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Space debris removal funding in the presence of numerous large satellites constellations: a mean field game approach*

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Abstract

Low Earth orbits (between 100 and 2,000 km above sea level) are currently the most congested regions of outer space. This congestion stems also from the presence of space debris, which, once it reaches a certain level, could trigger a chain reaction that would physically prevent the use of these orbits (i.e., the Kessler syndrome). The actual and planned deployment of numerous satellite mega-constellations significantly exacerbates this problem. In addition to minimizing the creation of new space debris, it has become necessary to remove some of the existing debris. To address the issue of funding these removals, we propose the creation of an international tax administered by an international agency. Given the current and future presence of a large number of mega-constellations, we propose a mean-field game model. We then compare two possible taxation systems: one proportional and the other one progressive. Within the framework of this model, we conclude that it is possible to finance the removal of the most dangerous space debris and thereby ensure a sustainable outer space without stopping the contributions and potential of the New Space economy.

Keywords: Mean-field game, linear-quadratic, outer space, space debris, mega constellations.

JEL Classification: C72, C61, H23, H32.

MSC Classification: 49N10, 49N80, 49N90, 91A16

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1 Introduction

The 1990s—particularly due to the U.S. government's decisions to authorize the commercialization of high-resolution satellites, to make the satellite positioning signal (i.e., GPS) publicly available and usable, and to allow for commercial agreements with foreign launch companies— laid the groundwork for what would become, at the dawn of the 21st century, the begining of a new era in outer space[1], since referred to as "New Space." This expression describes a paradigm shift both across all segments of the space sector (i.e., satellite manufacturing, satellite launch, satellite services, and ground equipment) and in the way we conceive our uses of outer space. All these developments are linked to the interplay of political, legal, and commercial changes, as well as to numerous scientific and technical innovations (e.g., digital technology, algorithms, microelectronics, CubeSats, 3D printing, reusable rockets) (for a general overview of these developments, see, in particular, Weinzierl (2018), Peeters (2021), Raswant et al. (2025).

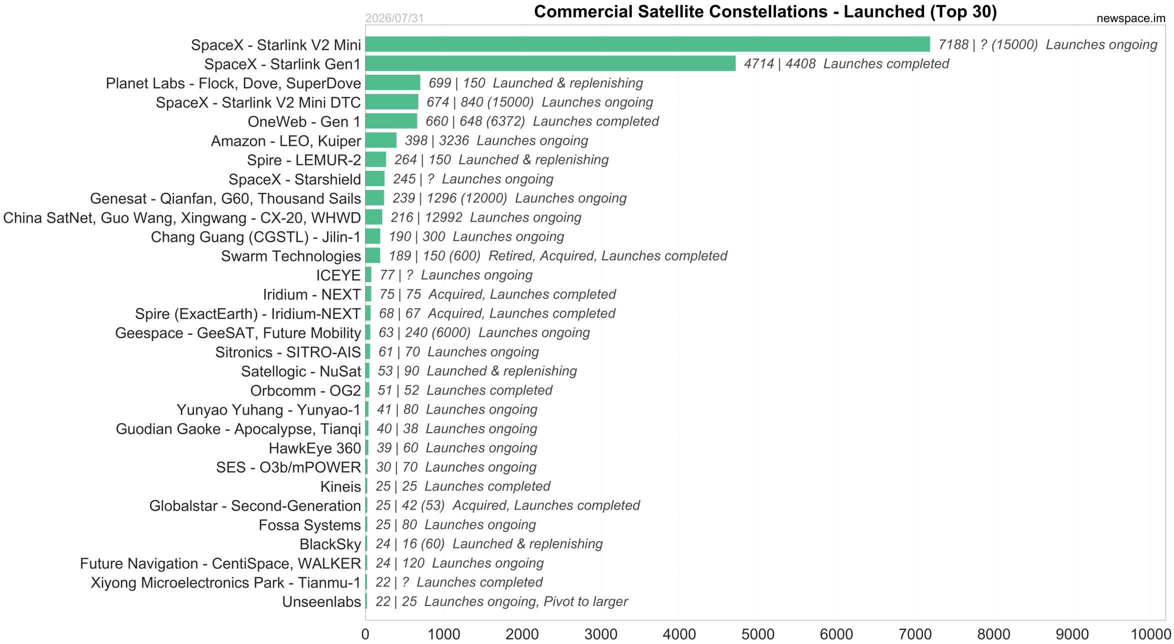
Since then, the benefits of these changes have been utilized on a daily basis in every country around the world. In fact, in addition to scientific and military applications, satellites are now widely used for civilian and commercial purposes, which are typically classified into three categories: Earth observation, Positioning-Navigation-Timing, and Communication. Furthermore, there are numerous technological spillovers from the space sector to all economic sectors, such as the thermal blankets initially designed to protect satellites and now used in emergency response (see, in particular, Mazzucato (2021) for numerous examples of everyday products originating from the space sector).

However, in recent years, the number of satellites launched and in operation has been growing exponentially. Basically, while there were 1000 active satellites in 2000, 1,500 in 2010, and 12,000 in 2025, by 2026 there were about 16,000 active satellites (ESA (2026)), most of which are located in some regions of low Earth orbits (i.e., between 250 and 1,500 km above sea level). This disruptive shift stems from the rise of a New Space economy, driven both by the growing number of public and private players in the space sector and, above all, by the advent of mega-constellations.

First, it should be noted that the idea of creating satellite constellations in low Earth orbit is not an innovation of New Space. In fact, the first constellations in these orbits —comprising dozens of satellites (max below 70)— emerged in the wake of telecommunications deregulation in the late 1990s, driven by industry players and funded by private capital. They were all focused on mobile telephony, data transmission, and Internet access (e.g., Iridium, Globalstar, Orbcomm, SkyBridge, Teledesic). All of these constellations were commercial failures. Nearly all went bankrupt, and none of those deployed at the time truly found a profitable market. Today, the new constellations that have been deployed and those currently in the planning stages share the same objectives, but they are on an entirely different scale. They consist

[1] There is currently no international treaty, nor international agreement defining the altitude at which outer space begins. Three main concepts coexist: (a) an arbitrary threshold of 100 km above sea level, (b) the "Kàrmàn line," which corresponds to the altitude at which aerodynamic flight becomes impossible and is located at approximately 80 km, and finally (c) the minimum altitude at which it is possible to remain in orbit, which corresponds to approximately 125 km. Although this issue may have legal implications, it has no bearing on our analysis.

of several hundred, or even tens of thousands of satellites for the largest among them, using satellites with a different technology, that are much smaller and less maneuverable than those of the 1990s. According to McDowell[2], nine "enormous" mega-constellations (i.e., more than 1,000 satellites) are currently in the planning stages (Starlink, OneWeb, Kuiper, Starshield, Xingwang, Qianfan, Yinhe, E-Space, and PWSA), and the NewSpace Index website[3] indicates that there are already 12 mega-constellations numbering more than 100 satellites:



Launching such a large number of satellites raises problems on a scale we have never experienced before. In addition to pollution issues related to the use of resources needed to manufacture and launch these satellites into space, they have major consequences in terms of light pollution, collision risk, availability and interference of frequencies used in orbit, space debris, and atmospheric and marine pollution when they re-enter Earth's atmosphere at the end of their lifespans. It must be emphasized, however, that it seems neither possible nor desirable to consider halting the launch of satellites, a move that would have truly dramatic consequences. These satellites serve all our societies at every moment and to an ever-greater extent, among other things to progressively achieve all seventeen of the UN's Sustainable Development Goals (SDGs) (Paravano et al. (2025)) and to help preserve the sustainability of our planet, particularly through Earth system monitoring (since of the 55 essential climate variables, 38 of them are exclusively or largely measured from outer space).[4]

Since we cannot address all of these issues at once, this article will focus solely on collision risk, its mitigation through the removal of space debris and the financing of these operations. We leave the discussion of all other issues open.

[2]See https://planet4589.org/space/con/conlist.html (August, 2026).

[3]See https://www.newspace.im/ (July, 2026).

[4]https://climate.esa.int/en/about-us-new/climate-change-initiative/Climate-Change-Initiative/(August 2026).

Although there is no legally binding definition, the expression “space debris” will be used here to refer to any man-made object that serves no purpose, as is the practice of all space agencies. This debris stems primarily from intentional jettisoning (e.g., launch vehicle upper stages), explosions (e.g., fuel tanks, batteries), unintentionally lost components (e.g., tools, bolts), inactive satellites, and collisions. The main dangers to satellites posed by debris stem from explosions[5] but, above all, from collisions, since most operators now deactivate their satellites at the end of their service life to prevent accidental explosions. In fact, in outer space, within low Earth orbits, collisions occur at relative speeds ranging from 25,000 to 57,000 km/h, and these collisions generate a multitude of new debris. For example, a piece of debris just a few centimeters in size can permanently disable a satellite, and a piece larger than 10 centimeters can completely destroy one. There is therefore a risk of triggering a chain reaction in which the amount of debris reaches a threshold such that it becomes virtually impossible to launch a satellite without it colliding and, in turn, creating new debris: this is known as the Kessler syndrome (Kessler and Cour-Palais (1978); Kessler (1991); Lewis and Kessler (2025)). According to the latest data from the European Space Agency (ESA)[6], there are currently 54,000 pieces of debris larger than 10 cm in space, 1.2 million pieces between 1 and 10 cm, and 140 million pieces between 1 mm and 1 cm. Furthermore, beyond the size and number of debris pieces present, the altitude at which they are located is very important because it determines how long they will remain in orbit. Thus, according to NASA[7]: “Debris left in orbits below 600 km normally fall back to Earth within several years. At altitudes of 800 km, the time for orbital decay is often measured in centuries. Above 1,000 km, orbital debris will normally continue circling the Earth for a thousand years or more.”

From a legal standpoint, the current general framework of space law is established by the treaty of October 1, 1967, which sets forth the principles governing human activity in outer space —that is, the principles that states have committed to uphold and must ensure are respected. Three major principles are set forth therein. First, outer space must be used for the benefit and in the interest of all countries, regardless of their level of economic or scientific development (art. 1). Second, States may not appropriate outer space, the Moon, nor any other celestial bodies, regardless of the method used (e.g., proclamation of sovereignty, use, occupation) (art. 2). Third, outer space must be used for peaceful purposes, and states must maintain international peace and security in outer space and promote international cooperation there (art. 3). More technical treaties address the international liability of states for damage caused by space objects and, to identify those responsible, require them to register objects launched into space.[8] However, to date there is no international treaty on space debris, and it must be noted that since the 1990s, despite the numerous charters, guidelines, and documents issued by space agencies (e.g., IADC), the UN (e.g., COPUOS), the OECD, the International Organization for Standardization (e.g., ISO 24113), the Davos Economic Forum, industry leaders, and, more recently, the G7 along with its seven Academies of Sci-

[5] This includes cases where such actions are deliberate, such as the anti-satellite missile tests that the United States, Russia, China, and India have already conducted against their own satellites. Some of these countries explicitly view space as a potential battlefield.

[6] See https://www.esa.int/Space_Safety/Space_Debris/Space_debris_by_the_numbers (August 2026).

[7] See https://orbitaldebris.jsc.nasa.gov/faq/# (August 2026).

[8] For a complete presentation of international space law, we refer to UN Office of Outer Space Affairs (https://www.unoosa.org).

ences, not only has the problem of space debris not been contained, but it continues to grow.

However, this problem is now: (a) becoming an urgent issue as it is escalating in scale with the deployment of all the mega-constellations, (b) explicitly on the agenda of all stakeholders, and (c) capable of being addressed since we have reached a stage where, technologically speaking, we are close to being able to begin attempting to remove space debris[9]. Our article will therefore focus on the question of how to finance these removals (to stabilize or reduce space congestion, depending on the chosen objective) while preserving the benefits associated with the New Space economy.

Since this problem requires an international response, we make the crucial assumption in this article that there will be an international agency in the future[10] tasked with this mission, funded by an international tax.[11] Our contribution aims to investigate the effects of two different taxing schemes on the equilibrium distribution of large constellations: either proportional to the number of satellites in the constellation or progressive, this while keeping in check the number of dangerous debris. Given the number of large constellations to be expected in the comming years, potentially several tens, we choose to use the framework of mean-field games.

Mean-Field Games (MFG) theory aims to approximate a game with a large number of identical players with a continuum of players. The main simplifying assumption is in representing the interactions of a player with the others as the mean effect of a global "field" of agents, the rest of the population being represented by a distribution of their characteristics. And because this population is infinite, and as in Aumann (1964), each player is "infinitesimal" and its actions are assumed to be without direct effect on this distribution. Yet, this distribution evolves according to the strategies of all players. As a matter of facts, all players being identical and sharing the same information on the state of the population, at equilibrium they use the same strategies that, together, shape their distribution. It should be noted that the practice of approximating the behavior of a large population as a continuum of agents has a long history in mathematical modeling. It dates back at least to the Lotka-Volterra prey-predator dynamics (Lotka (1924); Volterra (1926)) which use real numbers for densities of populations, and was largely extended in population games (Sandholm (2003, 2010)), epidemiology (Petrakova and Krivorotko (2022), Fabbri et al. (2024), Ghilli et al. (2024)), and related domains (Breban et al. (2007)). A somewhat non standard example is Lahkar et al. (2023), where in reverse of most of the MFG literature which approximates a N-player game with a continuum of players, such a game with a continuum of players is approximated by a N-person game to prove convergence of the logit dynamics in the former. Another domain where a large number of agents has been approximated by a continuum is in dynamics over

[9]Regarding the question of the order in which most hazardous space debris should be removed, we refer to Laruelle et al. (2025), who offer a recent overview perspective based on social choice.

[10]This is an assumption that has already been proposed in the literature (e.g., Béal et al. (2020); Bernhard et al. (2023) and which can be supported (e.g., Deschamps and Thomas (2025)), despite the pessimism that may exist regarding its implementation in the short term, given both the current state of international geopolitics and the situation of multilateral international organizations. Naturally, this could take the form of either a new institution created specifically for this purpose or a new mandate entrusted to an existing institution, such as the UN.

[11]Regarding the legal feasibility of an international tax on space activities, we refer in particular to the arguments put forward by Scuderi (2026).

a network. A so called "dense network" of communication nodes is represented by a density of nodes at everyplace over the network, or a (continuous) distribution of degrees accros the network. See e.g. Shin (2017). At least concerning communications networks, this literature seems to have been initiated by Jacquet (2004), a typical example is Altman et al. (2007). This has been followed by an explicit use of MFG theory, such as in Coron (2017) where an infinite network is used to propose an algorithm of classification according to K-means, or Leduc et al. (2017) where the mean effect over an homogeneous infinite network arises as a probability law on the degree of nodes adjacent to any of them. A modern description of Mean-Field Games analysis considers the *master equation*, a kind of Bellman equation in a state space product of $\mathbb{R}^d$ and a (probability) measure space, as the central tool (Lions (2026)). In this type of presentation, the method we use here is considered as a particular case where one may dispense with the infinite dimensional state space. It follows the seminal contributions of the theory by Lasry and Lions (2006a,b) and early examples such as Guéant et al. (2011), as well as the simultaneous and independent articles from Huang et al. (2006a,b).

Our contribution thus connects to the extensive literature on applications of MFG theory to economic problems, starting with some of the earliest examples such as Guéant (2009); Guéant et al. (2011) up to now. The breadth of MFG applications has since grown and is still growing. See Başar et al. (2026) for an extensive account or Bensoussan (2022) for examples in management science. In the present contribution, we also take advantage of the further simplification arising from the linear-quadratic character of our problem, following Huang et al. (2007); Bardi (2012), Bensoussan et al. (2013), Bensoussan et al. (2016) or Başar et al. (2026). This allows us to go all the way up to numerical solutions of the problems investigated, confirming the desired effect of the progressive tax. While a large part of the literature considers first a N-player game passing to the limit as $N \to \infty$, to simplify the exposition and the analysis and following the early lead of Aumann (1964) and, for mean-field games the theory of (Carmona and Delarue, 2018, pp 206–208), we start directly from the game with a continuous infinity of players. This spares us the complexity of a limiting process. The price to be paid is that we do not address the question of the quality of the MFG approximation of the real world N-player game.

To the best of our knowledge, we are the first to explore the issue of funding the removal of space debris through a tax within the context of a mean field game. In that respect, our contribution also connects to the growing economic literature on outer space (Bongers et al. (2024) provide the most up-to-date and detailed survey), whose origins can be traced back to seminal contributions of Snow (1975), O'Neill (1977), Wihlborg and Wijkman (1981), Sandler and Schulze (1981). It was this last contribution that marked the beginning of economic research on the risk of satellite collisions. However, it is only in the last little more than ten years that a small but growing body of economic literature has begun to address the problem of congestion (in terms of both orbits and spectrum) and, more specifically, space debris. Seminal papers investigating the economic consequences of space debris externality and measures that both prevent their production and finance their removal are Adilov et al. (2015, 2018) and Macauley (2015). They were followed among others by Grzelka and Wagner (2019), Béal et al. (2020), Rao et al. (2020), Rouillon (2020), Bongers and Torres (2023), Bernhard et al. (2023), Guyot and Rouillon (2023), Guyot et al. (2023), Nomura et al. (2024), and Bongers et al. (2025). Almost every one of these contributions mentions either the idea of a launch tax or an orbital tax. The articles most closely related to ours are those by Bernhard et al. (2023), Bernhard et al. (2026) as they examine the problem of space debris in a fully dynamic and strategic context. However, unlike those models, we propose a continuous-time model with a continuum of players.

In a nutshell, we show that with space debris removal funded we can have a sustainable exploitation of outer space, even in the presence of numerous large satellites constellations. The rest of the paper is organized as follows. In Section 2, we introduce our assumptions (on constellations, debris, collisions, and taxing schemes) and develop the model. In Section 3, we state it as a dynamic game with a continuum of players, whose precise form depends on the option regarding the constellation's management and the taxing scheme. In Section 4, we provide the mean-field solution and propose a numerical simulation. We conclude in Section 5. (Appendix detail closed-form solutions for the case with no concave term added in the revenue, and monopolies case with no constellations interferences.)

2 The model

2.1 Dynamics

Our model is to take place over the time interval $t \in [0, T]$. A classical probabilistic framework $(\Omega, \mathcal{A}, P)$ and a filtration indexed by time t allow one to introduce standard independant Brownian motions, i.e. adapted Wiener processes, $W_i(t)$ defined over $\mathbb{R}$.

2.1.1 Operators and their constellations

There is a continuously infinite measured set $\mathbf{M}$ of constellation operators, endowed with a σ-algebra $\mathcal{M}$ and a positive measure $\mathbb{M}$ of total mass $\mathbb{M}(\mathbf{M}) = 1$. At every instant $t \in [0, T]$ each operator $\mu \in \mathbf{M}$ operates a constellation of $x_\mu(t)$ satellites. We will need that the application $\mu \mapsto x_\mu(t)$ be measurable, and that this variable have a differentiable cumulative distribution function $M(t, x)$ with $\partial M(t, x)/\partial x = m(t, x)$ that we will be interested in.

Therefore, technically, $\mu \mapsto x_\mu(t)$ is a random variable of density $m(t, x)$. In keeping with the theory of field of particles, the various μ do not describe alternative possibilities among which Chance will eventually pick one *realization* as in classical probability theory, but are simultaneously active players, each operating a constellation with $x_\mu(t)$ satellites, allowing for interactions between their constellations, the source of the resulting McKean-Vlasov equation. (See, e.g. Lasry and Lions (2006a); Huang et al. (2006b)). This produces an infinite set, or *cloud*, of satellites, that we endow with a total measure or "*weight*"

$$X(t) = \int x m(t, x) \, \mathrm{d}x \, .$$

The processes $t \mapsto x_\mu(t)$ will be stochastic, specifically diffusions with identical independent initial distribution densities $m_0(\cdot)$, independent of the mutually independent driving brownian processes of these diffusions.

It follows from the theory of propagation of chaos (e.g., Benachour et al. (1998)) that these diffusions will remain independent identically distributed stochastic variables, and that their common distribution density $m(t, x)$ will also be that of the population, i.e. of the stochastic variable $\mu \mapsto x_\mu(t)$. So that any particular constellation may be regarded as a typical representative of the population.

Remark We chose to state our problem directly with a continuum of players, following the early lead of Aumann (1964). But this model may also be understood as the limit of one where $\mathbf{M}$ would be a finite set of N operators. Then $\mathbb{M}(t,\cdot)$ would be the classical stochastic *empirical measure* $\mathbb{M}_t = (1/N)\sum_{\mu=1}^{N} \delta_{x_\mu(t)}$, (with δ_{x_μ} the Dirac measure, and since x_μ is scalar, $M(t,\cdot)$ represented by a staircase function and $m(t,\cdot)$ a finite sum of Dirac impulses), and $X(t)$ the mean number of satellites per operator. Then (see next paragraph) we would let y be $1/N$ times the number of debris, and our collision model would be proportional to a Lotka-Volterra model. We will use this parallel to choose numerical values in our simulations.

2.1.2 Debris and collisions

The constellations considered evolve in a region of space where there is an infinite set, or "cloud", of mean weight $y(t)$ of passive debris. We assume that the international agreement is based upon an agreed desired future trajectory $\bar{y}(\cdot)$ of y, which necessarily obeys $\bar{y}(0) = y_0$ the weight of the debris cloud at initial time. A typical example might be a decreasing exponential, $\dot{\bar{y}} = -\delta\bar{y}$, with δ a small positive number, or a constant $\bar{y}(t) = y_0$, (i.e. $\delta = 0$).

Collisions We cannot adopt a strict model of collisions *à la* Lotka-Volterra, because with an infinite number of objects aloft, an isolated satellite would be almost surely instantaneously destroyed, its population suffering an infinite decrease rate. Instead, we introduce a small *congestion parameter* τ, of the order of 10^{-6} to 10^{-7} (see e.g., Le May et al. (2018)), which may depend on the nature of the objects considered, and we assume that an isolated object evolving in an infinite cloud of objects of weight Z has a probability τZ of colliding per unit time. Each collision is assumed to produce a number ν of debris, so that collisions between objects of a cloud of weight X and a cloud of weight y produce a flow of debris of weight $\tau\nu Xy$ per unit time, while collisions between objects within a single cloud of weight y produce a flow of debris of weight $\tau\nu y^2/2$ per unit time. Notice that this is completely in line with the classical formulation for these large population games (e.g., Bardi (2012)).

Constellation size dynamics The number $x(t)$ of satellites of a typical constellation varies due to

- a mean rate α of deorbitation, be it by natural decay or by active deorbitation, resulting in a decrease $\mathrm{d}x = -\alpha x\,\mathrm{d}t + \sigma_1\,\mathrm{d}W_1$
- a mean rate β of spontaneous breakdowns —these become debris—, resulting in a decrease $\mathrm{d}x = -\beta x\,\mathrm{d}t + \sigma_2\,\mathrm{d}W_2$,
- a mean rate $\tau_1 y$ of collisions with debris, (τ_1 is a small parameter, of the same order of magnitude as τ), creating a decrease $\mathrm{d}x = -\tau_1 yx\,\mathrm{d}t + \sigma_3\,\mathrm{d}W_3$,
- the launch of ℓ satellites per unit time by the constellation operator, leading to an increase $\mathrm{d}x = \ell(t)\,\mathrm{d}t$.

This leads to the following partial model

$$\mathrm{d}x = -(\alpha + \beta + \tau_1 y)x(t)\,\mathrm{d}t + \ell\;\mathrm{d}t + \sigma\,\mathrm{d}W_t\,, \tag{1}$$

where $\sigma^2 = \sigma_1^2 + \sigma_2^2 + \sigma_3^2$. Moreover, at time $t = 0$, the operators all have some satellites aloft, supposedly a small number, independently identically distributed according to the common density $m_0(x)$. Therefore, as in the theory of fields of particles, m_0 is both the distribution of initial sizes of the constellations and for each constellation the distribution of its initial conditions, independent of that of the other constellations.

At this point, we have not yet considered the interactions between the constellations. This is to come later.

Remark It is desirable, for the realism of the model, that $x(t)$ remain (almost surely) non-negative for all t. The dynamics (1) may let $x(t)$ become negative. Replacing this diffusion by a geometric diffusion would solve that issue, but on the one hand, it means having a launch rate $x\ell$, not very satisfactory either, and on the other hand, requiring that the launch costs be $cx\ell + dx^2\ell^2/2$, it ruins the quadratic nature of the resulting cost in our problem. Examination of the support of the density $m(t, x)$ obtained will let one know whether this is a real concern for the realism of this model. (The numerical results of our simulation section 4.2 show that this is not.)

Debris dynamics The mean weight y of space debris varies due to:

- a decrease at a mean rate α_0 by natural decay corrected by arrivals from higher orbits. Hence $\alpha_0 < \alpha$, and depending on higher orbits, α_0 may be very small, and even negative. It creates a mean variation $\mathrm{d}y = -\alpha_0 y \,\mathrm{d}t$,
- a mean increase of the weight of the cloud of debris $\mathrm{d}y = \beta X(t)\,\mathrm{d}t$ dead satellites,
- the creation of a mean weight $\mathrm{d}y = \nu\tau_1 X(t)y(t)\,\mathrm{d}t$ of new debris by collisions with the constellations' satellites,
- the creation of a mean weight $\mathrm{d}y = \nu\tau_0(y^2/2)\,\mathrm{d}t$ new debris by collisions of debris with debris (τ_0 is of the same order of magnitude as τ_1, probably larger). That term may make the weight of debris diverge, preventing the use of that orbital altitude (i.e., the Kessler syndrome),
- a decrease of $\mathrm{d}y = -h(t)\,\mathrm{d}t$ via Active Debris Removal (ADR). In our model, $h(t)$ is assumed to be chosen so as to keep $y(t) = \bar{y}(t)$, the cost of ADR being charged to the operators according to a taxing scheme to be discussed hereafter,
- possible collisions of satellites with satellites of other constellations. This possibility requires a further discussion, according to an hypothesis on constellations management.

This leads to the following partial model:

$$\dot{y} = \left[-\alpha_0 y + \beta X + \nu\tau_1 X y + \nu\tau_0 \frac{y^2}{2} - h\right]. \tag{2}$$

Constellation management Two main possibilities may occur (a third possibility will appear as a special case of both.)

1. One possibility is to assume that the constellations are watched over by their operators so that collisions do not occur: they are detected before hand and avoided through just-in-time collision avoidance (JCA) maneuvers. (This is the hypothesis made for the finite operators problem in Bernhard et al. (2023).) In that case, these are not reflected in the dynamics, but only in the profits of the operators through a cost of maneuvering. If a unit maneuver costs 2γ (through loss of time aloft due to orbit-keeping propellant usage), then the typical constellation operator bears on the average a cost $\gamma\tau_2 x(t)X(t)$, because only one of the potentially colliding satellites need maneuver, and thus each constellation will maneuver half of the times in the average.

   However, the frequency of these events is larger than that of actual collisions in the absence of JCA. This is because a maneuver is triggered each time a *risk* of collision is detected. Typically, if the trigger is that the collision probability reaches a threshold π, then τ_2 —which is typically of the same order of magnitude as τ_1— should be multiplied by $1/\pi$, a coefficient of the order of 10^2 according to space agencies. We will hereafter rather assume that this coefficient has been absorbed in γ.

2. On the contrary, if we assume that there are too many constellations, and collisions do occur between their satellites, this appears
   - in the dynamics (1) of x as a term $-\tau_2 x(t)X(t)\,\mathrm{d}t$,
   - in the dynamics (2) of y by a term $+\nu\tau_2 X^2/2$, to be added to h if y is to follow its desired trajectory, and therefore to the taxes collected.

Remark Setting $\tau_2 = 0$ makes the two management methods coincide and would model a situation where constellations are engineered in such a way as to have no physical interferences. See A.

2.2 Taxes and payoff

2.2.1 Taxing schemes

Remember that it is assumed that an international agreement has created an agency in charge of proceeding with the ADR in a way to ensure that the expectation $y(t)$ of the debris cloud weight follows a pre-defined trajectory. For the sake of simplicity, we will only consider the case where that trajectory is just $y(t) = y_0$. (Although we might easily consider, e.g., a decreasing exponential.) The agency is financed through a tax on the operators. In Bernhard et al. (2023), the tax charged to each operator is proportional to its contribution to the creation of debris, the common proportionality coefficient p being the unit cost of an ADR. Here, it will be the cost of removing as many debris as there are operators, to make y decrease by one unit. We will investigate this tax system in the present mean field context as a benchmark against which to compare the nonproportional weighting.

An aim of this contribution is to compare that policy with one penalizing more the constellations at the higher end of the distribution of satellite numbers, and less those at the lower end. To that end, we investigate a different tax system: the tax charged will then be proportional to the individual contribution to the creation of debris, but with an extra coefficient $k(x, m)$ chosen to be smaller for operators at the lower end of the distribution of satellite numbers, and larger for those at the upper end of that distribution. The coefficient will thus be a function of the rank of the operator among its peers. We write the total coefficient charged as $pk(x, m)$ to allow for a choice of the parameter p to match taxes and the cost of ADR. The case similar to Bernhard et al. (2023) is obtained by letting $k(x, m) = 1$.

In order to stay within the framework of linear quadratic problems, we choose

$$k(x, m) = \kappa(m)x$$

with a parameter $\kappa(m)$ only depending on the distribution m. For debris productions proportional to x, this means taxing them proportionally to x^2. For the comparison of the two taxing schemes, proportional and progressive, to be meaningful, we need that the total tax for the same (mean) number of debris creation be the same in both taxing schemes, i.e., again if debris creation is proportional to x, that

$$\int_0^\infty xk(x, m)m(t, x) \, \mathrm{d}x$$

be the same in the two taxing schemes. This is obtained if

$$\kappa(m) = \frac{X}{S}, \qquad S = \int_0^\infty x^2 m(t, x) \, \mathrm{d}x \, .$$

This will require us to compute S, or equivalently the covariance $\Sigma = S - X^2$, in addition to X.

(We may notice that with that weighted scheme, the "mean player" whose constellation size is X is taxed less heavily than in the proportional scheme.)

2.2.2 Unattributable costs

Left unanswered up to here is how to share the variations of debris numbers attributable to no specific constellation, specifically

$$\omega = -\alpha_0 \bar{y} + \nu \tau_0 \frac{\bar{y}^2}{2} - \dot{\bar{y}} \, . \tag{3}$$

Note that this term may be positive or negative, depending on α_0 and $\dot{\bar{y}}$. It will be shared between the operators as an added tax per satellite, or a bonus on the taxes, simply proportionally to x as a charge $p\omega x/X$ to each. It would not be difficult to consider the case where this is shared as h proportionally to $k(x, m)x$. (Notice that again, it is only thanks to the mean field approximation that this will fit onto the quadratic cost formalism.)

In the Appendix A concerning the case $\tau_2 = 0$, we will also consider a simpler scheme called "flat tax" that will let us have a simple closed form solution.

2.2.3 Revenue of constellations operations

We assume that the market for the services provided by the constellations obeys a competition *à la* Cournot with a linear inverse demand function. Thus the price of the services offered by any player would be, for a N-player game $P = \varphi - \chi \sum_i x_i$, leading, in the limiting process per our modelization approach to $P = \varphi - \chi X$ and to a revenue of $R_0(x) = (\varphi - \chi X)x$. However, in the methodology of mean field games, in optimizing its own decision, each player ignores the effect of this decision on the cumulated decision level. Therefore, the term in x^2 present in Cournot's oligopoly theory is absent from the above formula, creating a singularity. In recognition of that undesirable feature, we introduce a revenue function with an added term insuring its concavity. Let therefore the instantaneous revenue drawn by each player from the exploitation of its constelletion of x satellites be

$$R(x) = \left(\varphi - \chi X - \frac{\psi}{2}x\right)x$$

for some known parameters φ, χ, and ψ. A similar choice of model is used, e.g., by Graber and Sicar (2023). (The cases with either χ or ψ or both null would nevertheless be compatible with the rest of our theory.) In addition, each player bears a cost for the manufacture and launch of its satellites

$$C(\ell) = c\,\ell + \frac{d}{2}\ell^2\,.$$

In addition, there is a value of operational satellites at final time T (a bequest to later exploitation) $\Phi x(T)$, with $\Phi > c$.[12]

3 The dynamic game

We summarize the equations of the model with each of the two hypotheses about the interactions between constellations.

3.1 JCA maneuvers

Constellation dynamics (1):

$$\mathrm{d}x = [-(\alpha + \beta + \tau_1 y)x + \ell]\,\mathrm{d}t + \sigma\,\mathrm{d}W_t\,.$$

Debris creation, to be compensated for through costly ADR: let ω be given by (3),

$$h(t) = \beta X(t) + \nu\tau_1 y X(t) + \omega(t)\,. \tag{4}$$

Operator's payoff

$$\begin{aligned}\Pi =&\mathbb{E}\Bigg\{\Phi x(T) + \int_0^T \mathrm{e}^{-rt}\Big[\varphi x - \chi X x - \frac{\psi}{2}x^2 - c\,\ell - \frac{d}{2}\ell^2 \\ &\qquad - p\left(k(x,m)(\beta + \nu\tau_1 y) + \frac{\omega}{X}\right)x - \gamma\tau_2 x X\Big]\mathrm{d}t\Bigg\}\,.\end{aligned} \tag{5}$$

[12]Notice that without this term, there is no incentive to pay the cost of a launch at a late date since there is not enough time left to recover this cost from its exploitation. And the mathematically optimal srategy could be a "negative" amount of launches, which we wish to avoid to spare a theory of constrained, positive, strategies. See Bernhard (2026).

3.2 Colliding constellations

Constellation dynamics (1):

$$\mathrm{d}x = [-(\alpha + \beta + \tau_1 y + \tau_2 X)x + \ell(t)]\,\mathrm{d}t + \sigma\,\mathrm{d}W_t\,. \tag{6}$$

Debris creation, to be compensated for through costly ADR: let ω be given by (3),

$$h(t) = \beta X(t) + \nu\tau_1 y X(t) + \nu\tau_2 \frac{X^2}{2} + \omega(t)\,. \tag{7}$$

Operator's payoff

$$\begin{aligned}\Pi =& \mathbb{E}\Bigg\{\Phi x(T) + \int_0^T \mathrm{e}^{-rt}\Big[\varphi x - \chi X x - \frac{\psi}{2}x^2 - c\ell - \frac{d}{2}\ell^2 \\ &\qquad - pk(x,m)[\beta + \nu(\tau_1 y + \tau_2 X)]x - p\frac{\omega}{X}x\Big]\,\mathrm{d}t\Bigg\}.\end{aligned} \tag{8}$$

(And we notice again that the case with no interferences can be obtained by setting $\tau_2 = 0$ in any of the other two.)

3.3 Common formulation

Since all the particular problems we want to investigate are with linear (more precisely affine) dynamics and (nonhomogeneous) quadratic payoffs, we may use a common formulation with an appropriate choice of the meaning of the coefficients, adapted to each particular problem. All will be written

$$\mathrm{d}x = (-ax + \ell)\,\mathrm{d}t + \sigma\,\mathrm{d}W_t\,, \tag{9}$$

and

$$\Pi = \mathbb{E}\left\{\Phi x(T) + \int_0^T \mathrm{e}^{-rt}\left[qx - \frac{Q}{2}x^2 - c\,\ell - \frac{d}{2}\ell^2\right]\,\mathrm{d}t\right\}. \tag{10}$$

Parameters a, q, and Q are defined underneath in (11) or (12), depending which applies, recalling that ω is given by (3). Specifically, in the case with JCA maneuvers, substitute a, q, and Q as given by the following table (11), according to the choice of $k(x;m)$, into (9) and (10) to get (1) and (5) respectively,

	$a = \alpha + \beta + \tau_1 y\,,$
$k = 1$	$q = \varphi - \chi X - \gamma\tau_2 X - p\left(\beta + \tau_1\nu y + \frac{\omega}{X}\right)\,,$ $Q = \psi\,,$
$k = \kappa x$	$q = \varphi - \chi X - \gamma\tau_2 X - p\frac{\omega}{X}\,,$ $Q = \psi + 2p\frac{X}{S}(\beta + \tau_1\nu y)\,.$

(11)

In the case where satellites of different constellations can collide, the corresponding substitutions are ruled by the alternative table (12) to obtain (6) and (8) respectively:

	$a = \alpha + \beta + \tau_1 y + \tau_2 X \,,$
$k = 1$	$q = \varphi - \chi X - p\left[\beta + \nu(\tau_1 y + \tau_2 X) + \dfrac{\omega}{X}\right] \,,$ $Q = \psi \,,$
$k = \kappa x$	$q = \varphi - \chi X - p\dfrac{\omega}{X} \,,$ $Q = \psi + 2p\dfrac{X}{S}[\beta + \nu(\tau_1 y + \tau_2 X)] \,.$

(12)

4 Solution as a mean-field game

4.1 Mathematical analysis

The mean field game equations treat the optimization problem as if m, and hence X and Σ, were fixed. Therefore, we have here a classical LQ maximization problem. Let $\exp(-rt)V(t,x)$ be the Value function. The Hamilton-Jacobi-Bellman equation reads

$$\begin{aligned} 0 = \partial_t V - rV + \frac{\sigma^2}{2}\partial^2_{xx}V \\ + \max_{\ell}\left[qx - \frac{Q}{2}x^2 - c\,\ell - \frac{d}{2}\ell^2 + \partial_x V(-ax + \ell)\right] . \end{aligned} \tag{13}$$

We know that the solution will be of the form[13]

$$V(t,x) = \frac{1}{2}K(t)x^2 + L(t)x + M(t). \tag{14}$$

The maximum in ℓ is reached at

$$\ell^\star = \frac{1}{d}(\partial_x V - c) = \frac{1}{d}(Kx + L - c) \,. \tag{15}$$

The dynamics of the optimum process are thus

$$\mathrm{d}x = \left[\left(-a + \frac{K}{d}\right)x + \frac{L-c}{d}\right]\mathrm{d}t + \sigma\,\mathrm{d}W_t \,. \tag{16}$$

Placing (14) and (15) in (13) and identifying coefficients of like powers in x, in a classical way leads to three differential equations for K, L, and M:

$$\dot K = (r + 2a)K - \frac{1}{d}K^2 + Q \,, \qquad K(T) = 0 \,, \tag{17}$$

$$\dot L = (r + a - \frac{1}{d}K)L + \frac{c}{d}K - q \,, \qquad L(T) = \Phi \,, \tag{18}$$

$$\dot M = rM - \frac{1}{2d}(L-c)^2 - \frac{\sigma^2}{2}K \,, \qquad M(T) = 0 \,. \tag{19}$$

[13]This M not to be confused with the c.d.f. of the stochastic variable x_μ.

The coefficient K will be negative. If we set $K = -P$, we recognize in equation (17) the standard Riccati equation of a minimization problem, for which we know that a solution exists over any time interval $[0, T]$. Equations (18) and (19) are linear and always have a solution. Note that the condition $\Phi > c$ ensures that, in a neighborhood of final time T at least, $\ell^\star > 0$. This is not guaranteed for all time $t \in [0, T]$ and is left to be checked on each numerical simulation.

The parameters a, q, and Q depend on $m(\cdot, \cdot)$ via $X(t)$ and $\Sigma(t)$. The density m itself satisfies a Fokker-Planck equation where these parameters appear:

$$0 = \partial_t m - \frac{\sigma^2}{2}\partial^2_{xx} m + \partial_x \left[\left(-ax + \frac{K}{d}x + \frac{L-c}{d} \right) m \right] ,$$

and therefore

$$0 = \partial_t m - \frac{\sigma^2}{2}\partial^2_{xx} m + \left[\left(\frac{K}{d} - a \right) x + \frac{L-c}{d} \right] \partial_x m + \left(\frac{K}{d} - a \right) m \,, \tag{20}$$

with $m(0, x) = m_0(x)$ the initial distribution density. However, we do not need to know the density $m(t, x)$ itself. It suffices for our purpose to know $X(t)$ and $S(t)$ or $\Sigma(t)$. Note that in Fokker-Planck's equation, $\partial_x(-ax) = -a$, i.e., the individual player's trajectory $x(\cdot)$ has no direct effect on the mean X which appears in our a. And equation (16) is linear in x, leading to the differential equations:

$$\dot{X} = \left(-a + \frac{K}{d} \right) X + \frac{L-c}{d} \,, \qquad X(0) = X_0 \tag{21}$$

$$\dot{\Sigma} = 2\left(-a + \frac{K}{d} \right) \Sigma + \sigma^2 \,, \qquad \Sigma(0) = \Sigma_0 \,, \tag{22}$$

where X_0 and Σ_0 are respectively the mean and the covariance of the given initial distribution $m_0(\cdot)$. (Notice that we do not need here that m be gaussian.) Because of the inherent fixed-point character of a MacKean-Vlasov equation, in the case with constellations collisions that we consider here, equation (21) is in fact a Riccati equation. Again, as long as $L - c$ remains positive, which is necessary to have $\ell^\star$ positive, the signs ensure existence over an arbitrary time interval $[0, T]$.

We are left with a two-point boundary value problem for ordinary differential equations, which moreover can be split as a backward equation for (17)(18) and a forward equation for (21)(22), and (19) may be integrated separately after.

4.2 Numerical simulations

In order to show that this model can indeed be put to work, we give here an example of numerical results. We chose to make the computations for the case with colliding constellations (which is beyond the linear quadratic theory for the game with a finite number of players), comparing the two choices of $k(x, m)$.

4.2.1 Choice of parameters' values

We propose here a set of values for the many parameters of our model. We choose them not too far from realistic values for near circular orbits in the altitude range 700–1100 km, a very crowded Low Earth Orbit region.

- $r = .1$. Our time unit is the year. This says that the value of a profit is discounted by one half if it is seven years away.
- $\alpha = .6$. We mean to consider telecommunication constellations, with rather low orbits (to avoid latency), and a relatively large natural decay due to residual atmospheric drag. This α leads to 95% of the satellites inoperative or deorbited after 5 years. (See Inmarsat (2022).)
- $\alpha_0 = .05$. Very much dependant on the orbital altitude, might as well be negative. We choose it positive (i.e. more debris falling out of the orbital altitude than arriving from higher orbits), close to zero.
- $\beta = .1$ Comparable to, but significantly lower than, α.
- $\sigma = 100$. Was found to provide a reasonable spread of the constellations sizes.
- $\tau_0 = 10^{-7}$, as many debris are smaller than active satellites, and some collisions are harmless. Hence a smaller rate of significative collisions.
- $\tau_1 = \tau_2 = 5 \times 10^{-7}$. This is the right order of magnitude according to empirical evidence and several sources. (See e.g., Le May et al. (2018).)
- $y_0 = 10^3$. This is a mean number of debris per operator *at the altitude of the constellations considered.*[14]
- $\dot{\bar{y}} = -\delta\bar{y}$ with $\delta = 1.5 \times 10^{-2}$: removing one fourth of the large debris population over 20 years. (Already an ambitious goal.)
- $\nu = 500$. We mean to only consider debris large enough to create hazardous new debris. According to Lewis and Kessler (2025), close to 250 per collision. We chose it double to emphasize the effect of the collision risk.
- $\varphi = 10$. The baseline of all pricing coefficients.
- $\chi = 2 \times 10^{-3}$. We expect that X may be several hundreds to a few thousands. We mean to keep the price of service positive as long as $X < 5{,}000$.
- $\psi = .5 \times 10^{-3}$, meaning that, if $X = 500$ satellites beyond the 2000-th do not add value to the constellation.
- $\Phi = 10$, at least one more year of revenue.
- $c = 2$ to reflect the fact that multiple launches on a single launcher may bring the launch cost per satellite way below φ.

[14] This y_0 remains lower than the threshold $2\alpha_0/(\tau_0\nu) = 2 \times 10^3$ triggering the Kessler syndrome in our model.

- $d = 3.5 \times 10^{-3}$. Chosen small, but necessarily non zero for the standard linear quadratic technique to be possible.
- $p = 2$, chosen somewhat irrealistically small with the current level of ADR technology. We need to ensure $q > 0$, otherwise there is no possible profitable exploitation of a constellation.
- γ: this coefficient measures the cost of maneuvering in terms of number of years of operation lost due to propellant used in the maneuver multiplied by the inverse of the probability of collision triggering a maneuver. We did not specify it in our numerical simulations because we chose to deal with the case with no evasive maneuvers.

Furthermore, we chose the initial constellations to be characterized by $X = 50$, and $\Sigma = 100/3$, corresponding to a uniform distribution between 40 and 60. Typical of a proof-of-concept pre-operational large constellation.

4.2.2 Solving the two-point boundary value problem (TPBVP)

We integrated the differential equations with an Adams-Bashforth method of order two, initialized by a single Euler step, itself of order two, with a fixed step size, this last feature being imposed by the method we use to solve the TPBVP. Experimenting with this step size showed that a step of 1/24 (half a month) was sufficiently precise. We have integrated over 25 years (600 steps) and we display the first 20 years, to ignore an artifact due to the terminal time causing a sudden increase in $X(t)$ in the last years.

The method used is simple-minded, akin to Picard iterations: initialize $X(t)$ and $\Sigma(t)$ with their initial values X_0 and Σ_0 for all t. Then integrate the equations for K and L backward then the equations for X and Σ forward, and iterate, taking for values of X and Σ in the backward integration and for K and L in the forward integration the values obtained at the previous iteration. If the algorithm does not converge in this naive form, slow it by subrelaxation, i.e. taking for new values of the variables at each iteration a convex combination of the last obtained and the previous one. (Up to 1/2 each, and 100 Picard iterations.)

The numerical results obtained are

Taxing scheme	**Progressivity coefficient** $k(x,m)$	**Mean constellations size** $X(T)$	**Sizes dispersal** $\Sigma(T)$
proportional tax	1	1580	$6322 \simeq 79.5^2$
progressive tax	$(X/S)x$	1333	$4710 \simeq 68.6^2$

If this model is a mean field simplification of a situation with 20 operators, this means 31,600 or 26,660 satellites aloft in the altitude range considered, depending on the taxing scheme. These numbers seem actually realistic.

As expected, the coefficient penalizing large constellations more than proportionally, and small constellations less than proportionally, tends to lower the number of satellites aloft in the equilibrium, and this although the average charge per satellite is the same in both cases. The same weighting reduces the spread around the mean. We also see that the total mass of constellations with a negative x, (or alternatively the probability of the representative's player state becoming negative) is negligible.

Figure 1 shows the time dependence of the equilibrium constellation sizes depending on the taxing scheme.

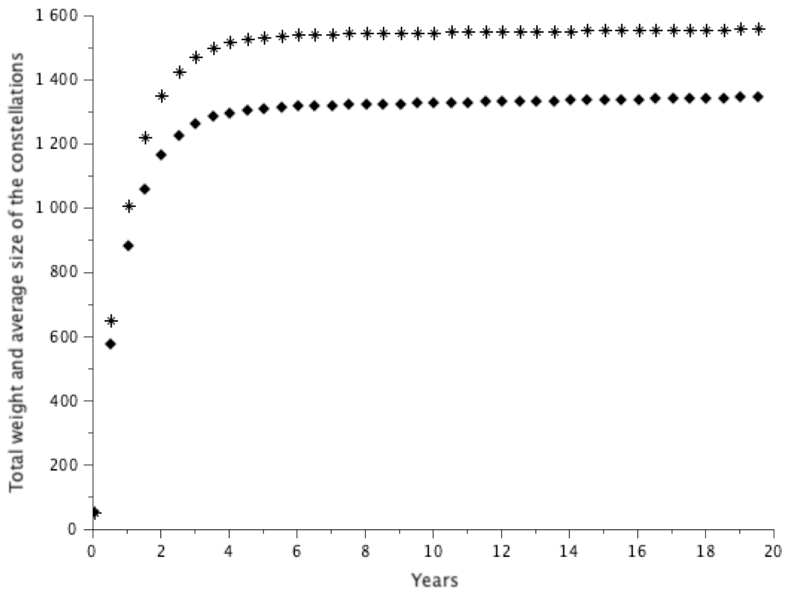


Figure 1: Mean constellation sizes for proportional taxes (top curve) and progressive taxes (bottom curve). The mean charge per satellite is the same in both cases.

Both graphs exhibit a rapid build-up of sizable constellations, followed by a slow growth of six or seven satellites per year. The smaller size of the equilibrium constellations in the progressive taxing scheme shows up since the early stages. In every cases, these numerical results point to a possibility of drastically reducing the number of large debris (by one fourth in 20 years) without sacrificing the overall economy of New Space.

5 Conclusion

Every day, all of our societies make use of outer space through satellites that, in real time and on a global scale, enable us, among other things, to observe, locate, and transmit a vast amount of information. Furthermore, given the substantial investments currently being made by firms and governments, all signs point to our use of outer space becoming even more widespread and intensive in the future. For this reason, —and also to preserve the ability of future generations to have access to outer space—, we must create and implement international institutional mechanisms as quickly as possible to ensure that outer space remains as usable and safe as possible. In this quest for a sustainable outer space, it is essential to secure the necessary funding to remove the most dangerous space debris.

This article is part of this effort to create financing mechanisms and develops a mean field game model to illustrate that it is possible to preserve both the economic benefits of outer space utilization and its sustainability. Furthermore, because the issue of space debris varies depending on the orbit, this model allows for different treatments of orbital altitude ranges (e.g., between 800 and 900 km and between 1,000 and 1,100 km), since τ and p depend on altitude while y_0 depends on the initial number of debris objects. And the choice of a desired trajectory $\bar{y}(t)$ of this number is left to the user (i.e., the international institution).

Nevertheless, according to us, our model has at least five limitations. First, concerning constellations' sizes, our model uses real numbers rather than integers. However, we do not view this choice as particularly problematic since, due to continuity, rounding the optimal strategies down to the nearest integer would yield profits close to those obtained with real numbers. Second, in our framework, all constellations are homogeneous (i.e., a single type of satellite occupies a given orbital altitude band). Third, constellations occupying the same orbital altitude band consist of satellites of the same type, with dynamics governed by the same control system and the same collision risk. Fourth, we assume that all debris are identical (i.e., same size, weight, shape, and physicochemical composition). Finally, fifth, in our model, each collision generates the same number of debris pieces, which is, of course, another simplifying assumption, since the number of debris pieces resulting from a collision is idiosyncratic because it depends on numerous specific parameters (apogee, perigee, geometry and relative velocity, composition of colliding objects, etc.).

These limitations naturally prompt us to continue our research. For now, we believe that the method proposed by Huang et al. (2006a) would quite naturally allow us to consider different classes of satellites ("shoe box size", "refrigerator size", "car size", "school bus size"), which might improve this model by distinguishing, this time, an identical number of debris pieces per collision, but varying according to the size of the satellites.

To conclude, given the stakes and the urgency of the issue, we hope that many other colleagues will take an interest in the problem of space debris and, above all, that this research will help quickly steer institutional and political discussions toward an international framework capable of solving it.

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A Monopolies case with no constellations interferences and constant desired debris count

Here we consider the simple cases where the markets addressed by the constellations are separate, each being a monopoly on its own market, i.e. $\chi = 0$, and $\tau_2 = 0$, $k(x, m) = 1$, and also where the desired $\bar{y}$ trajectory is just a constant $\bar{y}(t) = y_0$. We have a closed form solutions in two different cases.

A.1 Flat tax

We assume that the unattributable costs ω are recovered via a flat tax $p\omega$, independent of the size x of each constellation. It may therefore be ignored in the individual payoff to be maximized. This case yields the simplest solution of our problem. We summarize the various coefficients of the linear quadratic formulation (9)(10):

$$\begin{aligned} a &= \alpha + \beta + \tau_1 y \,, \\ q &= \varphi - p(\beta + \tau_1 \nu y) \,, \\ Q &= \psi \,. \end{aligned}$$

Equation (17) is uncoupled with any other one. Observe that, in a finite horizon problem, $K(T) = 0$ and $\dot{K}(T) = Q > 0$, so that $K(t)$ is negative, and its long time limit, as $T \to \infty$ is the only negative root of the asymptotic equation, i.e.

$$K = d \left[a + \frac{r}{2} - \sqrt{\left(a + \frac{r}{2}\right)^2 + \frac{\psi}{d}} \right] .$$

For more legibility, let

$$\rho = \sqrt{\left(a + \frac{r}{2}\right)^2 + \frac{\psi}{d}} \,, \qquad K = d\left(a + \frac{r}{2} - \rho\right) . \tag{23}$$

Knowing K, we also find the asymptotic value of

$$L = 2\frac{\varphi - p(\beta + \tau_1 \nu y) - c(a + r/2 - \rho)}{r + 2\rho} \,, \tag{24}$$

and thus

$$\frac{L - c}{d} = 2\frac{\varphi - p(\beta + \tau_1 \nu y) - c(a + r)}{d(r + 2\rho)} \,,$$

and finally,

$$M = \frac{1}{r}[(L - c)^2/(2d) + \frac{\sigma^2}{2}K] \,.$$

Hence

$$M = \frac{1}{r}\left[2\frac{(\varphi - p(\beta + \tau_1 \nu y) - c(a + r))^2}{d(r + 2\rho)^2} - \frac{\sigma^2}{2} d(\rho - a - \frac{r}{2}) \right] . \tag{25}$$

We may therefore state the following fact:

For the infinite horizon problem with a flat tax to recover the cost of actively deorbit the non attributable debris, one has $\ell^\star$ given by (15) and

$$\Pi = \frac{1}{2}Kx_0^2 + Lx_0 + M - \frac{p\omega}{1 - r} \,,$$

where K, L, and M are given by equations (23),(24), and (25) respectively.

A.2 Proportional taxes

We now allow for taxes proportional to x on all mean debris productions, including the unattributable costs. We consider the asymptotic regime of our system of equations. It holds for a very large or infinite horizon T, and considering a large time t. We still have a closed form solution. It is somewhat complicated, though, so that we need some short hand notation as follows. In these calulations a is always taken from any of the above tables with $\tau_2 = 0$. Let ρ be as above, and, if $\omega > 0$:

$$\begin{aligned}
A &= \rho - \frac{r}{2} > a > 0\,,\\
B &= \frac{\varphi - p(\beta + \tau_1 y) - c(a+r)}{d(r+2\rho)}\,,\\
C &= \frac{2p\omega}{d(r+2\rho)} > 0\,,\\
D &= B + \sqrt{B^2 - AC}\,.
\end{aligned}$$

We claim:
In the problem where live satellites cannot collide, for a large t and with infinite horizon, the problem has a stationary solution provided that, using the above notation:

$$B > \sqrt{AC}\,, \tag{26}$$

and then, one has

$$\begin{aligned}
\ell^\star(t,x) &= -(A-a)x + D\,,\\
V(t,x) &= -\frac{d}{2}(A-a)x^2 + (dD+c)x + \frac{d}{2r}[D^2 - \sigma^2(A-a)]\,.
\end{aligned}$$

Proof Equation (17) is the same as above, and yields (23) as above, or equvalently

$$K = d(a-A) < 0\,.$$

Now, the asymptotic regime of equation (18) yields

$$L = \frac{q - c\frac{K}{d}}{r + a - \frac{K}{d}} = \frac{\varphi - p(\tau_1 \nu y) - c(a-A)}{r+A} - \frac{p\omega}{(r+A)X}\,,$$

and thus
$$\frac{L-c}{d} = 2\frac{\varphi - p(\beta + \tau_1\nu y) - c(a+r)}{r+2\rho} - \frac{2p\omega}{(r+2\rho)X} = 2B - \frac{C}{X}\,.$$

Equation (21) yields for the asymptotic regime, i.e. large t:

$$\left(\frac{K}{d} - a\right)X + \frac{L-c}{d} = -AX + 2B - \frac{C}{X} = 0\,.$$

Observe that equation (21) now gives

$$X\dot{X} = \frac{1}{2}\frac{\mathrm{d}\|X\|^2}{\mathrm{d}t} = -AX^2 + 2BX - C$$

This equation has a stable point only if the right hand side has a root, i.e $B^2 > AC$. A positive root exists if $B > 0$, ensured by condition (26), and then the stable one is the largest one, i.e.

$$X = \frac{B + \sqrt{B^2 - AC}}{A} = \frac{D}{A}\,.$$

It follows that

$$\frac{L - c}{d} = AX = D\,,$$

and using the asymptotic regime of equation (19):

$$M = \frac{d}{2r}\left[\frac{D^2}{2} - \sigma^2(A - a)\right].$$

The proposition follows. □

Condition (26) has a qualitative explanation: a stable profitable situation is possible if the benefit of running satellites, as measured by φ, is sufficiently large, or conversely, if $p(\beta+\tau_1\nu y)$ is not too large, nor the cost c of launching a satellite.

B List of variables

Our model means to be adaptable to many different situations. As a result, it contains a large number of variables. We provide here an alphabetical list, with their meaning and the units in which they are to be expressed. A quantity meant "by satellite" will be denoted X^{-1}, we have also units of time T, and unit of currency denoted €, but it may mean dollars or rather megadollars. We denote 1 for dimensionless.[15]

Table 1: List of variables, roman letters

Variable	description	unit
a	Defined by table (11) or (12).	T^{-1}
c	Launch cost of the first satellite launched.	€X^{-1}
C	$C(\ell)$ cost of launching ℓ satellites.	€X^{-1}
d	Coefficient of a quadratic decrease in launch costs.	€X^{-2}
h	Weight of active debris removual per unit time.	XT^{-1}
k	$k(x,m)$ progressive tax rate.	1
K	Coefficient of the term in x^2 in $V(t,x) = K(t)x^2 + L(t)x + M(t)$.	€X^{-2}
L	Coefficient of the term in x in V.	€X^{-1}
M	Term independent of x in V.	€
ℓ	Number of satellites launched per unit time.	XT^{-1}
m	$m(t,x)$ density of the cloud of operators at x satellites and time t.	X^{-1}
	Equivalently, probability density of the representative operator at (t,x).	
p	Cost of actively removing one unit of weight of debris per unit time.	€$X^{-1}T^{-1}$
q	Defined by table (11) or (12).	€$X^{-1}T^{-1}$
Q	Defined by table (11) or (12).	€$X^{-2}T^{-1}$
r	Discounting coefficient.	T^{-1}
R	Revenue per unit time of the representative operator.	€T^{-1}
t	Current time.	T
V	Isaacs' Value or Bellman function.	€
W_i	A standard Wiener process.	$T^{1/2}$
x	Number of satellites in the representative player's constellation.	X
X	Total weight (measure) of the constellations.	X
	Equivalently, expected mean size of the constellations.	
y	Weight (measure) of the set of debris at the orbital altitude considered.	X
y_0	Initial value of y and $\bar{y}$	X
$\bar{y}$	$\bar{y}(t)$ agreed upon trajectory of y.	X

[15]We omitted M, $\mathcal{M}$, $\mathbb{M}$, M, and μ that do not appear in the calculations

Table 2: List of variables, greek letters

Variable	description	unit
α	Relative rate of satellites deorbitation.	T^{-1}
α_0	Relative natural rate of debris weight variation.	T^{-1}
β	Relative rate of satellites death by breakdown.	T^{-1}
γ	Half the cost of an evasive maneuver.	€
κ	Coefficient in the progressive term $k(x,m) = \kappa(m)x$.	X^{-1}
ν	Number of debris created by a collision.	1
σ_i	σ_i^2 covariance of W_i.	$XT^{-1/2}$
τ_0	$\tau_0 y^2/2$ mean rate of collision of debris with debris.	$X^{-1}T{-1}$
τ_1	$\tau_1 y$ mean rate of collision of a satellite with debris.	$X^{-1}T{-1}$
τ_2	Same as τ_1 for collision with an active satellite.	$X^{-1}T{-1}$
Φ	Value of a satellite operational at final time.	€X^{-1}
φ	Marginal value per unit time of the first satellite of any constellation.	€$X^{-1}T^{-1}$
χ	Slope of the (affine) inverse demand curve.	€$X^{-2}T^{-1}$
ψ	Coefficient of the concave term $-\psi x^2/2$ in the revenue.	€$X^{-2}T^{-1}$
ω	Rate of debris to actively remove not attributable. (See equation (3).)	XT^{-1}