \in S \subseteq T.$$

• Suppose $q(i) > q_S$. Then $C(S \cup \{i\}) = C(S)$ which implies that:

$$w_B^C(S) - w_B^C(S \setminus \{i\}) = 0.$$

Furthermore, by monotonicity of w_B^C, we have:

$$w_B^C(T) \geq w_B^C(T\backslash\{i\}) \Longleftrightarrow w_B^C(T) - w_B^C(T\backslash\{i\}) \geq 0,$$

as desired.

- Suppose $q(i) \leq q_S$. In this case, we have:

$$C(S\backslash\{i\}) = C(T\backslash\{i\}) = \left\{c_1, ..., c_{q(i)-1}\right\}$$

This implies:

$$\psi(S\backslash\{i\}) = \psi(T\backslash\{i\}).$$

Hence

$$\min\left\{F; \psi(S\backslash\{i\})\right\} = \min\left\{F; \psi(T\backslash\{i\})\right\},$$

or equivalently $w_B^C(S\backslash\{i\}) = w_B^C(T\backslash\{i\})$. Similarly as before, by monotonicity of w_B^C, we get:

$$w_B^C(T) \geq w_B^C(S).$$

Thus:

$$w_B^C(T) - w_B^C(T\backslash\{i\}) \geq w_B^C(S) - w_B^C(S\backslash\{i\}).$$

Conclude that w_B^C is convex. ■

This result implies other interresting properties. The cores of the games v_B^C and w_B^C (and also z_B) are non-empty (Shapley, 1971) and contain the Shapley value. It is therefore interesting to reward the healthcare professionals by means of the Shapley value. In the next section, we investigate some properties of the resulting allocations, but we also investigate two other allocation rules based on alternative (fairness) principles.

5. The allocation rules

5.1. The Shapley value

The Shapley value satisfies numerous desirable axioms. The desirability axiom (Maschler and Peleg, 1966) is one of them. It states that if a first healthcare professional has marginal contributions to coalitions at least as large as the marginal contributions of a second healthcare professional, then she should obtain a payoff at least as large as the payoff of the second healthcare professional. Let f be an arbitrary allocation rule on any class of games G.

Desirability: For each $v \in G$, for each pair of distinct players $i, j \in N$, such that for each $S \subseteq N\backslash\{i,j\}$, $v(S \cup \{i\}) \geq v(S \cup \{j\})$, then $f_i(v) \geq f_j(v)$.

We make use of this property to prove the following result.

Lemma 2. *The payoffs provided by the Shapley value of game* (N, v_B^C) *are ordered by the position of the first event involving each healthcare professional:*

$$q(i) < q(j) \Longrightarrow Sh_i(v_B^C) \geq Sh_j(v_B^C)$$

Proof. Consider a distinct pair of healthcare professionals $i, j \in N$ such that the first position of the healthcare professional i is previous the first position of the healthcare professional j in the chain, i.e.:

$$q(i) < q(j) \tag{4}$$

Let $S \subseteq N\backslash\{i,j\}$. From (4) we get

$$C(S \cup \{j\}) \subseteq C(S \cup \{i\}),$$

which implies

$$C\backslash C(S \cup \{j\}) \supseteq C\backslash C(S \cup \{i\}).$$

Hence

$$F - \sum_{c_q \in C\backslash C(S\cup\{i\})} p_{c_q} \geq F - \sum_{c_q \in C\backslash C(S\cup\{j\})} p_{c_q},$$

which is equivalent to

$$v_B^C(S \cup \{i\}) \geq v_B^C(S \cup \{j\}).$$

Therefore, the healthcare professional i is at least as desirable as j so that $Sh_i(v_B^C) \geq Sh_j(v_B^C)$ since Sh satisfies the desirability axiom. ∎

Thus, game v_B^C shows that the chronology of medical events is important if the Shapley value is used, because healthcare professionals at the beginning earn not less than professionals subsequent. This allocation can be used to provide incentives for the first healthcare professionals in the process to be particularly efficient.

The next result deals with the following specific situation. Consider the healthcare professional whose first intervention is the latest in the recovery process. Suppose that the total cost that follows this intervention in the chain is at least as large as the fee. Such a case is more likely to occur when the treatment is long. We show that the Shapley value of game v_B^C provides the same payoffs to all healthcare professionals.

Lemma 3. *Let* $q^* = \max_{j \in N} q(j)$. *Assume that* $\sum_{q \geq q^*} p_{c_q} > F$, *then the Shapley value of* v_B^C *provides equal payoffs to all healthcare professionals.*

Proof. Consider the case where q^* is the first position involving the healthcare professional who acts for the first time the latest in the chain:

$$q^* = \max_{j \in N} q(j).$$

Denote by i the healthcare professional who acts in this event q^*. The complementary chain of $C(N\backslash\{i\})$ is equal to the set of all events after the first intervention of i:

$$C \backslash C(N\backslash\{i\}) = \{c_q : q \geq q^*\}$$

Pick $S \neq N$ and consider the following two cases:

• Let $S \not\ni i$, then $\left\{c_q : q \geq q^*\right\} \subseteq C\backslash C(S)$. Thus, the sum of the prices charged from the event q^* to the end of the chain is greater than the fee F. Formally:

$$\sum_{c_q \in C \backslash C(S)} p_{c_q} \geq \sum_{q \geq q^*} p_{c_q} > F. \tag{5}$$

• Let $S \ni i$. The maximal chain of S is included in the maximal chain of $N\backslash\{i\}$. Formally:

$$C(S) \subseteq C(N\backslash\{i\}) \Rightarrow C\backslash C(N\backslash\{i\}) \subseteq C\backslash C(S). \tag{6}$$

From (5) and (6), this implies that whatever the events q occuring after the event q^*:

$$\left\{c_q : q \geq q^*\right\} \subseteq C\backslash C(S),$$

which means that:

$$\sum_{c_q \in C\backslash C(S)} p_{c_q} > F.$$

By definition on v_B^C, this implies that v_B^C is a symmetric game for each $S \subseteq N$:

$$v_B^C(S) = \begin{cases} 0 & \text{if } s \neq n, \\ F & \text{if } s = n. \end{cases}$$

All players are equal in a symmetric game. Since Sh satisfies the well-known axiom of equal treatment of equals, we get:

$$Sh_j(v_B^C) = \frac{F}{n}.$$

for each $j \in N$. ■

Note that Lemma 3 is compatible with Lemma 2 in the sense that the situation of Lemma 3 is extreme case in which all inequalities are weak.

The games u_B and z_B take into account the turnover of the healthcare professionals. The greater are your prices and number of events, the greater is your turnover and the greater is your payoff using Shapley value. That rewards better the most expensive healthcare professionals and the one who have the most number of events.

Lemma 4. *The payoffs provided by the Shapley value of games u_B and z_B are ordered by the amount of turnover involving each healthcare professionals. For each $j \in N \backslash \{i\}$:*

$$p_i|C^{-1}(i)| \geq p_j|C^{-1}(j)| \Longrightarrow \begin{cases} Sh_i(u_B) \geq Sh_j(u_B) \\ Sh_i(z_B) \geq Sh_j(z_B) \end{cases}$$

Proof. Consider $i, j \in N$ such that $p_i|C^{-1}(i)| \geq p_j|C^{-1}(j)|$. Let $S \subseteq N\backslash\{i,j\}$. Then:

$$\sum_{k \in S \cup \{i\}} p_k|C^{-1}(k)| \geq \sum_{k \in S \cup \{j\}} p_k|C^{-1}(k)|,$$

or equivalently,

$$\sum_{k \in S} p_k|C^{-1}(k)| + p_i|C^{-1}(i)| \geq \sum_{k \in S} p_k|C^{-1}(k)| + p_j|C^{-1}(j)|.$$

This shows that $u_B(S \cup \{i\}) \geq u_B(S \cup \{j\})$. The healthcare professional i is at least as desirable than the healthcare professional j. By the axiom of desirability:

$$Sh_i(u_B) \geq Sh_j(u_B).$$

Since $z_B = (u_B)^D$, $Sh(z_B) = Sh(u_B)$. Hence, the result also holds for z_B. ■

5.2. The priority rule

The priority rule (Moulin, 2000) is the allocation rule x^P which rewards the healthcare professionals in the order of their interventions until the fee F is depleted. Thus, all medical events are refunded from the beginning of the process as long as F can be used up. Denote by $\hat{q}$ the penultimate event which is refunded, so that $\hat{q}_{+1}$ will be the last (partially) refunded event. We have:

$$\hat{q} = \text{argmax}\left\{ q \in \{1, ..., k\} : \sum_{r=1}^{q} p_{c_r} < F \right\}.$$

The set of all healthcare professionals who act before the depletion of F is:

$$\hat{S} = \left\{ i \in N : q(i) \leq \hat{q}_{+1} \right\}.$$

For an healthcare professional $i \in N$, $x_i^P(B)$ refunds all medical events involving i before the depletion of F and possibly a part of a medical event if there is a residue of F in the remaining medical event involving i:

$$x_i^P(B) = \begin{cases} \sum_{q \leq \hat{q} : c_q = i} p_{c_q} & \text{if } c_{\hat{q}+1} \neq i, \\ \sum_{q \leq \hat{q} : c_q = i} p_{c_q} + F - \sum_{q=1}^{\hat{q}} p_{c_q} & \text{if } c_{\hat{q}+1} = i. \end{cases}$$

Lemma 5. *The payoffs provided by the priority rule x^P in problem B are in the core of the game w_B^C.*

Proof. Let $S \subseteq N$, we show that $\sum_{i \in S} x_i^P(B) \geq w_B^C(S)$. We consider three cases depending on the link between S and $\hat{S}$.

- Suppose that $S \cap \hat{S} = \varnothing$. Then $C(S) = \varnothing$, so that:

$$\sum_{i \in S} x_i^P(B) = w_B^C(S) = 0.$$

- Suppose that $\hat{S} \subseteq S$. Then $C(\hat{S}) \subseteq C(S)$, which implies, by definition of $\hat{S}$, that:

$$F \leq \sum_{q \leq q_{\hat{S}}} p_{c_q} \leq \sum_{q \leq q_S} p_{c_q}.$$

  Therefore, it holds that:

$$w_B^C(S) = F = \sum_{i \in \hat{S}} x_i^P(B) = \sum_{i \in S} x_i^P(B).$$

- Suppose that $S \cap \hat{S} \neq \{\varnothing; \hat{S}\}$. Then:

$$\sum_{i \in S} x_i^P(B) = \sum_{i \in S \cap \hat{S}} x_i^P(B) \geq \sum_{q \leq q_{S \cap \hat{S}}} p_{c_q} = w_B^C(S \cap \hat{S}) = w_B^C(S).$$

This concludes the proof. ■

We can extend the application of the priority rule to v_B^C.

Lemma 6. *The payoffs provided by the priority rule x^P in problem B are in the core of the game v_B^C.*

Proof. Let $S \subseteq N$, we show that $\sum_{i \in S} x_i^P(B) \geq v_B^C(S)$. We consider three cases:

- Suppose that $S \cap \hat{S} = \varnothing$. Then $C(S) = \varnothing$, so that:

$$\sum_{i \in S} x_i^P(B) = v_B^C(S) = 0.$$

- Suppose that $\hat{S} \subseteq S$. Then $C(\hat{S}) \subseteq C(S)$, which implies:

$$\sum_{i \in S} x_i^P(B) = \sum_{i \in \hat{S}} x_i^P(B) = F \geq v_B^C(S).$$

- Suppose that $S \cap \hat{S} \neq \{\varnothing; \hat{S}\}$. We have:

$$\sum_{i \in S} x_i^P(B) = \sum_{i \in \hat{S} \cap S} x_i^P(B) = \sum_{i \in \hat{S}} x_i^P(B) - \sum_{i \in \hat{S} \setminus S} x_i^P(B).$$

The difference above can be rewriten as:

$$F - \sum_{i \in \hat{S} \setminus S} x_i^P. \tag{7}$$

From (7):

$$F - \sum_{i \in \hat{S} \setminus S} x_i^P \geq F - \sum_{c_q \in C \setminus C(\hat{S} \cap S)} p_{c_q} \geq F - \sum_{c_q \in C \setminus C(S)} p_{c_q}.$$

Hence

$$\sum_{i \in S} x_i^P \geq v_B^C(S).$$

This concludes the proof. ∎

5.3. The proportional allocation rule

The proportional allocation rule y^P is the allocation rule which refunds the healthcare professionals in proportion to their turnover. This allocation rule refunds more healthcare professionals who have the highest turnover in the process of recovering. Formally:

$$y_i^P(S) = \frac{\sum_{i \in S} p_i |C^{-1}(i)|}{\sum_{j \in N} p_j |C^{-1}(j)|} \times F.$$

We will use the proportional rule in the next part to highlight whether it belongs to the core of the considered four games or not.

5.4. The three allocation rules and the core

This section comes back to the three allocation rules that we introduced and to their core membership. To this end, we use our results and examples. Recall that game u_B is as follows in this example:

S	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$u_B(S)$	75	50	240	125	315	290	345

Observe that $u_B(\{2\}) + u_B(\{1,3\}) > u_B(\{1,2,3\})$ which imples that the core is empty. As a consequence, the Shapley value, the priority rule and the proportional rule are not always core elements of game u_B.
We will consider another example to show that the proportional rule does not systematically belong to the core of w_B^C.

Example 3. Three healthcare professionals are involved in the process of recovering of a patient who needs the following chain:

$$1 \longrightarrow 3 \longrightarrow 2 \longrightarrow 2 \longrightarrow 1$$

The prices are $p_1 = \$10$, $p_2 = \$5$ and $p_3 = \$20$, and the fee $F = \$30$. The game w_B^C is given by the following table:

S	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$w_B^C(S)$	10	0	0	10	30	0	30

Concerning the coalition $\{1,3\}$, the proportional rule $y_1^P(w_B^C)+y_3^P(w_B^C) = 24 < 30 = w_B^C(\{1,3\})$. This shows that the proportional rule does not belong to the core of w_B^C. □

For the two other games, we conjecture that the proportional rule is in the core even if we failed to provide a proof so far.
The table below summarizes all these results:

	$C(v_B^C)$	$C(w_B^C)$	$C(u_B)$	$C(z_B)$
Sh	+	+	−	+
x^P	+	+	−	?
y^P	?	−	−	?

The symbol " + " means that the allocation rule belongs to the core of the considered game, the symbol " − " has the convere meaning and the symbol "?" means that it remains to prove whether te allocation rule is core element or not.

6. Conclusion

As a conclusion, cooperative game theory allows to apprehend the problem of integrated healthcare by assuming the cooperation of the different healthcare professionals involved in the process. The exogenous chain generated by the treatment of the disease and the four different games offer the possibility to the actor in charge of the final sharing to choose between different criteria in order to apprehend a same problem, either the financial criteria for which each healthcare professional would like to be refunded with the highest amount, or the timeline process for which the earliest heathcare professionals in the process are the most refunded. There are a lot a different allocation rules and among them, we applied three allocation rules with different principles, an allocation rule with marginality principles (the Shapley value), an allocation rule with priority principles (the Priority rule) and a proportional allocation rule. Despite their differences, we saw that the final sharing can be in the core of a same game (the three allocation rules may belong to the core of v_B^C) and can give some different choices to the actor in charge of the final sharing to respect simultaneously the stability and the fairness or priority criteria. The literature on integrated healthcare shows that patients should benefit because of the improvement of the quality of care (Rocks et al., 2020). Moreover, our results show that healthcare professionals could be refunded with payments that can be stable, fair or prioritised.

We conclude with some remarks. A current debate in the literature is on the identity of the institution in charge of the bundled payment problem and in particular on the ressource sharing among the healthcare professionals. Various options are considered in Hackbarth et al. (2008), Porter (2009) and HCAAM (2015). A first option is to make an healthcare professional responsible of this ressource sharing. Another option is to rely on an invitation to tender in order to select the institution in charge of the final sharing.

Our cooperative game approach should be considered as a first step towards a more complete model that would include strategic interactions between the healthcare professionals. In particular, such a model could be useful to determine how the healthcare professionals will take the chosen allocation rule into account. This is left for future researchs.

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