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Sharing the cost of cleaning up non-point source pollution⋆

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Abstract

We consider the problem of sharing the cost of cleaning the non-point source pollution of industrial sites among the firms that own these sites. The bilateral liabilities between firms are depicted by an undirected graph. We introduce and characterize axiomatically two allocation rules inspired by the celebrated Polluter pays and Beneficiary pays principles in environmental law. The first one shares evenly the cost of cleaning up a site among the firms that can have caused the corresponding environmental damage. The second one charges to each firm the entire cost of cleaning up its own production site. We also establish connections with cooperative game theory.

Keywords: Cooperative game theory; Cost allocation; Pollution; Liability.

JEL codes: C71

1. Introduction

1.1. Context and problematic

According to *the global Climate Litigation Report: 2023 Status Review*, the total number of court cases focused on climate action has more than doubled since 2017. In the coming years, environmental damage will be increasingly denounced as shown by the succession of rulings taken by the European Court of Human Rights (ECHR). The ECHR has delivered on April 9, 2024 two rulings in climate change cases. In the first case, *Verein KlimaSeniorinnen Schweiz and Others v. Switzerland*, the Court found that the Swiss Confederation had failed to comply with its obligation to act to reduce global warming. In the second case, *Duarte*

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Agostinho and Others v. Portugal and 32 Others, although the Court declared inadmissible the complaint, it recognizes that the 33 states are the cause of climate change and should respect their commitments to reduce pollution (see Paris Agreement adopted in 2015). These rulings highlight the fact that anyone (persons, firms[1], governments) is condemnable and can be jointly implicated in environmental damage, or pollution for short.

One of the objectives of environmental law is to force pollution clean up. One prominent principle is the **Polluter-pays principle**, which stipulates that the polluter pays for the cost of cleaning up the pollution. This principle is widely adopted in environmental law, for instance in Europe in the Single European Act[2] and in France in the Charter for the Environment[3]. An alternative to the Polluter-pays principle is the **Beneficiary-pays principle**, widely studied in philosophy (see, for instance, Page, 2012; Butt, 2014; Bazargan-Forward, 2021; Kim, 2023). It forces those who benefit from pollution to bear the cost of cleaning up the pollution. Both principles require a precise definition of what is meant by pollution.

Pollution can be divided into two main categories, the point-source pollution (PS) and the **non-point source pollution** (NPS). The PS pollution is a single identifiable source of air, water, thermal, noise or light pollution. For instance, air pollution from an industrial source or noise pollution from a jet engine. On the contrary, the NPS pollution is a diffuse source of water or air pollution, as land runoff or atmospheric deposition. It does not originate from a single discrete source, but from several sources whose exact contribution to NPS pollution is unknown. For the first category of pollution, it is not difficult to identify the polluter or the beneficiary of the pollution, which implies that the cost allocation is often immediate. We focus on NPS pollution problems in which several firms can be the source. In this case, the Article 9 of the European Directive[4] 2004/35/CE states that liability sharing should be determined in accordance with national law.

With regard to the specific case of France, Article L-162-18 of the Environmental code states that the cost of cleaning up NPS pollution is shared to the extent of the contribution of firms' activity to the pollution. However, by definition, it is not possible to determine the exact share of firms' liability in the NPS pollution. In this case, two allocation methods are used by the Courts, the Equal share rule and the Market share rule. Both were used in the Distilbène litigation[5]. The first shares the cost of cleaning up the pollution equally among the liable firms and the second proportionally to the market share of each liable firms. One

[1]In this paper, we focus on the firms that are most often sources of pollution.

[2]Single European Act, Article 25(2), " [...] environmental damage should as a priority be rectified at source and that the polluter should pay"

[3]French Charter for the Environment, Article 4, " [...] everyone must contribute to repairing the damage they cause to the environment."

[4]European Directive 2004/35/CE of 21 April 2004 on environmental liability with regard to the prevention and remedying of environmental damage.

[5]See Cour d'Appel of Paris, October 2012, n°10/18297 for Equal share rule and Tribunal de Grande Instance of Nanterre, April 2014, n°12/12349 for the Market share rule.

of the two allocation methods that we study implements the Polluter-pays principle and the Equal share rule. The other is inspired by the Beneficiary-pays principle.

1.2. Methodology and results

In our approach, the exact contribution of firms to NPS pollution is unknown but **the area of pollution influence** of each firm can be established, i.e. a firm can pollute its own site and the site of specific other firms., for instance neighboring firms. Although the NPS pollution is by definition diffuse, a multitude of factors can establish a causal relationship between firms and pollution, including geographical location and the presence of natural phenomena such as rivers, mountains, or the position of a firm within the production chain. We represent these liabilities in NPS pollution emissions by an undirected graph. The firms, or equivalently their production sites are represented by a set of vertices. The fact that the firms' activities can pollute the site of another firm is represented by a link between them. Hence, we envisage bilateral liabilities. The cost needed to clean up each production site is specified and the sum of these costs must be allocated among the firms. For this purpose, we use two allocation rules called the **Polluter-pays allocation rule** and the **Beneficiary-pays allocation rule**. The first is based on the Polluter-pays principle, i.e. firms should equally share the cost of cleaning up NPS pollution for which they can be liable. The second follows the Beneficiary-pays principle, the cost of cleaning up a site can be fully supported by the firm that owns the site because it is the only one to benefit from the site's clean-up.

In order to analyze the relevance of both allocation rules, we adopt a normative approach by introducing several desirable properties, called axioms. The first fives axioms highlight the importance of liable firms in the network, while the others are adaptations of classical axioms in our context. The axiom of **No liability** focuses on a situation in which the site of the firm and those of the firms that could have been polluted by its activities are free from pollution. Hence, such a firm is not involved in any NPS pollution, which we translate into requirement that it does not bear any cost. We provide a stronger version of the previous axiom, the axiom of **Strong no liability**. It requires that a firm should not bear any cost if its own production site is not polluted. These axioms follow the *no liability rule* in tort law, i.e. if no fault is established for any defendant, then non one is liable. The axiom of **Equal allocation under equal liabilities** focuses on a situation in which the area of pollution influence of two firms contains a unique common polluted site. Their respective areas of influence are everywhere else free from NPS pollution. The axiom imposes an identical share of the cost. The axiom of **Strong equal allocation under equal liabilities** relaxes the previous axiom by considering that the area of pollution influence of two firms contains the same polluted production sites. The axiom of **Fairness**, inspired by Myerson (1977), requires that if measures are taken to ensure that two firms initially liable to each other cannot pollute the other firm's site anymore, then the change in each firm's share of the costs is the same. This axiom encourages firms to jointly limit or control their NPS pollution emissions.

Then, the classical axiom **Cost efficiency** requires that the total cost of cleaning up NPS pollution should be entirely allocated among the liable firms. This axiom could also be applied within each industrial districts, i.e. firms connected to each other by links in the liability graph but not to the rest of the firms. Indeed, the axiom of **District cost efficiency** requires that each industrial district redistributes its total cost of cleaning up NPS pollution among the member firms. The axiom of **Area of influence monotonicity** imposes that if the costs within the area of pollution influence of a given firm weakly increases, then the cost share allocated to this firm should weakly increase as well. Finally, the axiom of **Additivity in costs** requires classically that an allocation rule is additive in the cost vector.

We demonstrate in Proposition 1 that the Polluter-pays allocation rule is the unique allocation rule satisfying the axioms of Cost efficiency, Additivity in costs, No liability and Equal allocation under equal liabilities. Proposition 2 shows that the Polluter-pays principle rule is the unique allocation rule satisfying the axioms of Cost efficiency, Strong equal allocation under equal liabilities and Area of influence monotonicity. Moreover, Proposition 3 states that the Beneficiary-pays allocation rule is the unique allocation rule that satisfies District cost efficiency and Fairness. In addition, the Beneficiary-pays allocation rule is the unique allocation rule that satisfies Cost efficiency, Strong no liability and either Additivity in costs or Area of influence monotonicity, see Proposition 4.

Our last axiomatic characterization involve a natural consistency axiom. This axiom is applicable when the proposed allocation designates unambiguously for which site firms pay. It requires that the cost shares of the firms are the same in the original problem and in a reduced game obtain by revising the problem data when a firm is no longer involved after paying its share. We call it **Unambiguous consistency**. Proposition 5 proves that the Polluter-pays allocation rule is the unique allocation rule satisfying Unambiguous consistency together with District cost efficiency, Additivity in costs and either Equal allocation under equal liabilities or Strong equal allocation under equal liabilities.

In order to analyze sustainability of both allocation rules, we rely on a cooperative game approach. We construct the **Polluters game** that assigns to each coalition of firms their extended cost of cleaning up pollution, i.e. the cost of cleaning up their sites and the sites for which they can be liable. We show that the Polluters game is concave (Proposition 7) and that the Polluter-pays allocation rule coincides with the allocation recommended by the Shapley value of the Polluters game (Proposition 6). Since Shapley (1971) showed that the Shapley value lies in the core of a concave game, we obtain that the Polluter-pays allocation rules yields a core allocation of the Polluters game. We demonstrate that the Polluter-pays allocation rule is the Shapley value of the Polluters game and thus it lies in the core. In addition, Proposition 7 further reveals that the Beneficiary-pays allocation rule is also in the core of the Polluters game. Therefore, both allocation rules can be seen as sustaining cooperation among the firms.

1.3. Related literature

Ni and Wang (2007) study the allocation of the costs of cleaning a polluted river among the players located along it. As our paper, they rely on the main advocated doctrines in international disputes. Applying these doctrines, they propose two allocation rules taking into account the player's position along the river. The first, the Local Responsibility Sharing rule, charges the player in a given segment her own local costs. The second, the Upstream Equal Sharing rule, considers that upstream players are liable for pollution discharged downstream, and charges a player a part of all downstream costs including her own local costs. For both allocation rules, a suitable cooperative game can be constructed to show that the associated allocation is a core allocation and coincides with that prescribed by the Shapley value. The authors also provide axiomatic characterizations with specific axioms inspired by the context. Dong et al. (2012) model this cost sharing problem on tree network and introduce another allocation rule named the Downstream Equal Sharing rule, which the same logic as the Upstream Equal Sharing rule but based on the upstream costs. These papers have given rise to a literature, see Alcade-Unzu et al. (2015); van der Laan and Moes (2016); Sun et al. (2019); Hou et al. (2020); Li et al. (2023), among other. Our paper differs from this literature in at least two points. First, a river is seen as a directed graph with one source and one spring. Players are located along the river and they are connected directly or indirectly by a link. On the contrary, we consider an undirected graph where the players may or may not be connected to each other, i.e. we allow the possibility to have multiple industrial districts. Moreover, pollution is not transitive: contrary to water pollution, we assume that pollution does not spread via the neighborhood on the graph. Second, these papers deal more with PS pollution than with NPS pollution, each player makes a pollution by using the river network then players' pollutant emissions can be identified.

Cooperative game theory and the axiomatic approach have also been used to analyze allocation problems within a tort law context. Dehez and Ferey (2013) use cooperative game theory to share a damage among multiple tortfeasors who have collectively caused an economic loss. They study the Shapley value and its weighted variants (see Kalai and Samet, 1987) of a suitable cooperative game. Similar to our approach, Ferey and Dehez (2016) consider an axiomatic approach to ensure that the damage sharing is in line with fundamental principles of tort law. Following this article, they deepen the axiomatic study to define an acceptable and fair sharing of the damage. They retain classical axioms characterizing the Shapley value and identify new ones more adapted to their cooperative framework. More recently, Oishi et al. (2023) axiomatize two solutions, including the Shapley value, with axioms derived from the legal concept of tort law. Other legal contexts are also analyzed through cooperative game theory. Crettez and Deloche (2019) demonstrate that the share of an elevator's costs given by the French law increases the risk of disputes between neighbors. Ambec and Sprumont (2002) focus on the fair distribution resulting from the optimal allocation of water among agents located along a river and rely on principles inspired by international water Agreements[6].

1.4. Plan

The rest of the paper is organized as follows. Section 2 defines the NPS pollution cost sharing problems. We study them by means of a normative approach in Section 3 and then by means of a cooperative game approach in Section 4.

2. Cost sharing problems and allocation rules

We consider a finite set N of **firms**. The production site of each firm $i \in N$ is polluted, and this NPS pollution may be caused by other firms. The **cost of cleaning up pollution** on the site of firm i is denoted as $c_i \geq 0$.[7] This is a NPS pollution situation as studied by Segerson (1988), meaning that the actions of possibly several polluters contribute to the NPS pollution, and only combined effects are observable. Thus, the exact level of liability of each firm in the global NSP pollution is unknown. However, we know that the activities of certain firms can pollute the sites of other firms. This situation is represented by a graph (N, L) whose set of vertices N is the set of participating firms and the set of links $L \subseteq L^N := \{\{i, j\} \in N \times N : i \neq j\}$ is such that that a link $\{i, j\} \in L$ means that i's activity can also harm j's site and vice versa. We often write ij instead of $\{i, j\}$. For each $i \in N$, the neighborhood of i in (N, L) is denoted by $L_i = \{j \in N : ij \in L\}$ and the set $L_i^+ = L_i \cup \{i\}$ is called the **area of pollution influence** of firm i. The connected components[8] of (N, L) are interpreted as independent (with respect to pollution) **industrial districts**. The objective is to share the total cost of cleaning up NPS pollution

$$\sum_{i \in N} c_i$$

across the entire industry. The triple (N, c, L), where $c = (c_i)_{i \in N}$, is called a **NPS pollution cost sharing with network externalities** or simply **problem**. We denote by CS^N the set of all such problems with a given set of firms N. Since N is fixed throughout this article (with the exception of subsection 3.3), we write (c, L) instead of (N, c, L). We often invoke

[6]For instance, revisited Convention on the navigation of the Rhine (1868), Colombia river Treaty (1961).

[7]The objective of environmental law is to force pollution clean up thus, we consider c_i as a clean-up cost. However, it could also represent damages paid to the victims of pollution, firms are both polluters and victims. We consider this cost as exogenous, it may be provided by firms or by external institutions such as associations or governmental institutions which have no incentive to lie.

[8]For any subset of vertices $S \subseteq N$, $L(S) = \{ij \in L : i, j \in S\}$ is the subset of links in L with endpoints in S. Two vertices $i, j \in N$ are connected in (N, L) if either $i = j$ or there is a sequence $(i_1, \ldots, i_k)$ such that $i_1 = i$, $i_k = j$ and for each $q \in \{1, ..., k-1\}$, $\{i_q, i_{q+1}\} \in L$. A set $S \subseteq N$ is connected in (N, L) if for each $i, j \in S$, i and j are connected. A connected component of (N, L) is a connected set S such that for each $i \in N \backslash S$, $S \cup \{i\}$ is not connected. We denote by N/L the set of connected components of (N, L).

the following two specific cost sharing problems: $(\mathbf{0}, L)$ and (c^i, L), for each $i \in N$, where $\mathbf{0}$ is the null $|N|$-dimensional cost vector and c^i is obtained from a given cost vector c by setting $c^i_i = c_i$ and $c^i_j = 0$ for each $j \in N\backslash\{i\}$. Our cost sharing problem is solved by relying on an allocation rule. More specifically, an **allocation rule** on CS^N is a function f which assigns to each problem $(c, L) \in CS^N$ a non-negative allocation $f(c, L) \in \mathbb{R}^{|N|}_+$ specifying the cost share $f_i(c, L)$ of each firm $i \in N$.

In order to design relevant allocation rules, we rely on principles of environmental law. The most famous one is the **Polluter-pays principle** (see Directorate, 1974, for instance), which makes the party responsible for producing pollution responsible for paying for the damage done to the natural environment. In our NPS pollution cost sharing problem, there is some uncertainty about who exactly contributes to the pollution beyond the possible liabilities summarized by the graph. Therefore, a natural interpretation of the Polluter-pays principle is that firms should equally share the pollution costs for which they can be responsible. The resulting **Polluter pays-allocation rule** is denoted by PP. Formally, for each problem $(c, L) \in CS^N$ and each firm $i \in N$,

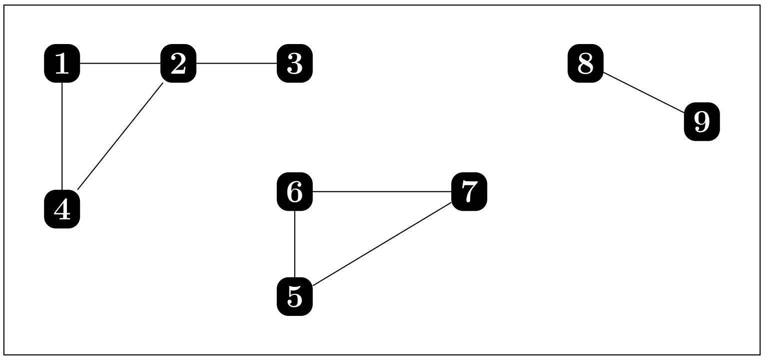
$$PP_i(c, L) = \sum_{j \in L^+_i} \frac{c_j}{|L^+_j|}.$$

An alternative to the Polluter-pays principle is the so-called **Beneficiary-pays principle**, according to which

> "The burdens involved [in addressing climate change] should be distributed amongst states according to the amount of benefit that each state has derived from past and present activities that contribute to climate change."(Page, 2012)

A plausible interpretation of this principle is that each firm should pay the full cost of cleaning up pollution on its own site. This is because cleaning up a firm's site will only benefit that firm and not others, or indeed to a significantly lesser extent. This gives rise to the **Beneficiary-pays allocation rule** denoted by BP, which assigns to each problem $(c, L) \in CS^N$ and each firm $\in N$ the cost share $BP_i(c, L) = c_i$.

Example 1. The set of firms is $N = \{1, ..., 9\}$. The area of pollution influence of each firm and their cost are given by Table 1. It easy to see that there are three industrial districts.



Firm i	1	2	3	4	5	6	7	8	9
L_i^+	{1,2,4}	{1,2,3,4}	{2,3}	{1,2,4}	{5,6,7}	{5,6,7}	{5,6,7}	{8,9}	{8,9}
c_i	12	24	6	12	24	24	12	6	6

Table 1: Area of pollution influence and costs

The Beneficiary-pays allocation rule charges c_i to each firm. The Polluter-pays allocation rule gives the following cost share to firm 1:

$$PP_1(c,L) = \frac{c_1}{|\{1,2,4\}|} + \frac{c_2}{|\{1,2,3,4\}|} + \frac{c_4}{|\{1,2,4\}|} = \frac{12}{3} + \frac{24}{4} + \frac{12}{3} = 14.$$

The cost shares going to the other firms according to the Polluter-pays allocation rule are given by Table 2. □

Firm i	1	2	3	4	5	6	7	8	9
$PP_i(c,L)$	14	17	9	14	20	20	20	6	6

Table 2: The PP allocation

3. A normative approach

In this section, we first present several axioms that are pertinent to the allocation of the costs associated with the remediation of pollution. The first three axioms are classical in the cost sharing literature, while most of the other axioms axioms highlight the way in which responsibilities are interpreted and taken into account in the allocation process. Then, we invoke these axioms to provide axiomatic characterizations of the two allocation rules introduced in Section 2.

3.1. Axioms

We start by the classical axiom of Cost efficiency, which requires that the set of participating firms should cover the total cost.

Cost efficiency (CE). For each $(c, L) \in CS^N$, it holds that $\sum_{i \in N} f_i(c, L) = \sum_{i \in N} c_i$.

A stronger condition than Cost efficiency is to require that each industrial district redistributes its total cost among the member firms.

District cost efficiency (DCE). For each district $S \in N/L$, it holds that $\sum_{i \in S} f_i(c, L) = \sum_{i \in S} c_i$.

This axiom makes sense for at least two reasons: (a) it imposes the cost efficiency condition on the whole industry (any allocation rule satisfying component District cost efficiency also satisfies Cost efficiency, while the converse is not true) and (b) the allocation to firms of a given district should not be influenced by remote firms whose activities cannot contribute to pollution in the district area. We also invoke the classical axiom of additivity in costs (see Moulin and Laigret, 2011, for instance).

Additivity in costs (ADD). For each pair of cost sharing problems $(c, L), (c', L) \in CS^N$, it holds that $f(c + c', L) = f(c, L) + f(c', L)$.

For our NPS pollution cost sharing problem, we can provide the following interpretation. NPS pollution can, by definition, harm several parts of environment: soil, air and water. Imagine that the cost of cleaning up NPS pollution consists in the cost of cleaning up water pollution and the cost of cleaning up air pollution. Additivity in costs says that there is no difference whether the firms share the two types of cost separately or together.

The next axioms highlight the importance of firms that are liable to each other. As for District cost efficiency, the next axiom is inspired by the work of Myerson (1977) on cooperative games with a communication structure. Consider two firms whose activities potentially pollute each other's production sites. Consider the counterfactual scenario in which measures are taken to prove that pollution emissions from one firm can no longer pollute the other, and vice versa. The axiom requires equal changes in each firm's share of costs.

Fairness (F). For each pair of firms $ij \in L$, it holds that $f_i(c, L) - f_i(c, L \backslash ij) = f_j(c, L) - f_j(c, L \backslash ij)$.

This axiom encourages firms to jointly invest in technologies to limit the spread of their pollution. Thus, they can limit their area of pollution influence and reduce their share of the cost.

The next axiom, called No liability, focuses on a situation in which the production site of the firm and those of the firms that could have been affected by its activities are free from pollution. It makes sense that this firm pays no cost since its production site is not involved in the pollution of other production sites.

No liability (NL). For each firm $i \in N$ such that $c_i = 0$ and $c_j = 0$ for each $j \in L_i$, it holds that $f_i(c, L) = 0$.

A stronger version of the previous axiom requires that a firm should not bear any cost if its own production site is not polluted.

Strong no liability (NL$^+$). For each $i \in N$ such that $c_i = 0$, it holds that $f_i(c, L) = 0$.

Through these two axioms, we consider two cases in which a firm is considered non-liable for NPS pollution. The No liability axiom deals with extensive non-liability, i.e. the firm is not liable for NPS pollution that its activity cannot impact, while Strong no liability deals with a local non-liability, i.e. the firm is not liable for NPS pollution that occurs outside the boundaries of its own site. These axioms interpret the *no liability rule* in tort law in two distinct ways: if no liability is established, then the firm does not bear any cost.

The next axiom, emphasizing the role of liable firms, considers a situation in which the responsibilities of two firms can be compared. More specifically, suppose that the area of pollution influence of two firms contains a unique common polluted production site. This means that the two considered firm can only be held responsible for the pollution of this production site since their respective areas of influence are everywhere else free from pollution. In such a case, the following axiom imposes an identical allocation for the two firms.

Equal allocation under equal liabilities (EAEL). For each pair of firms $i, j \in N$ such that $\{q \in L_i^+ : c_q > 0\} = \{k\} = \{q \in L_j^+ : c_q > 0\}$ for some $k \in N$, it holds that $f_i(c, L) = f_j(c, L)$.

The previous axiom can be strenghtened by relaxing the fact that the area of pollution influence of two firms contains a unique common polluted production site.

Strong equal allocation under equal liabilities (EAEL$^+$). For each pair of firms $i, j \in N$ such that $\{q \in L_i^+ : c_q > 0\} = \{q \in L_j^+ : c_q > 0\}$, it holds that $f_i(c, L) = f_j(c, L)$.

It is also possible to link these axioms to a rule of tort law. The *per capita rule*, if the firms polluted a production site with the same intensity then they share equally the associated cost. Obviously, Strong equal allocation under equal liabilities implies Equal allocation under equal liabilities, while the converse implication does not hold. The final axiom imposes that if the costs within the area of influence of a given firm weakly increase, then the cost share allocated to this firm should weakly increase as well.

Area of influence monotonicity (AIM). For each pair of problems $(c, L), (c', L) \in CS^N$ and each firm $i \in N$ such that $c'_j \geq c_j$ for each $j \in L_i^+$, it holds that $f_i(c', L) \geq f_i(c, L)$.

3.2. Axiomatic characterizations

We begin with two characterizations of the Polluter-pays allocation rules.

Proposition 1. *The unique allocation rule on CS^N that satisfies Cost efficiency (CE), Additivity in costs (ADD), No liability (NL) and Equal allocation under equal liabilities (EAEL) is the Polluter-pays allocation rule.*

Proof. Existence. For each $(c, L) \in CS^N$, we have

$$\sum_{i \in N} PP_i(c, L) = \sum_{i \in N} \sum_{j \in L_i^+} \frac{c_j}{|L_j^+|} = \sum_{j \in N} \sum_{i \in L_j^+} \frac{c_j}{|L_j^+|} = \sum_{j \in N} |L_j^+| \frac{c_j}{|L_j^+|} = \sum_{j \in N} c_j,$$

which proves that PP satisfies (CE). By definition, PP also satisfies (ADD) and (NL). Regarding (EAEL), consider a problem $(c, L) \in CS^N$ and two firms $i, j \in N$ such that $\{q \in L_i^+ : c_q > 0\} = \{k\} = \{q \in L_j^+ : c_q > 0\}$ for some $k \in N$. Then, $PP_i(c, L) = c_k / |L_k^+| = PP_j(c, L)$, as desired.

Uniqueness. Consider any allocation rule f on CS^N that satisfies all axioms. For any (c, L), it is clear that $c = \sum_{i \in N} c^i$. Hence (ADD) implies that

$$f(c, L) = \sum_{i \in N} f(c^i, L). \tag{1}$$

Let us show that $f(c^i, L)$ is uniquely determined for each $i \in N$. Pick $j \in N \backslash L_i^+$ and remark that $c_j^i = 0$ and $c_k^i = 0$ for each $k \in L_j^+$. Therefore, from (NL), we get

$$f_j(c^i, L) = 0 \tag{2}$$

for each $j \in N \backslash L_i^+$. Moreover, for each $j, k \in L_i^+$, it holds that $\{q \in L_j^+ : c_q > 0\} = \{i\} = \{q \in L_k^+ : c_q > 0\}$. Thus, (EAEL) yields

$$f_j(c^i, L) = f_k(c^i, L) \tag{3}$$

for each $j, k \in L_i^+$. Combining (2), (3) and (CE), we obtain

$$f_j(c^i, L) = \frac{c_i^i}{|L_i^+|} = \frac{c_i}{|L_i^+|} \tag{4}$$

for each $j \in L_i^+$. With (2), (4) and (1), the proof is complete ■

The second characterization does not involve the axiom of Additivity in costs.

Proposition 2. *The unique allocation rule on CS^N that satisfies Cost efficiency (CE), Strong equal allocation under equal liabilities (EAEL$^+$) and Area of influence monotonicity (AIM) is the Polluter-pays allocation rule.*

Proof. Since it is obvious that PP satisfies the three axioms on CS^N, we only prove the uniqueness part. So consider any allocation rule f on CS^N that satisfies the three axioms. We proceed by induction on the number $\gamma(c)$ of positive coordinates in vector c.
Initialization. If $\gamma(c) = 0$, i.e., if $(c, L) = \mathbf{0}, L)$, then $f_i(\mathbf{0}, L) \geq 0$ for each $i \in N$ and $\sum_{j \in N} f_j(\mathbf{0}, L) = 0$ from the definition of an allocation rule and (CE), which yields $f_i(c, L) = 0$ for each $i \in N$. If $\gamma(c) = 1$, then there is $i \in N$ such that $c_i > 0$ and $(c, L) = (c^i, L)$. By (AIM), we obtain that $f_j(c^i, L) = f_j(\mathbf{0}, L) = 0$ for each $j \in N \backslash L_i^+$. By (EAEL$^+$), it holds that $f_j(c^i, L) = f_k(c^i, L)$ for each $j, k \in L_i^+$, so that (CE) yields $f_j(c^i, L) = c_i / |L_i^+|$ for each $j \in L_i^+$.
Induction hypothesis. Assume that $f(c, L)$ is uniquely determined for each $(c, L) \in CS^N$ such that $\gamma(c) \leq q$ for some integer q such that $0 \leq q < n$.
Induction step. Consider any $(c, L) \in CS^N$ such that $\gamma(c) = q + 1$. Let $R = \{i \in N : c_i > 0\}$ be the coalition of firms with a positive cost. Let $P = \cap_{i \in T} L_i^+$. We distinguish two cases.

If $P = \varnothing$, then polluted sites are not neighbors in (N, L). For each $i \in R$ and each $j \in L_i^+$, from AIM and the Initialization, we get $f_j(c, L) = f_j(c^i, L) = c_i / |L_i^+|$ and for each $j \in N \backslash (\cup_{i \in R} L_i^+)$, $f_j(c, L) = f_j(\mathbf{0}, L) = 0$.

If $P \neq \varnothing$, consider first any $i \in N \backslash P$. Then, there is at least of firm $j \in N \backslash L_i^+$ such that $c_j > 0$. Now, consider the problem (c', L) such that $c'_k = c_k$ for each $k \in N \backslash \{j\}$ and $c'_j = 0$. Since $\gamma(c') = q$, by the Induction hypothesis, we have that $f(c', L)$ is uniquely determined. Moreover, an application of (AIM) to (c, L) and (c', L) yields $f_i(c, L) = f_i(c', L)$ and we conclude that $f_i(c, L)$ is uniquely determined for each $i \in N \backslash P$. Now, consider any $i \in P$. If $|P| = 1$, then $f_i(c, L)$ is uniquely determined by (CE). So assume that $|P| > 1$. Note that (EAEL$^+$) can be applied to each pair of firms $i, j \in P$ to obtain $f_i(c, L) = f_j(c, L)$. The previous cost shares are uniquely determined by (CE). This conclude the proof. ■

The axioms invoked in Propositions 1 and 2 are logically independent:

- The null allocation rule such that for each $(c, L) \in CS^N$ and each $i \in N$, $f_i(c, L) = 0$ satisfies (ADD), (NL), (EAEL), (AIM) and (EAEL$^+$) but not (CE).
- The allocation rule f such that for each $(c, L) \in CS^N$ and each $i \in N$,

$$f_i(c, L) = \begin{cases} \sum_{j \in N} c_j/n & \text{if } c_j > 0 \text{ for each } j \in N \\ PP_i(c, L) & \text{otherwise,} \end{cases}$$

  satisfies (CE), (NL) and (EAEL) but not (ADD).
- The allocation rule f such that for each $(c, L) \in CS^N$ and each $i \in N$, $f_i(c, L) = \sum_{j \in N} c_j/n$ satisfies (CE), (ADD), (EAEL) and (EAEL$^+$) but not (NL) and (AIM).
- Beneficiary-pays allocation rule BP satisfies (CE), (ADD), (NL) and (AIM) but not (EAEL) and (EAEL$^+$).

In the rest of this section, we present three characterizations of the Beneficiary-pays allocation rule.

Proposition 3. *The unique allocation rule on CS^N that satisfies District cost efficiency (DCE) and Fairness (F) is the Beneficiary-pays allocation rule.*

Proof. Fix any finite player set N. It is obvious that the Beneficiary-pays allocation rule BP satisfies (DCE) on CS^N. It also satisfies (F) since BP does not depend on L. Next, consider any allocation rule f satisfying the two axioms. This part of the proof mimics Myerson (1977) and is given for completeness. For each fixed cost vector c, we show that there is at most one such allocation rule. By contradiction, assume that f and g are two such allocation rules. Let L be a minimal set of links such that $f(c, L) \neq g(c, L)$. It must be that $L \neq \emptyset$ since (DCE) implies that $f_i(c, \emptyset) = g_i(c, \emptyset) = c_i$ for each $i \in N$. Now, consider any district $S \in N/L$ and any link $ij \in L(S)$. By minimality of L, $f(c, L \backslash \{ij\}) = g(c, L \backslash \{ij\})$, so that (F) implies

$$f_i(c, L) - f_j(c, L) = f_i(c, L\backslash\{ij\}) - f_j(c, L\backslash\{ij\}) = g_i(c, L\backslash\{ij\}) - g_j(c, L\backslash\{ij\}) = g_i(c, L) - g_j(c, L)$$

or equivalently

$$f_i(c, L) - g_i(c, L) = f_j(c, L) - g_j(c, L).$$

Therefore, there is a constant $a(S, L) \in \mathbb{R}$ such that $f_i(c, L) - g_i(c, L) = a(S, L)$ for each $i \in S$. Since (DCE) also implies that $\sum_{i \in S} f_i(c, L) = \sum_{i \in S} c_i = \sum_{i \in S} g_i(c, L)$, we get

$$0 = \sum_{i \in S} \big(f_i(c, L) - g_i(c, L) \big) = |S| a(S, L)$$

and thus $a(S, L) = 0$, which implies that $f_i(c, L) = g_i(c, L)$ for each $i \in S$ and each $S \in N/L$, a contradiction proving the result. ■

Proposition 4. *The unique allocation rule on CS^N that satisfies Cost efficiency (CE), Strong no liability (NL$^+$) and either Additivity in costs (ADD) or Area of influence monotonicity (AIM) is the Beneficiary-pays allocation rule.*

Proof. It is not necessary to prove that the BP allocation satisfies the four axioms. Regarding the uniqueness parts, consider firstly an allocation rule f on CS^N that satisfies (CE), (NL$^+$) and (ADD). Pick any $(c, L) \in CS^N$. Using the decomposition (1), it is enough to prove that $f(c^i, L)$ is uniquely determined for each $i \in N$. So choose any $i \in N$. For each $j \in N \backslash \{i\}$, (NL$^+$) yields $f_j(c^i, L) = 0$ so that (CE) implies that $f_i(c^i, L) = c^i_i = c_i$. This complete the first uniqueness part.

Secondly, consider an allocation rule f on CS^N that satisfies (CE), (NL$^+$) and (AIM). Obviously, (NL$^+$) yields $f_i(\mathbf{0}, L)$ for each $i \in N$. As before, combining (CE) and (NL$^+$) yields that $f_i(c^i, L) = c_i$ for each $i \in N$. Now, (AIM) implies that $f_i(c, L) \geq f_i(c^i, L)$ for each $i \in N$. Applying (CE) in (c, L), we get

$$\sum_{i \in N} f_i(c, L) = \sum_{i \in N} c_i = \sum_{i \in N} f_i(c^i, L),$$

which forces $f_i(c, L) = c_i$ for each $i \in N$ and completes the second uniqueness part. ■

The axioms invoked in Proposition 3 and 4 are logically independent:

- The Polluter-pays allocation rule PP satisfies (DCE), (CE), (ADD) and (AIM) but not (F) and (NL$^+$).
- The allocation rule f such that for each $(c, L) \in CS^N$ and each $i \in N$, $f_i(c, L) = \sum_{j \in N} c_j / n$ satisfies (F), (CE) and (ADD) but not (DC), (NL$^+$) and (AIM).
- The null allocation rule satisfies (NL$^+$), (AIM), (F) and (ADD) but not (DCE) and (CE).
- The allocation rule f such that for each $(c, L) \in CS^N$ and each $i \in N$,

  $$f_i(c, L) = \begin{cases} \sum_{j \in N} c_j / n & \text{if } c_j > 0 \text{ for each } j \in N \\ BP_i(c, L) & \text{otherwise,} \end{cases}$$

  satisfies (CE) and (NL$^+$) but not (ADD), (F), (DCE) and (AIM).

3.3. Consistency

In this section, we provide an alternative axiomatic approach by means of a consistency principle (see Thomson, 2011, for a review of the abundant use of the consistency principle in game theory, economics and political science). In our context, for each problem under consideration and each cost allocation, a reduced problem can be constructed by considering the departure of an arbitrary firm after paying its cost share according to this allocation and reassessing the options open to the remaining firms. Then, an allocation rule is consistent if it selects the same the cost shares for the remaining firms in the original problem and in this reduced problem.

In order to design our consistency principle, it is necessary to consider a cost sharing problem for which the payoffs given by a cost allocation can be associated to the clean up of a given site without any ambiguity. This can be achieved in a problem (N, c, L) if there is a single firm $i \in N$ such that $c_i \neq 0$, which we call unambiguous problem. In such a case, what the firms pay is necessarily used to clean up the only polluted site. Hence, any unambiguous problem can be written as (N, c^i, L) for some cost vector c and some firm $i \in N$. From any unambiguous problem (N, c^i, L), any firm $j \in N\backslash\{i\}$ and any allocation $x \in \mathbb{R}^N_+$, we can construct the **reduced game** induced from (N, c^i, L) by the leave of j after paying its cost share x_j. Its is denoted by $(N\backslash\{j\}, (c^i)^{j,x}, L(N\backslash\{j\}))$, where for each $k \in N\backslash\{j\}$,

$$(c^i)^{j,x}_k = \begin{cases} \max\{0; c_i - x_j\} & \text{if } k = i, \\ 0 & \text{if } k \in N\backslash\{i, j\}, \end{cases}$$

and $L(N\backslash\{j\})$ is the usual restriction of L to $N\backslash\{j\}$. Moreover, the use of a consistency principle imposes to consider a class of problems with variable sets of firms. Therefore, in this section only, we consider the set CS of all problems with a finite firm set and we denote an element of it by a triple (N, c, L) so that we can clearly identify the set of firms under consideration.

Unambiguous consistency (UC). For each unambiguous $(N, c^i, L) \in CS$ and each $j \in N\backslash\{i\}$, it holds that for each $k \in N\backslash\{j\}$,

$$f_k\big(N, c^i, L\big) = f_k\big(N\backslash\{j\}, (c^i)^{j, f(N, c^i, L)}, L(N\backslash\{j\})\big).$$

In words, the axiom says that if a firm pays its cost share and leaves (i.e. it can no longer be held responsible for the pollution on site i) and the other firms reevaluate their opportunities by sharing the rest of the pollution cost, then the cost share of each remaining firm is unaffected. While Unambiguous consistency is satisfied by the PP and BP rules, the next results shows how it can help to characterize the PP rule.

Proposition 5. *The unique allocation rule on CS that satisfies district cost efficiency (DCE), Additivity in costs (ADD), Unambiguous consistency (UC) and either Equal allocation under equal liabilities (EAEL) or Strong equal allocation under equal liabilities (EAEL$^+$) is the Polluter-pays allocation rule.*

Proof. EXISTENCE. From the previous results in section 3.2, it is enough to show that PP satisfies (UC) on CS. So pick unambiguous $(N, c^i, L) \in CS$, any $j \in N\backslash\{i\}$ and any $k \in N\backslash\{j\}$. We distinguish two cases according to whether firm j is in L_i or not. First, If $j \in N\backslash L_i$, since $j \neq i$, we have $PP_j(N, c^i, L) = 0$. Hence $(c^i)_i^{j,PP(N,c^i,L)} = c_i - 0 = c_i$, which immediately implies that

$$PP_k(N, c^i, L) = PP_k\big(N\backslash\{j\}, (c^i)^{j,PP(N,c^i,L)}, L(N\backslash\{j\})\big) = \frac{c_i}{|L_i^+|}$$

for each $k \in L_i^+$. Now, for each $k \in N\backslash(L_i^+ \cup \{j\})$, we have

$$PP_k(N, c^i, L) = PP_k\big(N\backslash\{j\}, (c^i)^{j,PP(N,c^i,L)}, L(N\backslash\{j\})\big) = 0. \tag{5}$$

Second, if $j \in L_i$, then (5) holds for each $k \in N\backslash L_i^+$. Now, for each $k \in L_i^+\backslash\{j\}$, we have:

$$\begin{aligned} PP_k\big(N\backslash\{j\}, (c^i)^{j,PP(N,c^i,L)}, L(N\backslash\{j\})\big) &= \frac{(c^i)_i^{j,PP(N,c^i,L)}}{|L_i^+| - 1} \\ &= \frac{c_i - PP_j(N, c^i, L)}{|L_i^+| - 1} \\ &= \frac{c_i - c_i/|L_i^+|}{|L_i^+| - 1} \\ &= \frac{c_i}{|L_i^+|} \\ &= PP_k(N, c^i, L). \end{aligned}$$

UNIQUENESS. Consider any f on CS that satisfies the four axioms. Pick any $(N, c, L) \in CS$. From (ADD), it is enough to show that f is uniquely determined for all (N, c^i, L), $i \in N$. Denote by S the district in N/L containing firm i. From (DCE), we get $f_k(N, c^i, L) = 0$ for each $k \in N\backslash S$. Regarding the firms in S, we distinguish two cases.

Case (a). Assume that $S = L_i^+$. Then combining (DCE) and either (EAEL) or (EAEL$^+$) yields immediately $f_k(N, c^i, L) = c_i/|L_i^+|$ for each $k \in S$.

Case (b). Assume that $L_i^+ \subsetneq S$. Consider any ordering $(i_1, \ldots, i_{|L_i|})$ of the firms in L_i (which is nonempty by assumption). We consider $|L_i|$ successive applications of (UC) to the firms in L_i. To save on notation, for each $q \in \{1, \ldots, |L_i|\}$, we denote by $(N^q, (c^i)^{q,f}, L^q)$ the reduced problem obtained from (N, c^i, L) after the successive applications of UC to i_1, ..., i_{q-1}. Then, for each $k \in S\backslash L_i^+$, we have

$$f_k(N, c^i, L) = f_k\big(N^1, (c^i)^{1,f}, L^1\big) = \cdots = f_k\big(N^{|L_i|-1}, (c^i)^{|L_i|-1,f}, L^{|L_i|-1}\big) = f_k\big(N^{|L_i|}, (c^i)^{|L_i|,f}, L^{|L_i|}\big). \tag{6}$$

In the problem $(N^{|L_i|}, (c^i)^{|L_i|,f}, L^{|L_i|})$, note that each firm $k \in S\backslash L_i^+$ now belongs to a district different from the one of firm i and that this district is by construction free of pollution.

Therefore, (DCE) implies that

$$f_k\big(N^{|L_i|}, (c^i)^{|L_i|,f}, L^{|L_i|}\big) = 0,$$

so that we get $f_k(N, c^i, L) = 0$ for each $k \in S \backslash L_i$ from (6). Finally, combining the previous equality with (DCE) and either (EAEL) or (EAEL$^+$) yields that $f_k(N, c^i, L) = c_i/|L_i^+|$ for each $k \in L_i^+$, concluding the proof. ■

The axioms invoked in Proposition 5 are logically independent:

- The null allocation rule satisfies all axioms except (DCE);
- The BP allocation rule satisfies all axioms except (EAEL) or (EAEL$^+$);
- The allocation rule f such that $f(N, c, L) = BP(N, c, L)$ if $c_j > 0$ for each $j \in N$ and $f(N, c, L) = PP(N, c, L)$ otherwise satisfies all axioms except (ADD);
- The allocation rule f such that $f(N, c, L) = BP(N, c, L)$ if $|N| > 10$ for each $j \in N$ and $f(N, c, L) = PP(N, c, L)$ if $|N| \leq 10$ satisfies all axioms except (UC).

4. A cooperative game approach

Let N be a nonempty and finite set of players. Each subset $S \in 2^N$ is referred to a coalition of cooperating players. The grand coalition N represents a situation in which all players cooperate. Coalition $\varnothing$ represents a situation in which no player cooperates, it is called the empty coalition.

A transferable utility game, or simply a TU-game, is a couple (N, v) consisting of a finite players set N and a characteristic function $v : 2^N \rightarrow \mathbb{R}$, with the convention that $v(\varnothing) = 0$. The real number $v(S)$ can be interpreted as the worth the players in S generate when they cooperate. This worth can be perceived by the players as desirable (like profits) or, on the contrary, undesirable (like costs). We will focus on the second case: the players share costs. Thus, from now on, the game (N, v) is interpreted as a cost game. For ease of writing the game (N, v) will be designated by its characteristic function v when N is fixed.

The basic issue in the theory of cooperative games is to divide fairly the cost of the grand coalition among its members. This issue may be addressed using allocations for TU-games. An allocation $x \in \mathbb{R}^{|N|}$ is a $|N|$-dimensional vector that assigns a share of the cost $x_i \in \mathbb{R}$ to each player $i \in N$.

An efficient allocation shares exactly $v(N)$ among the players and it is called coalitionally rational if no coalition would be better off by splitting from the grand coalition and paying its cost. The core (Shapley, 1955) of a game v is the set $Core(v)$ of efficient and coalitionally rational allocations:

$$Core(v) = \Big\{x \in \mathbb{R}^N : \sum_{i \in N} x_i = v(N) \text{ and for each } S \subseteq N, \sum_{i \in S} x_i \leqslant v(S)\Big\}.$$

In the context of cost sharing, an allocation in the core cannot be dominated by an alternative allocation in which some player or coalition of players can have their cost shares reduced. Thus, no coalition of players has the incentive to not cooperate in the grand coalition, which can be described as the fact that the allocation is stable. Shapley (1971) demonstrates that the core of a concave game is nonempty. Formally, a game v on N is concave if

$$v(S \cup \{i\}) - v(S) \geq v(T \cup \{i\}) - v(T), \quad \forall S \subseteq T \subseteq N \backslash \{i\}.$$

The core can contain several allocations from which it can be difficult to choose one and only one. Shapley (1971) proves that the Shapley value of a concave game lies in its core. The Shapley value (Shapley, 1953) assigns to each game v a unique allocation $Sh(v)$ such that for each $i \in N$:

$$Sh_i(v) = \sum_{S \subseteq N \backslash \{i\}} \frac{|S|!(|N| - |S| - 1)!}{|N|!} \Big(v(S \cup \{i\}) - v(S) \Big).$$

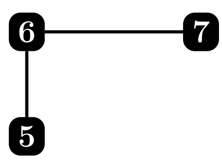
The Shapley value of a player can be seen as an average of the additional cost incurred by the player to each coalition not already containing her.

In the rest of this section, we prove that the Polluter-pays allocation rule coincides with the allocation prescribes by the Shapley value of a suitable cooperative game. More specifically, to each problem $(c, L) \in CS^N$ we associate the Polluters game $v_{c,L}$ on N such that, for each coalition $S \subseteq N$,

$$v_{c,L}(S) = \sum_{i \in L_S^+} c_i$$

where $L_S^+ = \cup_{j \in S} L_j^+$. The Polluters game $v_{c,L}$ is a global approach of pollution. The cost $v_{c,L}(S)$ can be interpreted as the cost of cleaning up the pollution that can be caused by the members of coalition S. Example 2 illustrates this game.

Example 2. The set of firms is $N = \{1, 2, 3\}$, the area of pollution influences are given by $L_1^+ = \{1, 2\}$, $L_2^+ = \{1, 2, 3\}$ and $L_3^+ = \{2, 3\}$. The costs of cleaning up NPS pollution are the following $c_1 = 4$, $c_2 = 3$ and $c_3 = 2$.



S	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$v_{c,L}(S)$	7	9	5	9	9	9	9

□

Proposition 6. *For each problem (c, L), the Polluter-pays allocation rule $PP(c, L)$ is the Shapley value of the game $v_{c,L}$.*

Proof. We show that the allocation rule f on CS^N such that for each $(c, L) \in CS^N$, $f^*(c, L) = Sh(v_{c,L})$ satisfies the four axioms invoked in Proposition 1.

Since the Shapley value is an efficient allocation rule, for each problem $(c, L) \in CS^N$, we have that

$$\sum_{i \in N} f_i^*(c, L) = \sum_{i \in N} Sh_i(v_{c,L}) = v_{c,L}(N) = \sum_{i \in L_N^+} c_i = \sum_{i \in N} c_i,$$

proving that f^* satisfies (CE).

Next, consider any two problem $(c, L), (c, L') \in CS^N$. Then,

$$v_{c+c',L}(S) = \sum_{i \in L_S^+} (c + c')_i = \sum_{i \in L_S^+} c_i + \sum_{i \in L_S^+} c'_i = v_{c,L}(S) + v_{c',L}(S)$$

for each $S \subseteq N$. From the additivity of the Shapley value, we get that

$$f^*(c + c', L) = Sh(v_{c+c',L}) = Sh(v_{c,L}) + Sh(v_{c',L}) = f^*(c, L) + f^*(c', L),$$

proving that f^* satisfies (AC).

Now, consider any problem $(c, L) \in CS^N$ and any firm $i \in N$ such that $c_i = 0$ and $c_j = 0$ for each $j \in L_i$. Pick any coalition $S \subseteq N \backslash \{i\}$. Then

$$v_{c,L}(S \cup \{i\}) = \sum_{j \in L_{S \cup \{i\}^+}} c_j = \sum_{j \in L_S^+} c_j + \sum_{j \in L_i^+ \backslash L_S^+} = \sum_{j \in L_S^+} c_j = v_{c,L}(S).$$

This means that firm i is a null player in the Polluters game $v_{c,L}$. Since the Shapley value satisfies the Null player axiom, it follows that $f_i^*(c, L) = Sh_i(v_{c,L}) = 0$, proving that f^* satisfies (NL).

Finally, consider a problem $(c, L) \in CS^N$ and two firms $i, j \in N$ such that $\{q \in L_i^+ : c_q > 0\} = \{k\} = \{q \in L_j^+ : c_q > 0\}$ for some $k \in N$. Pick any coalition $S \subseteq N \backslash i, j$. If $k \in L_S^+$, then

$$v_{c,L}(S \cup \{i\}) = \sum_{q \in L_{S \cup \{i\}}^+} c_q = \sum_{q \in L_S^+} c_q = v_{c,L}(S \cup \{j\}).$$

If $k \in N \backslash L_S^+$, then

$$v_{c,L}(S \cup \{i\}) = \sum_{q \in L^+_{S \cup \{i\}}} c_q = \sum_{q \in L^+_S} c_q + c_k = v_{c,L}(S \cup \{j\}).$$

Combining the previous two cases, we have that i and j are equal players in the Polluters game $v_{c,L}$. Since the Shapley value satisfies the axiom of Equal treatment of equal players, we obtain that $f_i^*(c,L) = Sh_i(v_{c,L}) = Sh_j(v_{c,L}) = f_j^*(c,L)$, proving that f^* satisfies (EAEL). We proved that f^* satisfies the four axioms invoked in Proposition 1. Since there is a unique allocation rule satisfying this set of axioms, we conclude that $f^* = PP$ as desired. ∎

Proposition 7. *The Polluters game $v_{c,L}$ is concave and thus its core contains the allocation prescribed by the Polluter-pays allocation rule. Moreover, the core contains also the allocation prescribed by the Beneficiary-pays allocation rule.*

Proof. Consider any problem $(c,L) \in CS^N$ and the associated Polluters game $v_{c,L}$. Pick any firm $i \in N$ and a pair of coalitions $S, T \subseteq N$ such that $S \subseteq T \subseteq N \backslash \{i\}$. Obviously, we have $L_S^+ \subseteq L_T^+$. Using this inclusion, we can write that

$$\begin{aligned} v_{c,L}(S \cup \{i\}) - v_{c,L}(S) &= \sum_{j \in L_{S \cup \{i\}^+}} c_j - \sum_{j \in L_S^+} c_j \\ &= \sum_{j \in L_i^+ \backslash L_S^+} c_j \\ &\geq \sum_{j \in L_i^+ \backslash L_T^+} c_j \\ &= \sum_{j \in L_{T \cup \{i\}^+}} c_j - \sum_{j \in L_T^+} c_j \\ &= v_{c,L}(T \cup \{i\}) - v_{c,L}(T), \end{aligned}$$

which proves that $v_{c,L}$ is concave.

The BP is obviously an efficient allocation in any problem (c,L). Moreover, for each $S \subseteq N$, we have

$$\sum_{i \in S} BP_i(c,L) = \sum_{i \in S} c_i \leq \sum_{j \in \cup_{i \in S} L_i^+} c_j = \sum_{j \in L_S^+} c_j,$$

where the inequality comes from the fact that $S \subseteq \cup_{i \in S} L_i^+$ for each nonempty coalition $S \subseteq N$. Hence, $BP(c,L)$ is coalitionnaly rational as well, proving that $BP(c,L)$ lies in the core of the Polluters game $v_{c,L}$. ∎

5. Conclusion

We conclude with a recap chart in which a "+" means that an axiom is satisfied by the corresponding allocation rule and a "–" has the converse meaning. Superscript numbers indicate the propositions in which the axioms are invoked for a characterisation.

	(CE)	(DCE)	(ADD)	(F)	(NL)	(NL^{+})	$(EAEL)$	$(EAEL^{+})$	(AIM)	(UC)
PP	$+^{1,2}$	$+^{5}$	$+^{1,5}$	$-$	$+^{1}$	$-$	$+^{1,5}$	$+^{2,5}$	$+^{2}$	$+^{5}$
BP	$+^{4}$	$+^{3}$	$+^{4}$	$+^{3}$	$+$	$+^{4}$	$-$	$-$	$+^{4}$	$+$

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