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**Abstract**

Health is one of the main components of well-being and medical progress has enabled many people to live better lives than at any time in history. Moreover, since the second half of the 20th century, the right to health has been recognized as a human right by international law as well as by many national laws. Unfortunately for many years now - and the phenomenon has become even more acute since COVID-19 pandemic - there has been a worldwide shortage of healthcare workers. This is particularly true for nurses, especially in poor countries. The aim of the paper is to help assess the number of nurses needed to ensure both healthier caregivers and healthier patients. To achieve this goal, we propose a model with random arrivals and exits of patients who may be of a single type (or several), and calculate the average care time they can receive. The results are given in closed form when arrivals follow a Poisson probability distribution. We also propose an analysis of the impact of working conditions on the average time that can be devoted to a patient.

*Keywords : Hospital, random arrivals, random exits, types of patients, available care time, nurses working conditions, Poisson distribution.*

Subject Classification: 60G50, 91A40, 91B32, 91B70

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# 1 Introduction

As part of his work on consumption, poverty, and well-being, which culminated in the award of the Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel in 2015, Angus Deaton wrote a book entitled "The Great Escape", providing an overview of developments in humanity over the last two hundred and fifty years. Referring explicitly to the title of John Sturges' 1963 film, Deaton describes how humanity has escaped hardship and premature death over this period: "Life is better now than at almost any time in history. More people are richer and fewer people live in dire poverty. Lives are longer and parents no longer routinely watch a quarter of their children die. Yet millions still experience the horrors of destitution and of premature death. The world is hugely unequal." [2013, p. 9].

This spectacular evolution, like the glaring inequalities we see between countries and populations, naturally lies in the evolution of various components but, as Deaton points out, income and health are two of the main components of human well-being. Moreover, in the labor-intensive healthcare sector, there is a correlation between the number of doctors and patient prognosis, as demonstrated, for example, in the study by Needleman *et al.* [2002]. The role of nurses is particularly crucial since, as stated by the World Health Organization (WHO [2022]), through their work they promote health, prevent disease, provide primary and community care, deliver care in emergency situations, and their participation is essential to achieving universal health coverage. Nurses are thus at the heart of any healthcare system.

Unfortunately, there is a worldwide shortage of healthcare personnel, and the COVID-19 crisis has exacerbated these shortages. This is particularly true for nurses and midwives, who account for over 50% of the world's current unmet need for healthcare personnel (there is currently a global shortage of 900,000 midwives and 6 million nurses); although the situation is highly uneven, with almost 90% of these shortages concentrated in low-income countries, particularly in Africa and South-East Asia, as the United Nations Regional Information Center (UNRIC [2021]) notes. Looking ahead, WHO estimates that an additional 9 million nurses and midwives will need to be recruited worldwide by 2030 (WHO [2022]) and, Scheffler and Arnold [2019] have predicted a shortfall of 2.5 million nurses in 2030 for 23 OECD countries. In short, demand exceeds supply everywhere.

Those shortages have two major implications.

First, they call into question decades of improvement in the provision of care and patient management, by increasing the risk of death and increasing the loss of opportunity (*e.g.*, Twigg *et al.* [2015], Griffiths *et al.* [2016], Haegdorens *et al.* [2019], Needleman *et al.* [2020], Keck School of Medicine of USC [2023]). By way of illustration, the Royal College of Emergency Medicine (RCEM [2023]) estimates the number of deaths in England at 442 per week for the year 2022 due to deficiencies in emergency care. Equally worrying, on January 8, 2023, the French Minister of Health declared in a televised interview: "We have fewer professionals, fewer nurses, fewer orderlies, so we have beds that are closed. We are not closing them for the sake of closing them, we are closing them because we have fewer staff" (France Info [2023]). And although staff shortages alone do not explain all the hospital bed closures, [2] they do contribute to them.

Second, through the degradation of their working conditions, those shortages also have negative impacts on the physical, emotional, and mental health of caregivers (sleep disorders, stress, ethical suffering, burnout, depression, injuries, etc.) (Rotenstein *et al.* [2016], Duarte *et al.* [2020], Hardy *et al.* [2020], Sexton *et al.* [2022]). These elements thus contribute to increased absenteeism, sick leave, and even abandonment of their profession by many nurses, as highlighted, for example in the USA by the National Council of State Boards of Nursing (NCSBN [2023]), which reports that 100,000 graduate nurses have left the profession since 2020, and that a further 600,000 intend to leave the profession by 2027 due to stress, burnout, and retirement. In other words, we are in a vicious circle where understaffing leads to deterioration, which in turn leads to understaffing.

In view of these realities and their public health implications, all countries around the world are simultaneously seeking to train, retain and recruit nurses, including, at least for the past twenty years, by engaging in international competition, in particular by recruiting those working in low-income countries (*e.g.*, Brush *et al.* [2004], Morgan [2022]), which both the WHO (WHO [2021]) and the International Council of Nurses (ICN [2023]) deplore and try to discourage so as not to further accentuate inequalities between countries and populations.

It is against this backdrop that several countries have begun to consider, without seeing it as a panacea, the creation of ratios of caregivers to patients in hospitals, with the aim of improving both the quality of care and caregivers' working conditions. To our knowledge, state of California (USA) was the first to enacted a law of this type (Assembly Bill n°394, Chap. 945, 1999), followed by the state of Victoria (Autralia), Wales (United Kingdom), Scotland (United Kingdom), Ireland, and the state of Queensland (Australia).[3] This subject is also currently under discussion in France, thanks to *Proposition de loi* n°105 tabled on November 8, 2022 by Senator Bernard Jomier, aimed at modifying the Public Health Code by creating hospital benchmarks in the form of ratios of caregivers per patient.

The aim of our article is to contribute to the analysis of patient/nurse ratios. We propose a way for assessing the number of nurses needed in a hospital department to achieve a given average time for each patient, while complying with legislation on working hours. To this end, we calculate, for a given (potentially random) resource, with random arrivals and exits of patients (which may be of several types), the average time allowed for each patient. To be more precise, let us consider a hospital care unit. During a time step (typically a day), we consider a global care time that medical staff can offer. This global time, which can be random (due to some absenteeism for example) is divided into the patients in the unit. Consider now an additional patient who is coming during the

[2] There is in particular the reorganization of the hospital with the shift to ambulatory care.
[3] For a positive assessment of this last case, we refer the reader to McHugh *et al.* [2021].

time period under study. The question we raise is to know if the medical staff can deal with this additional patient, given that this patient needs a minimum time of care.

Within the theoretical literature, our paper is in line with the game-theoretic dynamic models with random arrivals that Pierre Bernhard has developed with his co-authors in the context of biology (Bernhard and Hamelin [2016]) and economics (Bernhard and Deschamps [2017][2020][2021]). However, our article differs from these in considering that the resource may be random and that there are different types of arrivals (following Biard and Deschamps [2021]).

The paper is organized as follows. In *Section 2*, we present our model in the most simple case where there is only one type of patients (*i.e.*, patients are perfect clones of each other). The arrivals and exits of these patients are random, and we calculate the average time that can be spent on a patient in a hospital service. The results are obtained in closed form for the case of Poisson arrivals. In *Section 3*, the previous results are generalized to the case where there are two types of patients, and it helps to understand how to deal with cases where it would be appropriate to consider a greater number of patient types. Finally, in *Section 4*, we study the influence of working conditions (summarized as a linear function linking the evolution of the resource from period $i$ to period $i+1$) on the average time that can be devoted to a patient.

# 2 One profile of patients

## 2.1 General results

We denote by $\tau_i$ the available care time during time step $i \in \mathbb{N}$ in the hospital unit we focus on. The sequence $(\tau_i)_{i\geq 1}$ can be either deterministic or a sequence of independent random variables with expectation $\bar{\tau}_i := \mathbb{E}[\tau_i]$. In the first case, we have $\bar{\tau}_i = \mathbb{E}[\tau_i] = \tau_i$ so we can use the common notation $\bar{\tau}_i$ for both cases. The main question is to compute the care time available for an additional patient who is coming in the unit during the period under study. We assume in this section that all patients are identical, that is to say that the total care time $\tau_i$ is split equally into the patients. Mathematically, let $N_i$ be the number of other patients in this unit at time $i \in \mathbb{N}$. So, at time $i$, there are $N_i + 1$ patients in the unit. Finally, we assume that the $(\tau_i)_{i\in\mathbb{N}}$ and the $(N_i)_{i\in N}$ are independent, that is to say that the global care time by time step is independent from the number of patients in the unit. The mean care time by patient at time $i \in \mathbb{N}$, denoted by $\bar{t}_i$, is given by

$$\bar{t}_i = \mathbb{E}\left[\frac{\tau_i}{N_i+1}\right]. \tag{1}$$

**Remark 2.1.** *A naive approach could state that the mean care time by patient at step i would be equal to the mean global care time divided by the mean number of patients present at time i. However, it is well known by Jensen inequality (Jensen [1906]) that*

$$\frac{\mathbb{E}[\tau_i]}{\mathbb{E}[N_i+1]} \leq \mathbb{E}\left[\frac{\tau_i}{N_i+1}\right].$$

*As a consequence, the naive approach undervalues the mean care time by patient.*

Let us start with $N_0$ patients in the unit. We assume that $N_0$ is a random variable over $\mathbb{N}$, independent from all other random variables. At each time $i \in \mathbb{N}^*$, $A_i$ new patients arrive in the unit. We assume that $(A_i)_{i\geq 1}$ is a sequence of independent and identically distributed (iid) random variables in $\mathbb{N}$, with common probability-generating function $G(x) := \mathbb{E}[x^{A_1}]$. At time $i \in \mathbb{N}^*$, each patient in the unit at time $i-1$ can leave the unit with probability $q$. Consequently, given $N_{i-1}$, the number of departures at time $i \geq 1$, denoted by $D_i$, follows a binomial distribution with parameters $N_{i-1}$ and $q$:

$$D_i | N_{i-1} \sim Bin(N_{i-1}, q)\,. \tag{2}$$

Finally, we have:

$$N_{i+1} = N_i + A_{i+1} - D_{i+1}\,,\; i \in \mathbb{N}\,. \tag{3}$$

**Proposition 2.2.** *Denote by $\bar{n}_i := \mathbb{E}[N_i]$ the mean number of patients at time $i$, for $i \in \mathbb{N}$. Let us define $\mu := \mathbb{E}[A_1]$ as the mean of the number of arrivals by time step. We have, for all $i \in \mathbb{N}$,*

$$\bar{n}_i = p^i \left(\bar{n}_0 - \frac{\mu}{q}\right) + \frac{\mu}{q}\,,$$

*where $p = 1-q$. By consequence,*

- *if $\bar{n}_0 = \frac{\mu}{q}$, then $\bar{n}_i = \frac{\mu}{q}$ for all $i \in \mathbb{N}$;*
- *if $\bar{n}_0 < \frac{\mu}{q}$, then $\bar{n}_i$ is an increasing sequence;*
- *if $\bar{n}_0 > \frac{\mu}{q}$, then $\bar{n}_i$ is a decreasing sequence.*

*In the three cases, the sequence is convergent with limit equal to*

$$\lim_{i\to\infty} \bar{n}_i = \frac{\mu}{q}\,.$$

*Proof.* From (3), we have

$$\bar{n}_{i+1} = \bar{n}_i + \mu - \mathbb{E}[D_{i+1}]\,.$$

From (2), $\mathbb{E}[D_{i+1}] = q\bar{n}_i$, so

$$\bar{n}_{i+1} = (1-q)\bar{n}_i + \mu\,.$$

All results come from the fact that $(\bar{n}_i)_{i\in\mathbb{N}}$ is a linear sequence. □

This result states that the mean number of patients in the care unit is quite regular and tends to the ratio between the mean number of arrivals at each time over the probability of leaving. The behavior of this mean number only depends on the initial value. If it is lower (resp. higher) than the limit ratio, then the mean number of patients increases (resp. decreases). The particular case where the initial value is equal to the limit gives a constant mean number of patients (see Figure 1 for an illustration).

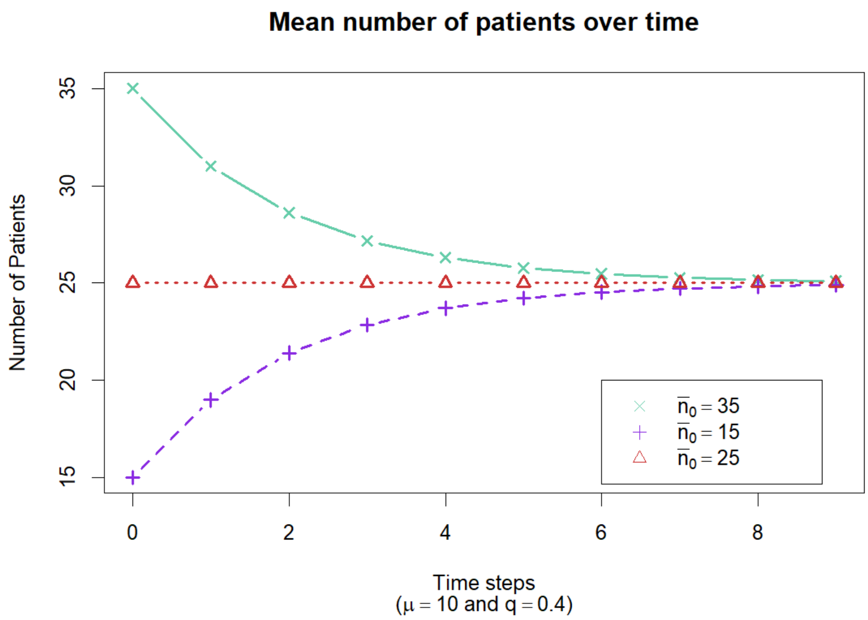


Figure 1: The mean number of patients is either constant if $\bar{n}_0 = \frac{\mu}{q}$, decreasing if $\bar{n}_0 > \frac{\mu}{q}$, or increasing if $\bar{n}_0 < \frac{\mu}{q}$. See Proposition 2.2.

**Remark 2.3.** *Assume here that $N_0$ is distributed as $A_1$, that is to say that the number of patients at time 0 is distributed as the number of incoming patients during other times. In this case, since $\bar{n}_0 = \mu < \mu/q$, the sequence $(\bar{n}_i)_{i\geq 0}$ is increasing.*

**Lemma 2.4.** *For $i \in \mathbb{N}$, denote by $G_i(x) := \mathbb{E}[x^{N_i}]$ the probability-generating function of $N_i$. We have, for all $i \in \mathbb{N}$,*

$$G_{i+1}(x) = G(x)G_i(px + q)$$

*Proof.* From (2), $D_{i+1}$ can be rewritten as:

$$D_{i+1} = \sum_{k=1}^{N_i} I_k \,,$$

where $(I_k)_{k\geq 1}$ is an iid sequence of Bernoulli random variables with parameter $q$. Thus, from (3),

$$N_{i+1} = \sum_{k=1}^{N_i} (1 - I_k) + A_{i+1} \,.$$

Since $A_{i+1}$ and $N_i$ are independent, we have:

$$G_{i+1}(x) = G(x) G_i \circ G_{1-I_1}(x) \,,$$

where $G_{1-I_1}(x) := \mathbb{E}[x^{1-I_1}] = px + q$ since $1 - I_1$ follows a Bernoulli distribution with parameter $1 - q = p$. $\square$

**Lemma 2.5.** *For $i \in \mathbb{N}^*$, we have*

$$G_i(x) = G_0(p^i(x-1)+1) \prod_{k=0}^{i-1} G\left(p^k(x-1)+1\right).$$

*Proof.* By induction. For $i = 0$, using Lemma 2.4,

$$G_1(x) = G(x)G_0(px+q) = G_0(p^1(x-1)+1) \prod_{k=0}^{0} G(p^0(x-1)+1).$$

From step $i+1$ given step $i$, we use again Lemma 2.4:

$$\begin{aligned}
G_{i+1}(x) &= G(x)G_i(px+q)\,, \\
&= G(x)G_0(p^i(px+q-1)+1) \prod_{k=0}^{i-1} G\left(p^k(px+q-1)+1\right), \\
&= G(x)G_0(p^i(px-p)+1) \prod_{k=0}^{i-1} G\left(p^k(px-p)+1\right), \\
&= G_0(p^{i+1}(x-1)+1)G(x) \prod_{k=0}^{i-1} G\left(p^{k+1}(x-1)+1\right).
\end{aligned}$$

Since

$$G(x) = G(p^0(x-1)+1),$$

and

$$\prod_{k=0}^{i-1} G\left(p^{k+1}(x-1)+1\right) = \prod_{k=1}^{i} G\left(p^k(x-1)+1\right),$$

we get the result. □

**Proposition 2.6.** *We have*

$$\bar{t_0} = \bar{\tau}_0 \int_0^1 G_0(x)dx,$$

*and for $i \geq 1$,*

$$\bar{t}_i = \bar{\tau}_i \int_0^1 G_0(p^i(x-1)+1) \prod_{k=0}^{i-1} G\left(p^k(x-1)+1\right) dx.$$

*Proof.* From (1),

$$\bar{t}_i = \mathbb{E}\left[\frac{\tau_i}{N_i+1}\right].$$

Since $\tau_i$ and $N_i$ are independent,

$$t_i = \bar{\tau}_i \mathbb{E}\left[\frac{1}{N_i+1}\right].$$

We have

$$\mathbb{E}\left[\frac{1}{N_i+1}\right] = \mathbb{E}\left[\int_0^1 x^{N_i}dx\right] = \int_0^1 \mathbb{E}\left[x^{N_i}\right] dx\,.$$

Since $\mathbb{E}[x^{N_i}] = G_i(x)$ and using Lemma 2.5, we get the result. □

As a consequence, to get the mean care time by patient at time $i$, we only need the probability generating functions of the initial number and the number of arrivals by time step since the integral is easy to get numerically. Note that two different distributions for the arrivals give two different results even if the mean numbers of arrivals are equal.

## 2.2 The Poisson case

In this subsection, we assume that $N_0$ is Poisson distributed with parameter $\lambda_0$ and the $A_i$'s are Poisson distributed with parameter $\lambda$.

**Proposition 2.7.** *We have, for all $i \in \mathbb{N}$,*

$$\bar{n}_i = p^i \left( \lambda_0 - \frac{\lambda}{q} \right) + \frac{\lambda}{q}\,.$$

*By consequence,*

- *if $\lambda_0 = \frac{\lambda}{q}$, then $\bar{n}_i = \frac{\lambda}{q}$ for all $i \in \mathbb{N}$;*
- *if $\lambda_0 < \frac{\lambda}{q}$, then $\bar{n}_i$ is an increasing sequence;*
- *if $\lambda_0 > \frac{\lambda}{q}$, then $\bar{n}_i$ is a decreasing sequence.*

*In the three cases, the sequence is convergent with limit equal to*

$$\lim_{i \to \infty} \bar{n}_i = \frac{\lambda}{q}\,.$$

*Proof.* The results are the direct consequence of Proposition 2.2 since the mean number of new arrivals is equal to $\lambda$ and the mean number of patients present at time 0 is $\lambda_0$. □

**Proposition 2.8.** *We have,*

$$\bar{t_0} = \frac{\bar{\tau}_0}{\lambda_0} \left(1 - e^{-\lambda_0}\right)\,, \tag{4}$$

*and for $i \in \mathbb{N}^*$,*

$$\bar{t}_i = \frac{q\bar{\tau}_i}{(q\lambda_0 - \lambda)p^i + \lambda} \left( 1 - \exp\left\{ -\frac{(q\lambda_0 - \lambda)\,p^i + \lambda}{q} \right\} \right)\,. \tag{5}$$

*In the case where $\lambda_0 = \lambda$, we have, for all $i \in \mathbb{N}$,*

$$\bar{t}_i = \frac{q\bar{\tau}_i}{\lambda\,(1 - p^{i+1})} \left( 1 - \exp\left\{ -\lambda \frac{1 - p^{i+1}}{q} \right\} \right)\,. \tag{6}$$

*Proof.* Since the $A_i$'s are Poisson distributed, we have

$$G(x) = e^{\lambda(x-1)}\,.$$

For $N_0 \sim \mathcal{P}(\lambda_0)$, we also have

$$G_0(x) = e^{\lambda_0(x-1)} .$$

So we easily get (4) from Proposition (2.6) and we have

$$G_0(p^i(x-1)+1) = e^{\lambda_0 p^i(x-1)} .$$

Since

$$\prod_{k=0}^{i-1} \exp\left\{\lambda\left((p^k(x-1)+1)-1\right)\right\} = \exp\left\{\lambda(x-1)\frac{1-p^i}{q}\right\} ,$$

we get

$$G_0(p^i(x-1)+1)\prod_{k=0}^{i-1} G\left(p^k(x-1)+1\right) = \exp\left\{\left(\lambda_0 p^i + \lambda\frac{1-p^i}{q}\right)(x-1)\right\} .$$

Then, we apply Proposition 2.6 to get (5). Taking $\lambda_0 = \lambda$ in (4) and (5) gives (6). $\square$

In the Poisson case, results are totally explicit. In particular, that allows us to study of the behavior of the mean care time by patient over time (see Proposition 2.9).

**Proposition 2.9.** *Assume $\bar{\tau}_i$ is constant over time and denote by $\bar{\tau}$ their common value. We have*

- *if $\lambda_0 = \dfrac{\lambda}{q}$, then $\bar{t}_i = \dfrac{q\bar{\tau}}{\lambda}\left(1-e^{-\frac{\lambda}{q}}\right)$ for all $i \in \mathbb{N}$;*
- *if $\lambda_0 < \dfrac{\lambda}{q}$, then $\bar{t}_i$ is a decreasing sequence;*
- *if $\lambda_0 > \dfrac{\lambda}{q}$, then $\bar{t}_i$ is an increasing sequence.*

*In the three cases, the sequence is convergent with limit equal to*

$$\lim_{i\to\infty} \bar{t}_i = \frac{q\bar{\tau}}{\lambda}\left(1-e^{-\frac{\lambda}{q}}\right) .$$

*Proof.* Letting $\tau_i = \tau$ for all $i$ in Proposition 2.8 gives

$$\bar{t}_i = \frac{q\bar{\tau}}{(q\lambda_0-\lambda)p^i+\lambda}\left(1-\exp\left\{-\frac{(q\lambda_0-\lambda)\,p^i+\lambda}{q}\right\}\right) ,$$

which can been rewritten as

$$\bar{t}_i = \bar{\tau} f(X(i)),$$

with $X(i) = \frac{(q\lambda_0-\lambda)p^i+\lambda}{q} > 0$ and $f(X) = \frac{1-e^{-X}}{X}$. Since $X'(i) = \frac{\ln(p)p^i(q\lambda_0-\lambda)}{q}$ and $\ln(p) < 0$, we have that

- $X(i)$ is constant if $\lambda_0 = \dfrac{\lambda}{q}$;

- $X(i)$ is increasing if $\lambda_0 < \dfrac{\lambda}{q}$;

- $X(i)$ is decreasing if $\lambda_0 > \dfrac{\lambda}{q}$.

Since $f'(X) = \frac{e^{-X}X-(1-e^{-X})}{X^2} = \frac{X+1-e^X}{e^X X^2}$, the sign of $f'(X)$ is the sign of $X + 1 - e^X$. Since $(X + 1 - e^X)' = 1 - e^X$ which is non positive on $(0, +\infty)$, $X \mapsto X + 1 - e^X$ is decreasing on $(0, +\infty)$, so $X + 1 - e^X \leq 0$ for $X \in (0, +\infty)$ since $0 + 1 - e^0 = 0$. As a consequence, $f'(X) < 0$ for $X \in (0, +\infty)$ and $f$ is decreasing on $(0, +\infty)$, which ends the proof. $\square$

In other words, if the mean care time is constant over time, the care time per patient is quite regular and its long-term behavior does not depend on the initial conditions. This initial value determines the monotony of this individual time only through the comparison between the initial mean number of patients and the ratio of the mean new patients per period and the probability of a departure. Figure 2 represents the three behaviors of the sequence $(\bar{t}_i)_{i\in\mathbb{N}}$ and its convergence.

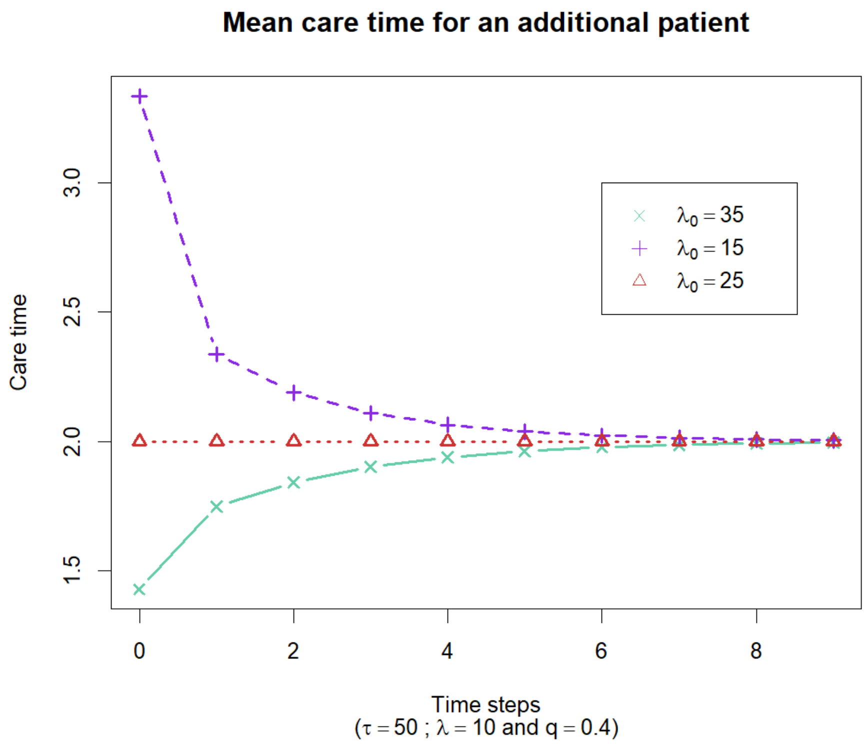


Figure 2: The mean care time for an additional patient is either constant if $\lambda_0 = \frac{\lambda}{q}$, decreasing if $\lambda_0 < \frac{\lambda}{q}$, or increasing if $\lambda_0 > \frac{\lambda}{q}$. See Proposition 2.9.

# 3 Two profiles of patients

In this section, only two profiles of patients are investigated. Note that the extension to more than two patients types could be obtained using the same methods. Nevertheless, expressions for more than two patients types are quite tedious and that is the reason why it is decided not to present them for the sake of clarity.

## 3.1 General results

We assume throughout this section that two different types of patients arrive in the unit. The two profiles are called type $A$ and type $B$. A $B$-patient needs $\alpha$ more care time than a $A$-patient. We furthermore assume that $\alpha \in (1, +\infty)$. Denote by $N_i^{(A)}$ (resp. $N_i^{(B)}$) the number of $A$-patients (resp. $B$-patients) who are present in the unit at time $i \in \mathbb{N}$. We also consider that both $\left(N_i^{(A)}\right)_{i\geq 1}$ and $\left(N_i^{(B)}\right)_{i\geq 1}$ are iid sequences and independent from one another. As previously, the number of patients are independent from the available care time $(\tau_i)_{i\in\mathbb{N}}$. The mean care time for an additional $A$-patient who arrives at time $i \in \mathbb{N}$, denoted by $\bar{t}_i^{(A)}$, is given by

$$\bar{t}_i^{(A)} = \mathbb{E}\left[\frac{\tau_i}{(N_i^{(A)}+1) + \alpha N_i^{(B)}}\right], \tag{7}$$

and the time of care for an additional $B$-patient, denoted by $\bar{t}_i^{(B)}$, is

$$\bar{t}_i^{(B)} = \mathbb{E}\left[\frac{\alpha\tau_i}{N_i^{(A)} + \alpha(N_i^{(B)}+1)}\right].$$

The processes $(N_i^{(A)})_{i\geq 0}$ and $(N_i^{(B)})_{i\geq 0}$ evolve similarly to that of $(N_i)_{i\geq 1}$ in Section 2. Explicitly, let us start with $N_0^{(A)}$ of $A$-patients and $N_0^{(B)}$ of $B$-patients in the unit. We assume that $N_0^{(A)}$ and $N_0^{(B)}$ are two random variables in $\mathbb{N}$, independent from all other random variables, and mutually independent. Let $A_i^{(A)}$ (resp. $A_i^{(B)}$) the number of new $A$-patients (resp. $B$-patients) at time $i$. Both sequences $(A_i^{(A)})_{i\geq 1}$ and $(A_i^{(B)})_{i\geq 1}$ are iid sequences of positive random variables in $\mathbb{N}$ and mutually independent. Denote by $G_{A^{(A)}}(x)$ (resp. $G_{A^{(A)}}(x)$) the common probability-generation function of the $A_i^{(A)}$'s (resp. $A_i^{(B)}$'s). Each $A$-patient (resp. $B$-patient) present at time $i-1$ can leave the unit at time $i \in \mathbb{N}^*$ with probability $q^{(A)}$ (resp. $q^{(B)}$). Consequently, the number of departures of $A$-patients (resp. $B$-patients) at time $i$, denoted by $D_i^{(A)}$ (resp. $D_i^{(B)}$), given $N_{i-1}^{(A)}$ (resp. $N_{i-1}^{(B)}$), follows a binomial distribution with parameters $N_{i-1}^{(A)}$ and $q^{(A)}$ (resp. $N_{i-1}^{(B)}$ and $q^{(B)}$):

$$D_i^{(A)}|N_{i-1} \sim Bin(N_{i-1}^{(A)}, q^{(A)})\,, \tag{8}$$

and

$$D_i^{(B)}|N_{i-1} \sim Bin(N_{i-1}^{(B)}, q^{(B)})\,. \tag{9}$$

Finally, we have:

$$N_{i+1}^{(A)} = N_i^{(A)} + A_{i+1}^{(A)} - D_{i+1}^{(A)}\,,\; i \in \mathbb{N}\,, \tag{10}$$

and

$$N_{i+1}^{(B)} = N_i^{(B)} + A_{i+1}^{(B)} - D_{i+1}^{(B)}\,,\; i \in \mathbb{N}\,. \tag{11}$$

**Proposition 3.1.** *Denote by $\bar{n}_i^{(A)} := \mathbb{E}[N_i^{(A)}]$ the mean number of $A$-patients at time $i$ and $\bar{n}_i^{(B)} := \mathbb{E}[N_i^{(B)}]$ the mean number of $B$-patients at time $i$. Let $\bar{n}_i := \bar{n}_i^{(A)} + \bar{n}_i^{(B)}$ the total number of patients at time $i$. Let us define $\mu^{(A)} := \mathbb{E}[A_1^{(A)}]$ and $\mu^{(B)} := \mathbb{E}[A_1^{(B)}]$ the mean number of arrivals for $A$-patients and $B$-patients by time step, respectively. We have*

$$\bar{n}_i^{(A)} = {p^{(A)}}^i \left(\bar{n}_0^{(A)} - \frac{\mu^{(A)}}{q^{(A)}}\right) + \frac{\mu^{(A)}}{q^{(A)}},$$

*and*

$$\bar{n}_i^{(B)} = {p^{(B)}}^i \left(\bar{n}_0^{(B)} - \frac{\mu^{(B)}}{q^{(B)}}\right) + \frac{\mu^{(B)}}{q^{(B)}},$$

*where $p^{(A)} = 1 - q^{(A)}$ and $p^{(B)} = 1 - q^{(B)}$. So*

$$\bar{n}_i = {p^{(A)}}^i \left(\bar{n}_0^{(A)} - \frac{\mu^{(A)}}{q^{(A)}}\right) + {p^{(B)}}^i \left(\bar{n}_0^{(B)} - \frac{\mu^{(B)}}{q^{(B)}}\right) + \frac{\mu^{(A)}}{q^{(A)}} + \frac{\mu^{(B)}}{q^{(B)}}\,.$$

*Letting $n$ goes to infinity gives*

$$\lim_{i\to\infty} \bar{n}_i^{(A)} = \frac{\mu^{(A)}}{q^{(A)}}\,,\; \lim_{i\to\infty} \bar{n}_i^{(B)} = \frac{\mu^{(B)}}{q^{(B)}},\ \textit{and}\ \lim_{i\to\infty} \bar{n}_i = \frac{\mu^{(A)}}{q^{(B)}} + \frac{\mu^{(B)}}{q^{(B)}}\,.$$

*Proof.* The proof is similar to the one of Proposition 2.2 since (10) and (11) are similar to (3). □

The behaviors of the mean number of $A$-patients or $B$-patients are similar to the mean number of patients in the one-profile case (see Proposition 2.2). In particular, the decrease or the increase of theses numbers only depends on the position of the initial mean numbers with respect to the limit values, which are equal to the ratio between the mean number of arrivals and the probability of leaving for each type of patients. The discussion on the total number of patients is done in Remark 3.2.

**Remark 3.2.** *If $\mu^{(A)}/q^{(A)} < (>) \bar{n}_0^{(A)}$, then the population of $A$-patients increases (decreases) with time. The same results hold for the $B$-patients. Obviously, if the two populations increase (decrease), the total number of patients increases (decreases) too. It is more complicated when one population increases and the other decreases. Indeed, many situations can appear (an increase of the total number of patients all along the time, or a decrease all along the time, or a decrease then an increase or an increase then a decrease). However, we can show (studying $i \mapsto \bar{n}_i$) that there is one change at most and the monotony changes after time:*

$$T_C = \left\lceil \frac{\ln\left(-\frac{\ln(p^{(B)})}{\ln(p^{(A)})} \frac{\bar{n}_0^{(B)} - \mu^{(B)}/q^{(B)}}{\bar{n}_0^{(A)} - \mu^{(A)}/q^{(A)}}\right)}{\ln\left(p^{(A)}/p^{(B)}\right)} \right\rceil,$$

*where $\lceil x \rceil$ is the ceiling of $x$. This configuration can be seen in Figure 3.*

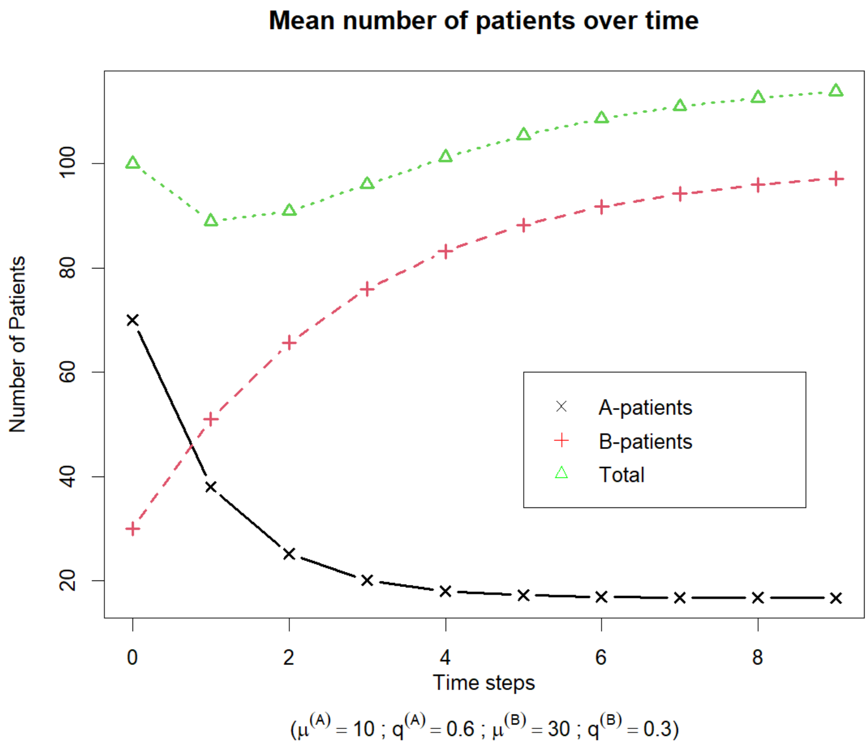


Figure 3: One situation with decreasing mean number of $A$-patients, increasing mean number of $B$-patients, and a fluctuation in the total mean number (see Remark 3.2).

**Proposition 3.3.** *We have*

$$\bar{t}_0^{(A)} = \bar{\tau}_i \int_0^1 G_0^{(A)}(x) G_0^{(B)}(x^\alpha) dx,$$

*and*

$$\bar{t}_0^{(B)} = \alpha \bar{\tau}_i \int_0^1 x^{\alpha-1} G_0^{(A)}(x) G_0^{(B)}(x^\alpha) dx.$$

*For $i \geq 1$, we have*

$$\bar{t}_i^{(A)} = \bar{\tau}_i \int_0^1 G_0^{(A)}(p^i(x-1)+1) G_0^{(B)}(p^i(x^\alpha - 1)+1) \\ \prod_{k=0}^{i-1} G^{(A)}\left(p^k(x-1)+1\right) G^{(B)}\left(p^k(x^\alpha-1)+1\right) dx,$$

*and*

$$\bar{t}_i^{(B)} = \alpha\bar{\tau}_i \int_0^1 x^{\alpha-1} G_0^{(A)}(p^i(x-1)+1) G_0^{(B)}(p^i(x^\alpha-1)+1) \prod_{k=0}^{i-1} G^{(A)}\left(p^k(x-1)+1\right) G^{(B)}\left(p^k(x^\alpha-1)+1\right) dx.$$

*Proof.* From (7),

$$\bar{t}_i^{(A)} = \mathbb{E}\left[\frac{\tau_i}{(N_i^{(A)}+1)+\alpha N_i^{(B)}}\right].$$

Since $\tau_i$, $N_i^{(A)}$ and $N_i^{(B)}$ are independent,

$$\bar{t}_i^{(A)} = \tau_i \mathbb{E}\left[\frac{1}{(N_i^{(A)}+1)+\alpha N_i^{(B)}}\right].$$

We have

$$\mathbb{E}\left[\frac{1}{(N_i^{(A)}+1)+\alpha N_i^{(B)}}\right] = \mathbb{E}\left[\int_0^1 x^{N_i^{(A)}+\alpha N_i^{(B)}} dx\right] = \int_0^1 \mathbb{E}\left[x^{N_i^{(A)}+\alpha N_i^{(B)}}\right] dx\,.$$

For $\bar{t}_i^{(B)}$, from (3.1),

$$\bar{t}_i^{(B)} = \mathbb{E}\left[\frac{\alpha\tau_i}{N_i^{(A)}+\alpha(N_i^{(B)}+1)}\right].$$

Since $\tau_i$, $N_i^{(A)}$ and $N_i^{(B)}$ are independent,

$$\bar{t}_i^{(B)} = \alpha\tau_i \mathbb{E}\left[\frac{1}{N_i^{(A)}+\alpha N_i^{(B)}+\alpha}\right].$$

We have

$$\begin{aligned}\mathbb{E}\left[\frac{1}{N_i^{(A)})+\alpha N_i^{(B)}+\alpha}\right] &= \mathbb{E}\left[\int_0^1 x^{N_i^{(A)}+\alpha N_i^{(B)}+\alpha-1} dx\right] \\ &= \int_0^1 x^{\alpha-1}\mathbb{E}\left[x^{N_i^{(A)}+\alpha N_i^{(B)}}\right] dx\,.\end{aligned}$$

Since $N_i^{(A)}$ and $N_i^{(B)}$ are independent,

$$\mathbb{E}\left[x^{N_i^{(A)}+\alpha N_i^{(B)}}\right] = \mathbb{E}\left[x^{N_i^{(A)}}\right]\mathbb{E}\left[x^{\alpha N_i^{(B)}}\right] = G^{(A)}(x)G^{(B)}(x^\alpha)\,.$$

Since (10) and (11) are similar to (3), then Lemma 2.5 applies for $G^{(A)}(x)$ and $G^{(B)}(x^\alpha)$, which ends the proof. □

Given the probability-generating functions of the number of new incomers of both types and the probability-generating functions of initial numbers of both type, these results give the care time that a new patient of both type could expect at step $i$, $\bar{t}_i^{(A)}$ and $\bar{t}_i^{(B)}$. Let us recall here that a $B$-patient needs twice as much care time as a $A$-patient. We notice that $\bar{t}_i^{(B)}$ is not equal to $\alpha\bar{t}_i^{(A)}$ but also depends on the dynamics of both numbers of $A$-patients and $B$-patients.

## 3.2 The Poisson case

In this subsection, we assume that $N_0^{(A)}$ is Poisson distributed with parameter $\lambda_0^{(A)}$, $N_0^{(B)}$ is Poisson distributed with parameter $\lambda_0^{(B)}$, the $A_i^{(A)}$'s are Poisson distributed with parameter $\lambda^{(A)}$, and the $A_i^{(B)}$'s are Poisson distributed with parameter $\lambda^{(B)}$.

**Proposition 3.4.** *We have*

$$\bar{n}_i^{(A)} = {p^{(A)}}^i \left(\lambda_0^{(A)} - \frac{\lambda^{(A)}}{q^{(A)}}\right) + \frac{\lambda^{(A)}}{q^{(A)}}\,,$$

*and*

$$\bar{n}_i^{(B)} = {p^{(B)}}^i \left(\lambda_0^{(B)} - \frac{\lambda^{(B)}}{q^{(B)}}\right) + \frac{\lambda^{(B)}}{q^{(B)}}\,.$$

*So*

$$\bar{n}_i = {p^{(A)}}^i \left(\lambda_0^{(A)} - \frac{\lambda^{(A)}}{q^{(A)}}\right) + {p^{(B)}}^i \left(\lambda_0^{(B)} - \frac{\lambda^{(B)}}{q^{(B)}}\right) + \frac{\lambda^{(A)}}{q^{(A)}} + \frac{\lambda^{(B)}}{q^{(B)}}\,.$$

*Letting $n$ goes to infinity gives*

$$\lim_{i\to\infty} \bar{n}_i^{(A)} = \frac{\lambda^{(A)}}{q^{(A)}}\,,\ \lim_{i\to\infty} \bar{n}_i^{(B)} = \frac{\lambda^{(B)}}{q^{(B)}}\ ,\ \textit{and}\ \lim_{i\to\infty} \bar{n}_i = \frac{\lambda^{(A)}}{q^{(B)}} + \frac{\lambda^{(B)}}{q^{(B)}}\,.$$

*Proof.* The results are direct consequences of Proposition 3.1 since the mean numbers of new arrivals are respectively equal to $\lambda^{(A)}$ and $\lambda^{(B)}$ and the mean numbers of patients present at time 0 are, respectively, $\lambda_0^{(A)}$ and $\lambda_0^{(B)}$ for $A$-patients and $B$-patients. □

**Proposition 3.5.** *We have*

$$\bar{t}_0^{(A)} = \bar{\tau}_i e^{-\left(\lambda_0^{(A)}+\lambda_0^{(B)}\right)} \int_0^1 \exp\left\{\lambda_0^{(A)} x + \lambda_0^{(B)} x^\alpha\right\} dx, \tag{12}$$

*and*

$$\bar{t}_0^{(B)} = \alpha\bar{\tau}_i e^{-\left(\lambda_0^{(A)}+\lambda_0^{(B)}\right)} \int_0^1 x^{\alpha-1} \exp\left\{\lambda_0^{(A)} x + \lambda_0^{(B)} x^\alpha\right\} dx. \tag{13}$$

*For $i \geq 1$, we have*

$$\bar{t}_i^{(A)} = \bar{\tau}_i e^{-\left(C_i^{(A)}+C_i^{(B)}\right)} \int_0^1 \exp\left\{C_i^{(A)} x + C_i^{(B)} x^\alpha\right\} dx,$$

*and*

$$\bar{t}_i^{(B)} = \alpha\bar{\tau}_i e^{-\left(C_i^{(A)}+C_i^{(B)}\right)} \int_0^1 x^{\alpha-1} \exp\left\{C_i^{(A)} x + C_i^{(B)} x^\alpha\right\} dx,$$

*with*

$$C_i^{(A)} = \lambda_0^{(A)} {p^{(A)}}^i + \lambda^{(A)} \frac{1 - {p^{(A)}}^i}{q^{(A)}},$$

*and*

$$C_i^{(B)} = \lambda_0^{(B)} {p^{(B)}}^i + \lambda^{(B)} \frac{1 - {p^{(B)}}^i}{q^{(B)}}.$$

*Proof.* The results are the consequences of Propositions 3.3. To get the desired results, we just have to explicit

$$G_0^{(A)}(x)G_0^{(B)}(x^\alpha),$$

and

$$G_0^{(A)}(p^i(x-1)+1)G_0^{(B)}(p^i(x^\alpha-1)+1)\prod_{k=0}^{i-1}G^{(A)}\left(p^k(x-1)+1\right)G^{(B)}\left(p^k(x^\alpha-1)+1\right).$$

Since the $A_i^{(A)}$'s and the $A_i^{(B)}$'s are Poisson distributed, we have

$$G^{(A)}(x)=e^{\lambda^{(A)}(x-1)} \text{ and } G^{(B)}(x)=e^{\lambda^{(B)}(x-1)}.$$

For $N_0^{(A)}\sim\mathcal{P}(\lambda_0^{(A)})$ and $N_0^{(B)}\sim\mathcal{P}(\lambda_0^{(B)})$, we also have

$$G_0^{(A)}(x)=e^{\lambda_0^{(A)}(x-1)} \text{ and } G_0^{(B)}(x)=e^{\lambda_0^{(B)}(x-1)}.$$

So we easily get (12) and (13) from Proposition 3.3 and we have

$$G_0^{(A)}(p^i(x-1)+1)=e^{\lambda_0^{(A)}p^i(x-1)}.$$

Since

$$\prod_{k=0}^{i-1}\exp\left\{\lambda^{(A)}\left(({p^{(A)}}^k(x-1)+1)-1\right)\right\}=\exp\left\{\lambda^{(A)}(x-1)\frac{1-{p^{(A)}}^i}{q^{(A)}}\right\},$$

we get

$$G_0^{(A)}({p^{(A)}}^i(x-1)+1)\prod_{k=0}^{i-1}G^{(A)}\left({p^{(A)}}^k(x-1)+1\right)=$$
$$\exp\left\{\left(\lambda_0^{(A)}{p^{(A)}}^i+\lambda^{(A)}\frac{1-{p^{(A)}}^i}{q^{(A)}}\right)(x-1)\right\},$$

which is equal to $e^{C_i^{(A)}(x-1)}$. Following the same way gives

$$G_0^{(B)}({p^{(B)}}^i(x-1)+1)\prod_{k=0}^{i-1}G^{(B)}\left({p^{(B)}}^k(x-1)+1\right)=e^{C_i^{(B)}(x-1)},$$

which ends the proof. □

The Poisson case gives more tractable expressions, which allows us to give the asymptotic behavior (see Proposition 3.6). Figure 4 represents one example of these results where a $B$-patient needs twice as much care time as a $A$-patient and the global care time is constant.

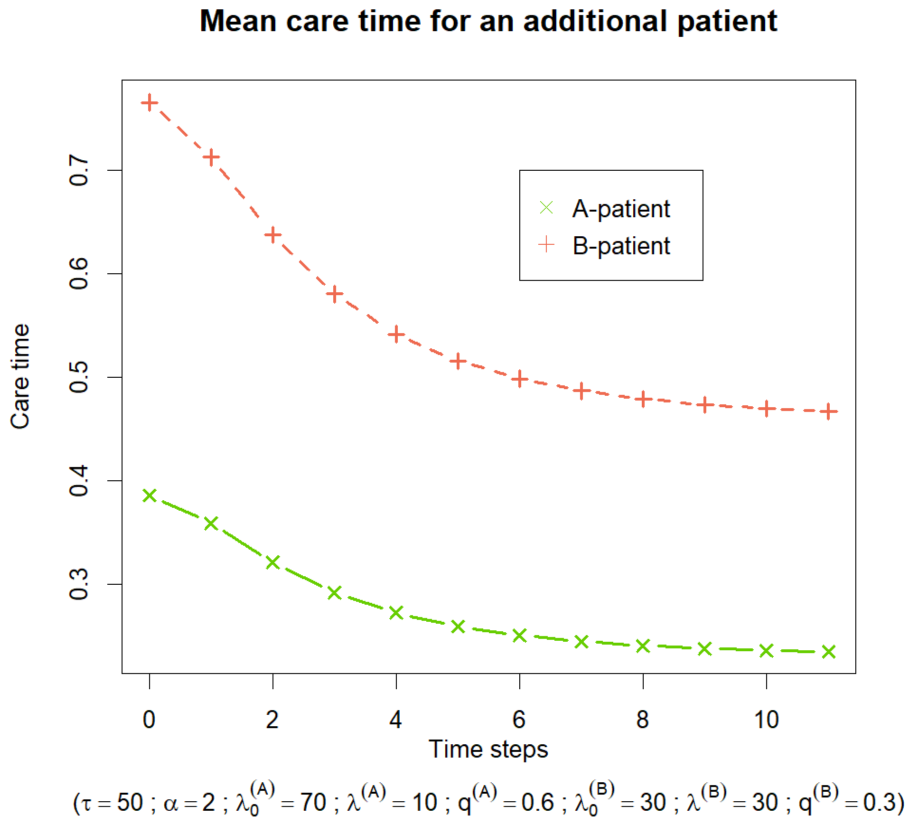


Figure 4: One situation where a $B$-patient needs twice as much care time as a $A$-patient (see Proposition 3.5).

**Proposition 3.6.** *Assume $\bar{\tau}_i$ is constant over time and denote by $\bar{\tau}$ their common value. We have*

$$\lim_{i\to\infty} \bar{t}_i^{(A)} = \bar{\tau} e^{-\left(\frac{\lambda^{(A)}}{q^{(A)}}+\frac{\lambda^{(B)}}{q^{(B)}}\right)} \int_0^1 \exp\left\{\frac{\lambda^{(A)}}{q^{(A)}}x + \frac{\lambda^{(B)}}{q^{(B)}}x^\alpha\right\} dx\,,$$

*and*

$$\lim_{i\to\infty} \bar{t}_i^{(B)} = \alpha\bar{\tau} e^{-\left(\frac{\lambda^{(A)}}{q^{(A)}}+\frac{\lambda^{(B)}}{q^{(B)}}\right)} \int_0^1 x^{\alpha-1}\exp\left\{\frac{\lambda^{(A)}}{q^{(A)}}x + \frac{\lambda^{(B)}}{q^{(B)}}x^\alpha\right\} dx\,.$$

*In other words, both individual care times are convergent in the case of constant global care time.*

*Proof.* Suppose $\bar{\tau}_i = \bar{\tau}$ for all $i$ in Proposition 3.5. We have

$$\lim_{i\to\infty} C_i^{(A)} = \frac{\lambda^{(A)}}{q^{(A)}},$$

and

$$\lim_{i\to\infty} C_i^{(B)} = \frac{\lambda^{(B)}}{q^{(B)}},$$

which ends the proof. □

# 4 Working conditions impact

In this section, we focus on the quantity $\bar{\tau}_i$, that is the available mean care time at step $i$. We aim to model the impact of a degradation in the working conditions by assuming a decreasing care time along the periods. Mathematically, let

$$\bar{\tau}_{i+1} = a\bar{\tau}_i + b\,,\ i \in \mathbb{N}\,.$$

Such a sequence is not necessarily decreasing. Let us discuss the cases.

- If $a = 1$, then $b$ has to be negative and greater than $-\bar{\tau}_0/N$, where $N$ is the final time of the study to guarantee the positivity of $\bar{\tau}_i$. In this case, we have a linear decrease for the available care time.
- If $0 < a < 1$, the sequence decreases if $\bar{\tau}_0 > b/(1-a)$ and converges to $b/(1-a)$.
- If $a > 1$, the sequence diverges.
- If $a < 0$, the sequence oscillates.

Only the two first cases are relevant. The first case could be seen as a constant degradation without reaction. The second case could model a situation where the degradation is controlled by managers to ensure a minimum service. In this situation, the particular case $b = 0$ could model an abrupt degradation without reaction (since the limit is zero).

For both cases of one profile and two profiles, we are going in this section to compare four different situations with respect to the sequence $(\bar{\tau}_i)_{i\in\mathbb{N}}$:

**Model 1** A constant situation: $\bar{\tau}_i = \bar{\tau}_0$ for all $i \in \mathbb{N}$.

**Model 2** A constant degradation: $\bar{\tau}_{i+1} = \bar{\tau}_i - 0.1$ for $i \in \mathbb{N}^*$.

**Model 3** A brutal degradation without reaction: $\bar{\tau}_{i+1} = 0.99\bar{\tau}_i$ for $i \in \mathbb{N}^*$.

**Model 4** A brutal degradation with reaction: $\bar{\tau}_{i+1} = 0.99\bar{\tau}_i + 0.2$ for $i \in \mathbb{N}^*$.

The four cases are plotted in Figure 5.

## 4.1 One profile of patients

In this subsection, we consider the setting of Proposition 2.8, that is to say that there exists one profile of patients and the arrivals are Poisson distributed. Let us recall here that in the case where the global care time is constant, Proposition 2.9 states that the mean care time by patient converges and the behavior is regular. In particular, this time is either increasing or decreasing (or constant). For the four models of this section, Figure 6 and Figure 7 depict the two situations for the mean global care time $\bar{\tau}_i$. In Figure 6, the individual mean care time is increasing at the beginning but, for the three cases that model a

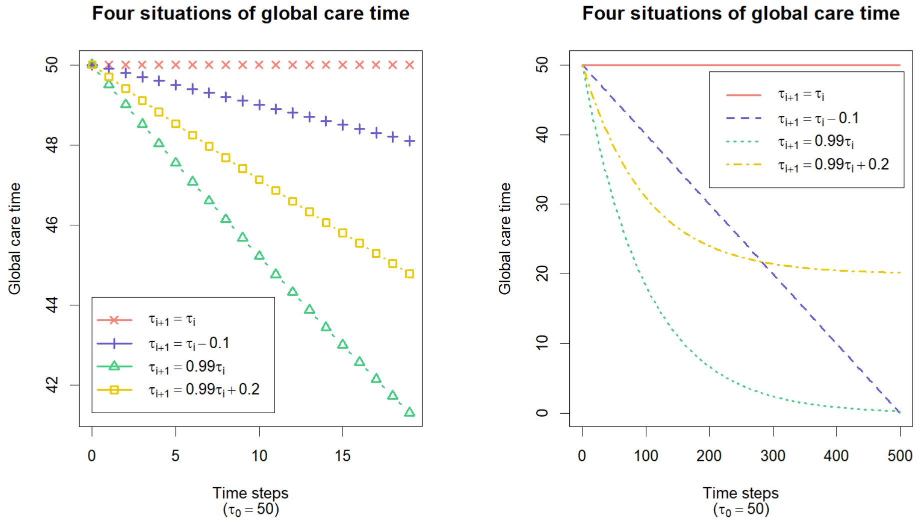


Figure 5: The four situations for the $(\tau_i)_{i\in\mathbb{N}}$ during a short period (left-hand side graph) and a long period (right-hand side graph).

degradation of the working conditions, this time is decreasing afterwards. In Figure 7, the individual mean care time is decreasing and continue to decrease for the three models with a degradation on working conditions but stabilizes when the global mean care time is constant. In both figures, we observe two phases. The first one ends when the constant case stabilizes and the behavior of the four cases are quite similar. In a second phase, we clearly see differences. The individual mean care time in the two cases without reaction goes to zero (linearly when the degradation is constant and exponentially when the degradation is brutal). When there exists a reaction, the individual mean care time stabilizes and stay positive. The shape of curves in the second phase is similar to the shape of curves of $\bar{\tau}_i$ (Figure 5).

## 4.2 Two profiles of patients

In this subsection, we consider the setting of Proposition 3.5 where there exist two profiles of patients ($A$ and $B$) and the arrivals are Poisson distributed. Here a $B$-patient needs twice as much care time as a $A$-patient. Figure 8 represents the individual mean care time for an additional patient in the case where this patient is of type $A$ or type $B$. These two individual mean care times are plotted for the four cases of global mean care time. We retrieve the two phases we have mentioned in the previous subsection. We observe the same behaviors as in the one profile case but we can notice that the difference of individual mean care time between a $A$-patient and a $B$-patient reduces with time for the three cases where there exists a degradation of working conditions even in the case where there exists a reaction. Figure 9 gathers the four models for both types.

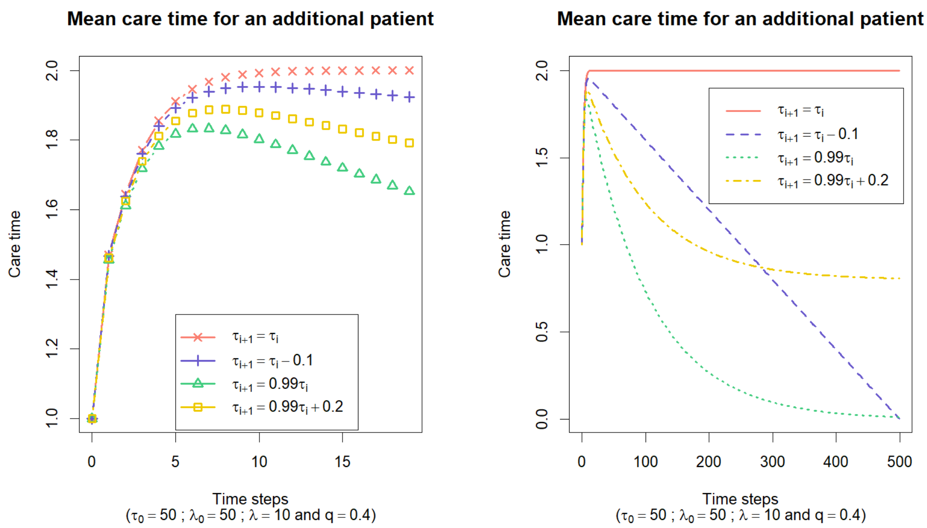


Figure 6: The four situations for the $(\tau_i)_{i\in\mathbb{N}}$ during a short period (left-hand side graph) and a long period (right-hand side graph).

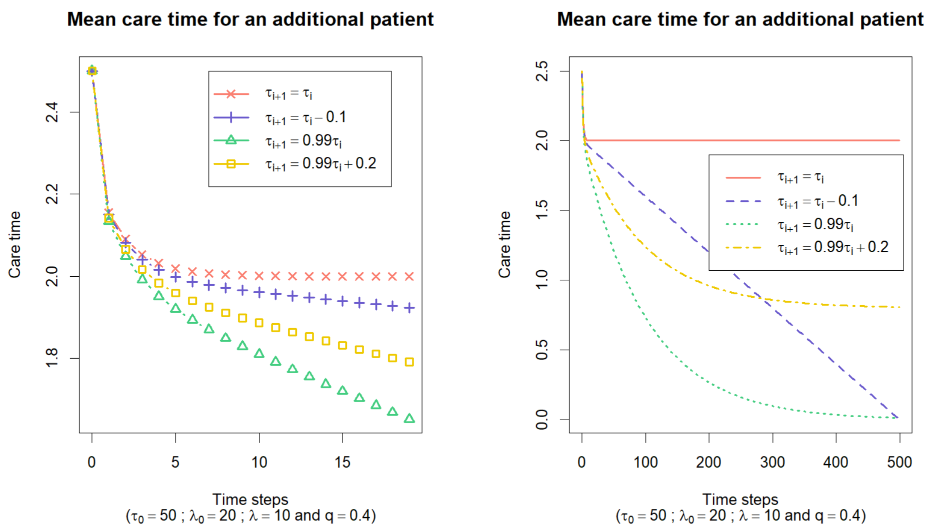


Figure 7: The four situations for $(\tau_i)_{i\in\mathbb{N}}$ during a short period (left-hand side graph) and a long period (right-hand side graph).

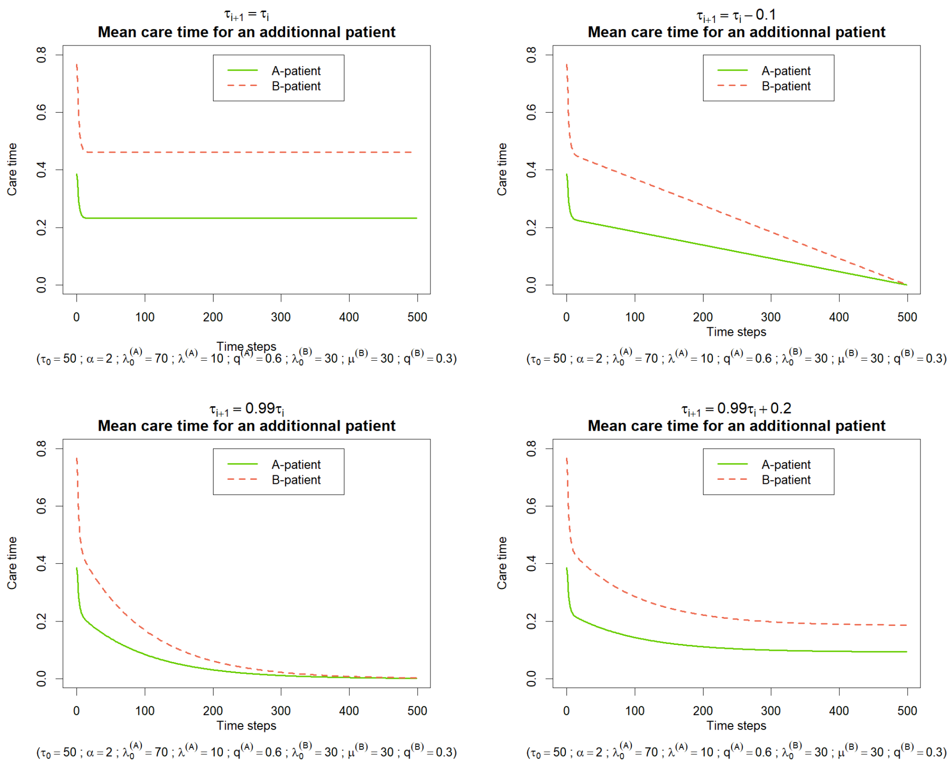


Figure 8: The four situations for $(\tau_i)_{i\in\mathbb{N}}$ during a short period (left-hand side graph) and a long period (right-hand side graph).

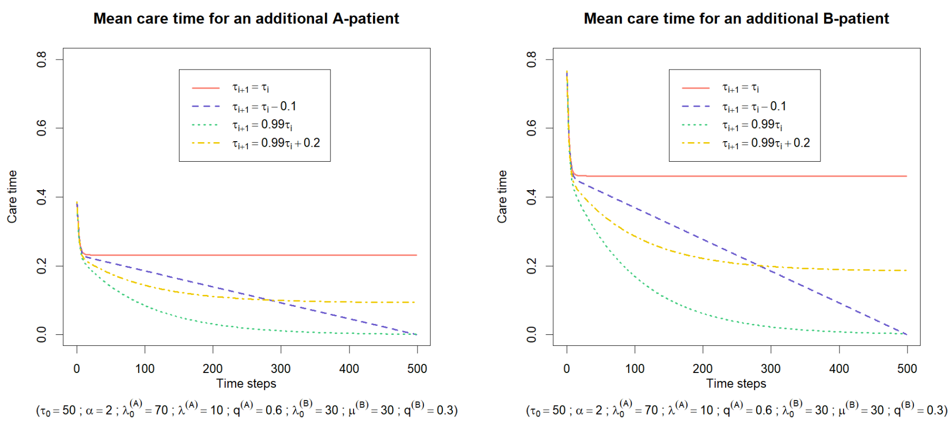


Figure 9: The four situations for $(\tau_i)_{i\in\mathbb{N}}$ during a short period (left-hand side graph) and a long period (right-hand side graph).

## 5 Conclusion

The main objective of the paper at hand has been to propose a model in order to help assess the number of nurses needed to ensure both healthier caregivers and healthier patients. More exactly, we have proposed a simple model where there is, for everyone, an unknown exact number of patients at each time step. We have assumed that: 1/ all nurses are perfect substitutes (*i.e.*, one hour of a nurse is equal to one hour of another), 2/ patients could be categorized into several types (once and for all), and 3/ entry and exit devices are exogeneous and common knowledge. In this setting we are able to calculate the mean available care time and thereby provide assistance in assessing the potential nursing shortage in a hospital. It should also be noted that we have made the assumption that patient types correspond to pathology types. However, in the event of severe resource scarcity, issues of social choice concerning the distribution of this resource could arise, and patient types could then be reinterpreted as indicating orders of priority. In other words, the model could be reinterpreted in situations where patients need to be sorted.

We left for future research the cases where nurses have different skills and must cooperate to care of a given type of patient (*i.e.*, the case where nurses are complement).

To our mind, our analysis has three limits. First, the available care time during time step $i$ (*i.e.*, $\tau_i$) is independent of the number of other patients in the same unit at time $i$ (*i.e.* $N_i$), which means that we do not consider situations where the hospital adjusts the number of nurses on the number of patients present at previous periods. Second, our model is only relevant in steady state situations because our random parameters are constant over time. Third, we do not differentiate between the different exit situations (*i.e.*, we do not distinguish the case where the patient leaves because: she is dying, she leaves cured, or she is transported to another service).

Finally, our analysis only focused on the quantity of human resources, but we are aware that there is in fact an interplay between these and, the quality of human ressources (*e.g.*, education level and nursing experience), the material resources, the organization of care services and their cooperation (*e.g.*, Tamata and Mohammadnezhad [2022]).

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