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Emanuel Namaiwa Barke

September 2026

Working paper No. 2026 – 10

CRESE
30, avenue de l'Observatoire
25009 Besançon
France
http://crese.univ-fcomte.fr/

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Abstract

This paper studies mutual outsourcing between competing firms in a differentiated network duopoly and examines how positive network externalities interact with the timing of quantity competition. Firms first choose the outsourcing prices charged to their rivals and then compete either simultaneously à la Cournot or sequentially à la Stackelberg. We show that stronger network effects increase equilibrium output and profits under both competitive regimes, but have non-monotonic effects on outsourcing prices. Under Cournot competition, the equilibrium outsourcing price increases with network intensity when network effects are weak and decreases once they become sufficiently strong. Under Stackelberg competition, outsourcing incentives become asymmetric: the leader's outsourcing price is non-increasing in network intensity, while the follower's price may exhibit a threshold effect. The comparison across market structures further reveals that network effects can reverse both outsourcing-price and profit rankings. In particular, the Stackelberg leader always earns more than under Cournot competition, whereas sufficiently strong network effects allow the follower first to outperform the Cournot benchmark and eventually to earn more than the leader. The analysis highlights a strategic trade-off specific to network industries: firms balance the incentive to raise a rival's input cost against the need to preserve the common installed base that supports market demand.

Keywords: Mutual outsourcing; positive network externalities; Cournot competition; Stackelberg competition; outsourcing prices; strategic input pricing

JEL Classification: D43; L13; L14

*Université Marie et Louis Pasteur, CRESE (UR 3190), F-25000 Besançon, France. Email: emanuel.namaiwa_barke@univ-fcomte.fr.

1 Introduction

Mutual outsourcing has become an increasingly important feature of technology-intensive industries, where firms may compete vigorously in final-good markets while simultaneously supplying one another with critical inputs, components, or technologies. The smartphone industry provides a prominent example. Google supplies the Android operating system used in Samsung's Galaxy devices, while Samsung has, in turn, manufactured Google-branded Nexus smartphones and supplied components such as OLED displays for Google's Pixel devices (Arai and Matsushima, 2023). More broadly, the production of smartphones relies on highly specialized inputs supplied by firms that may themselves interact strategically with downstream competitors. Qualcomm, for instance, has long occupied a central position in the modem-chip market and has supplied major smartphone manufacturers including Apple, Google, and Samsung (Srivastava, 2017; Leswing, 2019); in 2017, its market share in modem chips exceeded one half (Ai and Lu, 2019, p. 645). These arrangements illustrate how mutual outsourcing has evolved beyond a simple cost-saving device into a strategic component of interfirm relationships, linking firms' input-pricing decisions to competition in final-product markets.

A growing literature has examined outsourcing and mutual outsourcing as strategic devices in oligopolistic markets. Early contributions emphasized that outsourcing may alter not only production costs, but also the strategic structure of downstream competition. Arya et al. (2008b) show that when a key input is outsourced to a vertically integrated retail rival with upstream market power, standard Cournot–Bertrand rankings can be overturned: Bertrand competition may generate higher prices and industry profits, together with lower consumer and total surplus, because outsourcing weakens both retail and wholesale competition. In a related vein, Chen (2010) studies outsourcing directly between downstream rivals and shows that outsourcing can serve as a commitment device by revealing a firm's future output choice to its competitor. This strategic commitment changes the rival's downstream behavior and implies that the profitability of outsourcing depends crucially on whether firms compete in quantities or prices. Arya et al. (2008a) further analyze the make-or-buy decision in the presence of a common monopolistic supplier and show that a firm may rationally outsource even at a price exceeding its own in-house production cost. By becoming a customer of the common supplier, the firm can weaken the supplier's incentive to offer particularly favorable terms to its rival, thereby using outsourcing as an indirect means of raising the rival's cost.

Subsequent studies have developed these strategic mechanisms in richer oligopolistic environments. Pun (2015) focuses explicitly on mutual outsourcing between competing manufacturers and shows that reciprocal outsourcing can arise endogenously when firms possess a cost advantage and downstream competition is sufficiently moderate. Chang et al. (2018) examine outsourcing between a subcontractor and an outsourcer in a differentiated duopoly and find that outsourcing input production can be more profitable than in-house production. Arai and Matsushima (2023) extend the analysis to organizational design by combining mutual outsourcing with managerial delegation. They show that mutual outsourcing can induce firms to behave less aggressively in the downstream market and may thereby facilitate more collusive outcomes. Introducing quality investment, Wang et al. (2024) demonstrate that the profitability of outsourcing is not unambiguous: outsourcing can reduce manufacturers' profits depending on production efficiency and the effectiveness of quality investment.

More recent contributions have turned to the interaction between mutual outsourcing and the structure of market competition. Choi and Lee (2025) analyze the endogenous choice of competition mode among mutual-outsourcing firms and show that the equilibrium regime depends on product substitutability: Bertrand competition emerges when products are relatively differentiated, whereas Cournot competition arises when substitutability is sufficiently high, with corresponding implications for Pareto efficiency and prisoners' dilemma outcomes. Milliou and Serfes (2025) examine mutual outsourcing in a setting with complementary inputs and supplier encroachment and show that reciprocal outsourcing may emerge when input-production costs are sufficiently low, although it can adversely affect the incumbent firm. Finally, Xu et al. (2026) incorporate strategic environmental corporate social responsibility into a vertically related market with mutual outsourcing. They show that ECSR adoption reduces the profits of mutual-outsourcing firms but improves welfare and environmental performance, while also affecting the endogenous choice between Cournot, Bertrand, and asymmetric modes of competition.

Despite these advances, the interaction between mutual outsourcing and network externalities has received remarkably little attention, even though mutual outsourcing is particularly relevant in technology-intensive industries where network effects are pervasive. A positive network externality arises when consumers' valuation of a product increases with the number of users adopting the same product, platform, or compatible technology, so that firms' strategic decisions affect not only direct competition but also the size of the network supporting demand (Katz and Shapiro, 1985). This feature may fundamentally alter outsourcing incentives: raising a rival's input cost can weaken its competitive position, but the resulting contraction in its sales may simultaneously reduce the value of the common network. The smartphone ecosystem illustrates this interaction particularly well. Google reported more than three billion active Android devices worldwide in 2021,[1] while Samsung accounted for approximately 20% of global smartphone shipments in the first quarter of 2025.[2] At the same time, Google and Samsung have maintained a close technological and sourcing relationship for more than a decade: Google supplies the Android technology embedded in Samsung's Galaxy devices, while Samsung has provided manufacturing capabilities and hardware components for Google-branded devices (Arai and Matsushima, 2023). Their collaboration included the joint introduction of the Galaxy Nexus in 2011,[3] and has more recently expanded to artificial-intelligence technologies through a multi-year partnership announced in 2024 between Samsung and Google Cloud for the Galaxy S24 series.[4] These observations suggest that understanding mutual outsourcing in network industries requires explicitly accounting for the demand-side externalities generated by the size of the network.

In the spirit of Katz and Shapiro (1985), a substantial literature has examined positive network externalities in oligopolistic markets and has shown that they can substantially modify firms' strategic incentives. Hoernig (2012) studies strategic delegation under price competition and shows that sufficiently strong network effects may reverse the conventional incentives associated with managerial delegation. Chirco and Scrimitore (2013) endogenize the choice between price and quantity competition in a differentiated duopoly and find that, under managerial delegation, strong network externalities can select price competition as the unique equilibrium. Turning to firms' social objectives, Fanti and Buccella (2018) show that network effects can make corporate social responsibility profitable, overturning the prisoners' dilemma result typically obtained in standard industries. More recently, Buccella et al. (2025a) examine unilateral cross-ownership in a network duopoly and show that reduced competitive intensity may lower industry profits or even increase social welfare, depending on product compatibility. Buccella et al. (2025c) further demonstrate that, under interlocking cross-ownership, the profit-maximizing ownership share can be substantially below the conventional benchmark and may even be zero, while Buccella et al. (2025b) show that endogenous partial cross-ownership is profitable only over a limited range of ownership shares, particularly when network effects are strong and compatibility is low. In an asymmetric-cost setting, Namaiwa Barke (2026) shows that network effects also reshape firms' incentives to adopt

[1]Google, "Android 12 Beta: Designed for you," May 18, 2021. Available at: https://blog.google/products-and-platforms/platforms/android/android-12-beta/.

[2]Counterpoint Research, "Global Smartphone Market Share: Quarterly," 2025. Available at: https://www.counterpointresearch.com/insight/global-smartphone-share/.

[3]Samsung Electronics, "Samsung and Google Introduce GALAXY Nexus," October 19, 2011. Available at: https://news.samsung.com/global/samsung-and-google-introduce-galaxy-nexus.

[4]Samsung Electronics, "Samsung and Google Cloud Join Forces to Bring Generative AI to Samsung Galaxy S24 Series," January 18, 2024. Available at: https://news.samsung.com/uk/samsung-and-google-cloud-join-forces-to-bring-generative-ai-to-samsung-galaxy-s24-series.

strategic corporate social responsibility, with implications for market survival, concentration, prices, and welfare.

Despite the growing literature on both strategic outsourcing and network industries, to the best of our knowledge, the interaction between mutual outsourcing and positive network externalities has not yet been examined. This paper addresses this gap by asking three related questions. First, how do network externalities affect firms' incentives to set outsourcing prices when competing firms mutually supply essential inputs? Second, how does the timing of quantity decisions—simultaneous Cournot competition versus sequential Stackelberg competition—alter these outsourcing incentives? Third, how do network effects reshape the resulting rankings of outsourcing prices and profits across the two competitive regimes?

To address these questions, we develop a differentiated network duopoly in which two competing firms—which may be interpreted, for instance, as firms such as Google and Samsung—mutually supply essential inputs to one another while competing in the final-good market. Each firm first chooses the outsourcing price charged to its rival and then competes in quantities. We consider two alternative timing structures in the product market: simultaneous quantity competition à la Cournot and sequential quantity competition à la Stackelberg, with one firm acting as leader and the other as follower.[5] This framework allows us to examine jointly how network externalities and the timing of market competition shape mutual outsourcing decisions and their consequences for firms' market performance.

Our analysis yields three main findings. First, network externalities expand market activity under both Cournot and Stackelberg competition: stronger network effects increase equilibrium quantities and profits. Their effect on outsourcing prices is nevertheless more subtle. Under Cournot competition, the equilibrium outsourcing price initially increases with the network effect but decreases once network intensity exceeds a threshold $\widehat{n}_C$. Under Stackelberg competition, the leader's outsourcing price is non-increasing in the network effect, whereas the follower's price may display a similar threshold pattern. Thus, stronger network effects progressively shift firms' incentives away from extracting input-market rents toward preserving the size of the network.

Second, the timing of quantity decisions changes the ranking of equilibrium outsourcing prices. The Stackelberg follower always charges a lower outsourcing price than under Cournot competition, while the leader's outsourcing price exceeds the Cournot level only when network effects are sufficiently weak; above a threshold $\widehat{n}_1$, this ranking is reversed. Moreover, within

[5]Quantity competition provides a natural framework for technology-intensive and manufacturing industries in which production capacity, component availability, and supply constraints play an important role in firms' market decisions. Comparing Cournot and Stackelberg competition further allows us to isolate how strategic commitment in output decisions modifies firms' incentives to use outsourcing prices.

the Stackelberg regime, the leader initially charges more than the follower, but sufficiently strong network effects reverse this ordering beyond a second threshold $\widehat{n}_2$. Hence, network externalities can overturn the conventional asymmetry generated by quantity leadership in the input market.

Third, the profit comparison reveals a distinct effect of network intensity on the value of sequential competition. The Stackelberg leader always earns more than under Cournot competition, whereas the follower initially earns less and overtakes the Cournot benchmark only when the network effect exceeds a threshold $\widehat{n}_3$. For still stronger network effects, above $\widehat{n}_4$, the follower also becomes more profitable than the Stackelberg leader. Network externalities therefore do not eliminate the advantage of sequential competition relative to Cournot, but they can substantially redistribute its gains and eventually reverse the usual first-mover profit advantage.

We can also draw several managerial implications from these findings. In network industries, outsourcing prices should not be treated solely as instruments for extracting input-market rents or weakening a rival. When network effects are strong, aggressive input pricing may reduce the rival's output and thereby contract the common installed base that supports demand for both firms. Managers should therefore jointly consider outsourcing prices, market timing, and network expansion. The results also indicate that firms occupying leader and follower positions face different pricing incentives, suggesting that reciprocal sourcing agreements should be tailored to the firm's strategic position rather than designed symmetrically across partners.

The remainder of the paper is organized as follows. Section 2 presents the model. Section 3 derives the equilibrium outcomes under Cournot and Stackelberg competition and compares them. Section 4 concludes.

2 The model

We consider a duopoly in which firms $i \in \{1, 2\}$ produce differentiated network goods. In the spirit of Katz and Shapiro (1985), the inverse demand function (see, among others, Fanti and Buccella, 2018; Buccella et al., 2023; Namaiwa Barke, 2026) faced by firm i is given by[6]

$$p_i = a - q_i - bq_j + n(y_i + y_j), \qquad i \neq j, \quad i, j \in \{1, 2\}. \tag{1}$$

[6]We assume that the two product networks are perfectly compatible. More generally, network compatibility can be explicitly parameterized. For instance, Buccella et al. (2025a) consider inverse demands of the form $p_i = a - q_i - q_j + n(y_i + ky_j)$, where $k \in [0, 1]$ measures the degree of cross-network compatibility. Our specification corresponds to the perfectly compatible case $k = 1$.

where p_i denotes the price of firm i's final product, while q_i and q_j denote the quantity of the goods produced by the two firms, y_i and y_j denote the consumers' expectation about the firms' sales. The parameter $a > 0$ captures the size of the market, while $b \in (0, 1)$ measures the degree of substitutability between the two differentiated products. A larger value of b indicates that the products are closer substitutes. The parameter $n \in [0, 1)$ indicates the strength of network effects (i.e. the higher the value of the parameter the stronger the network effects).

We consider mutual outsourcing between the two firms. Each firm requires one unit of a specialized input supplied by its rival to produce one unit of its final good. Firm i supplies this input to firm j at the linear outsourcing price r_i, while firm i purchases the corresponding input from firm j at price r_j. In addition to the outsourcing payment, both firms incur the same constant marginal production cost $c \in (0, a)$. Hence, the effective marginal cost borne by firm i in the final-good market is $c + r_j$.

Thus, firm i's profit is given by

$$\Pi_i = (p_i - c - r_j)\, q_i + r_i q_j. \tag{2}$$

The first term represents firm i's profit from selling its differentiated final good, whereas the second term captures the revenue earned from supplying the specialized input to firm j.

We consider a two-stage game. In the first stage, firms simultaneously choose their linear outsourcing prices, r_i and r_j. In the second stage, given these outsourcing prices, they compete in the final-good market. To examine how the timing of quantity decisions shapes the strategic use of mutual outsourcing, we consider two alternative market configurations. Under Cournot competition, firms simultaneously choose their output levels. Under Stackelberg quantity competition, one firm acts as the quantity leader and the other as the follower.

3 Model analysis

We solve the game by backward induction and characterize the equilibrium outcomes under the two alternative modes of oligopolistic competition.

3.1 Cournot competition

Throughout this subsection, the superscript "C" denotes equilibrium outcomes under Cournot competition. We begin the analysis by imposing the following assumption.

Assumption 1. *We assume that*

$$n < \bar{n}_C(b) = \frac{4 + 2b - b^2}{6 - 2b}.$$

Assumption 1 ensures the existence of an interior Cournot equilibrium with strictly positive outsourcing prices, quantities, final-good prices, and profits, and guarantees the concavity of each firm's first-stage profit with respect to its own outsourcing price.

In the second stage of the game, firms simultaneously choose their output levels. Given the outsourcing prices r_i and r_j, as well as consumers' expected sales y_i and y_j, firm i chooses q_i to maximize its profit in (2). Using the inverse demand function in (1), firm i's profit can be written as

$$\Pi_i = [a - q_i - bq_j + n(y_i + y_j) - c - r_j]\, q_i + r_i q_j.$$

The first-order condition is[7]

$$\frac{\partial \Pi_i}{\partial q_i} = a - c - r_j + n(y_i + y_j) - 2q_i - bq_j = 0,$$

the Cournot reaction function of firm i is

$$q_i(q_j, y_i, y_j) = \frac{a - c - r_j + n(y_i + y_j) - bq_j}{2}. \tag{3}$$

Following Hoernig (2012); Chirco and Scrimitore (2013); Fanti and Buccella (2018); Buccella et al. (2023), we adopt the fulfilled-expectations approach, whereby consumers correctly anticipate the realized sales of each firm in equilibrium, so that $y_i = q_i$ and $y_j = q_j$. Solving the two reaction functions jointly under fulfilled expectations yields the second-stage outputs and profits reported in the following lemma.

Lemma 1. *For given outsourcing prices r_i and r_j, the equilibrium output of firm i under Cournot competition is*

$$q_i^C = \frac{(2 - b)(a - c) + (b - n)r_i - (2 - n)r_j}{(2 - b)(2 + b - 2n)}, \tag{4}$$

[7]The second-order condition is satisfied since $\frac{\partial^2 \Pi_i}{\partial q_i^2} = -2 < 0$.

and firm i's second-stage profit is

$$\Pi_i^C = \frac{[(2-b)(a-c)+(b-n)r_i-(2-n)r_j]^2}{(2-b)^2(2+b-2n)^2} + \frac{r_i\,[(2-b)(a-c)+(b-n)r_j-(2-n)r_i]}{(2-b)(2+b-2n)}. \tag{5}$$

Proof. Under fulfilled expectations, the reaction functions form the linear system

$$(2-n)q_i+(b-n)q_j = a-c-r_j, \qquad (b-n)q_i+(2-n)q_j = a-c-r_i.$$

Solving this system yields (4) and substituting into $\Pi_i^C = (q_i^C)^2 + r_i q_j^C$ gives (5). □

Before turning to the firms' optimal outsourcing decisions, it is useful to examine how a unilateral change in r_i affects the Cournot equilibrium characterized in Lemma 1.

Corollary 1. *Under Cournot competition:*

(i) Firm i's output satisfies

$$\frac{\partial q_i^C}{\partial r_i} = \frac{b-n}{(2-b)(2+b-2n)} \gtreqless 0 \iff b \gtreqless n. \tag{6}$$

(ii) Firm j's output and total industry output satisfy

$$\frac{\partial q_j^C}{\partial r_i} = -\frac{2-n}{(2-b)(2+b-2n)} < 0, \qquad \frac{\partial (q_i^C+q_j^C)}{\partial r_i} = -\frac{1}{2+b-2n} < 0. \tag{7}$$

Proof. The results follow straightforwardly by differentiating the equilibrium expressions in Lemma 1. □

Corollary 1 highlights the two channels through which firm i's outsourcing price affects competition in the final-good market. A higher r_i raises firm j's effective marginal cost and therefore reduces q_j^C. This direct cost effect is unambiguous. By contrast, the effect on firm i's own output depends on the relative strength of product substitutability and network externalities. When $b > n$, the strategic-substitution effect dominates. By making the rival less competitive, a higher r_i induces firm i to expand its own output. When $n > b$, however, the contraction of the rival's output reduces the realized size of the common network sufficiently strongly to offset this competitive gain, so that firm i's own output also falls. At the threshold $b = n$, these two effects exactly cancel and q_i^C is locally unaffected by r_i.

Regardless of this threshold effect on firm i's own quantity, total industry output always decreases with r_i. Hence, a unilateral increase in the outsourcing price reallocates output toward firm i only when product substitutability is sufficiently strong relative to the network effect, but it always contracts the overall market.

We now turn to the first stage of the game. Anticipating the Cournot equilibrium characterized in Lemma 1, the two firms simultaneously choose their outsourcing prices, r_i and r_j, to maximize their respective profits in (5). The corresponding outsourcing-price reaction functions are

$$r_i(r_j) = \frac{\begin{array}{c}(2-b)(a-c)\left(4+2b-b^2+2bn-6n\right)\\ +(b-n)\left(2bn-b^2-2n\right)r_j\end{array}}{2\left[(2-b)(2+b-2n)(2-n)-(b-n)^2\right]}. \tag{8}$$

Solving the two outsourcing-price reaction functions (8) yields:

$$r_i^{C*} = r_j^{C*} = \frac{(a-c)\left(4+2b-b^2+2bn-6n\right)}{8+4b-b^2+bn+2n^2-12n}. \tag{9}$$

Substituting the equilibrium outsourcing prices in (9) into the second-stage equilibrium characterized in Lemma 1 yields the Cournot outcomes associated with the subgame-perfect Nash equilibrium.

Lemma 2. *Under Assumption 1, the subgame-perfect Nash equilibrium under Cournot competition is characterized as follows. The equilibrium quantities are*

$$q_i^{C*} = q_j^{C*} = \frac{(a-c)(2-n)}{8+4b-b^2+bn+2n^2-12n}, \tag{10}$$

and the equilibrium profits are

$$\Pi_i^{C*} = \Pi_j^{C*} = \frac{(a-c)^2(2-n)\left(6+2b-b^2+2bn-7n\right)}{\left(8+4b-b^2+bn+2n^2-12n\right)^2}. \tag{11}$$

Proof. The results follow straightforwardly by substituting the equilibrium outsourcing prices in (9) into the quantities and profits reported in Lemma 1. □

We now examine the equilibrium characterized in Lemma 2 through the following results.

Proposition 1. *The effects of the network externality on the Cournot equilibrium outcomes are given by*

$$\frac{\partial r_i^{C*}}{\partial n} \gtreqless 0 \iff n \lesseqgtr \widehat{n}_C(b), \qquad \frac{\partial q_i^{C*}}{\partial n} > 0, \qquad \frac{\partial \Pi_i^{C*}}{\partial n} > 0,$$

where

$$\widehat{n}_C(b) = \frac{4+2b-b^2-(2-b)\sqrt{4-b}}{2(3-b)}, \quad \bar{n}_C(b) = \frac{4+2b-b^2}{6-2b}, \quad 0 < \widehat{n}_C(b) < \bar{n}_C(b).$$

Proof. See Appendix A. □

Proposition 1 shows that the network externality has an unambiguously expansionary effect on market activity, but a non-monotonic effect on the firms' outsourcing decisions. In particular, a stronger network effect increases each firm's equilibrium quantity and profit. The increase in output reflects the direct demand-enhancing effect of a larger installed base: fulfilled expectations reinforce consumers' willingness to pay, thereby supporting higher equilibrium sales. The associated increase in profits indicates that this expansion in market demand more than offsets the competitive effects induced by the firms' mutual outsourcing decisions.

By contrast, the equilibrium outsourcing price initially increases with n, but decreases once the network effect exceeds $\widehat{n}_C(b)$. For relatively weak network externalities, the increase in market demand allows each firm to charge a higher price for the specialized input supplied to its rival. When the network effect becomes sufficiently strong, however, a higher outsourcing price has a more pronounced contractionary effect on the rival's output and, consequently, on the size of the common network. Beyond the threshold $\widehat{n}_C(b)$, preserving network expansion becomes relatively more valuable, which induces firms to reduce their outsourcing prices.

The role of product substitutability can be further examined through its effect on the threshold identified in Proposition 1.

Corollary 2. *The threshold $\widehat{n}_C(b)$ is strictly increasing in b:*

$$\frac{\partial \widehat{n}_C(b)}{\partial b} > 0.$$

Moreover,

$$\lim_{b \to 0^+} \widehat{n}_C(b) = 0, \qquad \lim_{b \to 1^-} \widehat{n}_C(b) = \frac{5-\sqrt{3}}{4}.$$

Thus, greater product substitutability extends the range of network externalities for which $\partial r_i^{C*}/\partial n > 0$.

Proof. See Appendix B. □

Corollary 2 clarifies how product substitutability shapes the relationship between network effects and outsourcing prices. When products become more substitutable, a change in the rival's effective marginal cost has a stronger impact on the allocation of demand between the two firms. In this case, the strategic role of the outsourcing price becomes more important, and the positive effect of n on r_i^{C*} persists over a wider range of network intensities.

Conversely, when products are weak substitutes, increasing the rival's outsourcing cost generates only a limited demand reallocation toward firm i. The contraction in the rival's output therefore translates more directly into a reduction in the realized network size. As a result, the incentive to moderate the outsourcing price emerges at relatively low values of n.

The corollary should not be interpreted as implying that greater substitutability necessarily raises the equilibrium outsourcing price. Rather, it shows that substitutability delays the point at which the marginal effect of the network externality on that price changes sign. In the limiting case $b \to 0$, this sign reversal occurs immediately, whereas for $b \to 1$, the outsourcing price remains increasing in n over a substantially larger portion of the admissible parameter range.

3.2 Stackelberg competition

Throughout this subsection, the superscript "S" denotes equilibrium outcomes under Stackelberg competition. We restrict attention to the following interior parameter region.

Assumption 2. *We assume that*

$$\begin{aligned} b^4n^2 - 14b^4n + 40b^4 + 22b^3n^2 - 88b^3n - 8b^2n^2 + 168b^2n - 144b^2 \\ - 104bn^2 + 160bn + 96n^2 - 256n + 128 > 0. \end{aligned}$$

Assumption 2 ensures an interior Stackelberg equilibrium with strictly positive outsourcing prices, quantities, and profits, together with the relevant concavity conditions.

For given outsourcing prices r_1 and r_2, firm 2 observes the quantity selected by firm 1 and chooses q_2 to maximize

$$\Pi_2 = [a - q_2 - bq_1 + n(y_1 + y_2) - c - r_1]\, q_2 + r_2 q_1.$$

The follower's first-order condition is[8]

$$\frac{\partial \Pi_2}{\partial q_2} = a - c - r_1 + n(y_1 + y_2) - 2q_2 - bq_1 = 0,$$

[8]The second-order condition is satisfied: $\frac{\partial^2 \Pi_2}{\partial q_2^2} = -2 < 0$.

it follows that firm 2's quantity reaction function is

$$q_2(q_1) = \frac{a - c - r_1 + n(y_1 + y_2) - bq_1}{2}. \tag{12}$$

Anticipating the follower's reaction in (12), firm 1 substitutes $q_2(q_1)$ into its profit and chooses q_1, while taking y_1 and y_2 as given. Its profit can be written as

$$\Pi_1 = \left[\frac{2-b}{2}\left(a - c + n(y_1 + y_2)\right) - r_2 - \frac{2-b^2}{2}q_1\right] q_1 + \frac{r_1}{2}\left[a - c - r_1 + n(y_1 + y_2)\right].$$

The leader's first-order condition is[9]

$$\frac{\partial \Pi_1}{\partial q_1} = a - c - r_2 + n(y_1 + y_2) - \frac{b}{2}\left[a - c + n(y_1 + y_2)\right] - \left(2 - b^2\right) q_1 = 0. \tag{13}$$

Solving (13) yields the leader's optimal quantity for given expectations:

$$q_1^S(y_1, y_2) = \frac{(2-b)\left[a - c + n(y_1 + y_2)\right] - 2r_2}{2\left(2 - b^2\right)}. \tag{14}$$

Substituting (14) into the follower's reaction function (12) gives

$$q_2^S(y_1, y_2) = \frac{\left(4 - 2b - b^2\right)\left[a - c + n(y_1 + y_2)\right] - 2\left(2 - b^2\right) r_1 + 2br_2}{4\left(2 - b^2\right)}. \tag{15}$$

Imposing fulfilled expectations, $y_1 = q_1$ and $y_2 = q_2$, on (14) and (15) yields the second-stage outputs and profits reported in the following lemma.

Lemma 3. *For given outsourcing prices r_1 and r_2, the equilibrium output of the leader under Stackelberg competition is*

$$q_1^S = \frac{2(2-b)(a-c) - (2-b)nr_1 - 2(2-n)r_2}{8 - 4b^2 + b^2 n + 4bn - 8n}, \tag{16}$$

while the equilibrium output of the follower is

$$q_2^S = \frac{(4 - 2b - b^2)(a-c) - \left(4 - 2b^2 + bn - 2n\right) r_1 + 2(b-n)r_2}{8 - 4b^2 + b^2 n + 4bn - 8n}. \tag{17}$$

[9]The second-order condition is satisfied: $\frac{\partial^2 \Pi_1}{\partial q_1^2} = -\left(2 - b^2\right) < 0$.

The leader's second-stage profit is

$$\begin{aligned}\Pi_1^S = {} & \frac{(2-b^2)\left[2(2-b)(a-c)-(2-b)nr_1-2(2-n)r_2\right]^2}{2\left(8-4b^2+b^2n+4bn-8n\right)^2} \\ & + \frac{br_1\left[2(2-b)(a-c)-(2-b)nr_1-2(2-n)r_2\right]}{2\left(8-4b^2+b^2n+4bn-8n\right)} \\ & + \frac{r_1\left[(4-2b-b^2)(a-c)-\left(4-2b^2+bn-2n\right)r_1+2(b-n)r_2\right]}{8-4b^2+b^2n+4bn-8n}, \end{aligned} \tag{18}$$

and the follower's second-stage profit is

$$\begin{aligned}\Pi_2^S = {} & \frac{\left[(4-2b-b^2)(a-c)-\left(4-2b^2+bn-2n\right)r_1+2(b-n)r_2\right]^2}{\left(8-4b^2+b^2n+4bn-8n\right)^2} \\ & + \frac{r_2\left[2(2-b)(a-c)-(2-b)nr_1-2(2-n)r_2\right]}{8-4b^2+b^2n+4bn-8n}. \end{aligned} \tag{19}$$

Proof. Straightforward. □

The following corollary summarizes how each outsourcing price affects the allocation of output between the Stackelberg leader and follower.

Corollary 3. *Under Stackelberg competition, the effects of the outsourcing prices on the firms' second-stage equilibrium quantities are given by*

$$\frac{\partial q_1^S}{\partial r_1} \leq 0, \qquad \frac{\partial q_1^S}{\partial r_2} < 0, \qquad \frac{\partial q_2^S}{\partial r_1} < 0,$$

and

$$\frac{\partial q_2^S}{\partial r_2} \gtreqless 0 \quad \Longleftrightarrow \quad b \gtreqless n.$$

Moreover, either outsourcing price reduces total industry output:

$$\frac{\partial\left(q_1^S+q_2^S\right)}{\partial r_1} < 0, \qquad \frac{\partial\left(q_1^S+q_2^S\right)}{\partial r_2} < 0.$$

Proof. See Appendix C. □

Corollary 3 shows that the two outsourcing prices affect the allocation of output asymmetrically because the firms occupy different positions in the quantity game. An increase in r_1, which raises the follower's effective marginal cost, reduces both q_2^S and q_1^S. The decline in the follower's output is direct, while the reduction in the leader's output operates through the contraction of the realized network. By contrast, an increase in r_2 always reduces the leader's quantity, since it directly raises its input cost, whereas its effect on the follower's quantity depends on the relative magnitude of product substitutability and the network externality. When $b > n$, the reduction in the leader's output generated by a higher r_2 creates a sufficiently strong strategic reallocation of demand toward the follower, so that q_2^S increases. When $n > b$, the associated contraction of the common network dominates this reallocation effect, and the follower's quantity also decreases.

This mechanism is closely related to that identified under Cournot competition in Corollary 1. In both market structures, a firm's own outsourcing price raises its quantity only when product substitutability is sufficiently strong relative to the network effect. The Stackelberg structure nevertheless introduces an asymmetry: the leader's quantity is non-increasing in its own outsourcing price, whereas the follower retains the same threshold effect as in the Cournot case. Despite these differences in output allocation, both outsourcing prices reduce total industry output.

Anticipating the Stackelberg equilibrium characterized in Lemma 3, firms 1 and 2 simultaneously choose their outsourcing prices, r_1 and r_2, to maximize their respective reduced-form profits in (18) and (19).

The corresponding outsourcing-price reaction functions are

$$r_1(r_2) = \frac{2\left(2-b^2\right)(a-c)\left[8-4b^2+b^2n+4bn-8n-(2-b)^2n\right]+n^2(2-b)^3r_2}{\begin{array}{l}\left(8-4b^2+b^2n+4bn-8n\right)\left[b(2-b)n+2\left(4-2b^2+bn-2n\right)\right]\\ \quad -\left(2-b^2\right)(2-b)^2n^2\end{array}}. \tag{20}$$

$$r_2(r_1) = \frac{\begin{array}{l}2(a-c)\Big[(2-b)\left(8-4b^2+b^2n+4bn-8n\right)\\ \qquad +2(b-n)\left(4-2b-b^2\right)\Big]-\Big[(2-b)n\left(8-4b^2+b^2n+4bn-8n\right)\\ \qquad +4(b-n)\left(4-2b^2+bn-2n\right)\Big]r_1\end{array}}{4\left[(2-n)\left(8-4b^2+b^2n+4bn-8n\right)-2(b-n)^2\right]}. \tag{21}$$

Solving the two outsourcing-price reaction functions (20) and (21) simultaneously yields

$$r_1^{S*} = \frac{2(a-c)\begin{pmatrix}b^4n^2-14b^4n+40b^4+22b^3n^2-88b^3n\\ \quad -8b^2n^2+168b^2n-144b^2\\ \quad -104bn^2+160bn+96n^2-256n+128\end{pmatrix}}{\mathcal{D}_S(b,n)}, \tag{22}$$

$$r_2^{S*} = \frac{4(a-c)\begin{pmatrix} 2b^5n - 8b^5 - 2b^4n^2 + 16b^4n + 24b^4 \\ - 5b^3n^2 - 86b^3n + 24b^3 \\ + 52b^2n^2 + 52b^2n - 80b^2 \\ - 92bn^2 + 136bn - 16b \\ + 48n^2 - 128n + 64 \end{pmatrix}}{\mathcal{D}_S(b,n)}, \tag{23}$$

where, for notational convenience,

$$\begin{aligned}\mathcal{D}_S(b,n) = {} & b^4n^3 - 56b^4n + 160b^4 + 8b^3n^3 + 84b^3n^2 - 352b^3n \\ & - 60b^2n^3 + 64b^2n^2 + 672b^2n - 576b^2 + 112bn^3 - 624bn^2 + 640bn \\ & - 64n^3 + 512n^2 - 1024n + 512.\end{aligned}$$

Substituting the equilibrium outsourcing prices in (22) and (23) into the second-stage equilibrium characterized in Lemma 3 yields the Stackelberg outcomes associated with the subgame-perfect Nash equilibrium.

Lemma 4. *Under Assumption 2, the subgame-perfect Nash equilibrium under Stackelberg competition is characterized by the quantities*

$$q_1^{S*} = \frac{4(a-c)(4-3b)\begin{pmatrix} b^2n - 4b^2 - bn^2 + 4bn \\ + 2n^2 - 8n + 8 \end{pmatrix}}{\mathcal{D}_S(b,n)}, \tag{24}$$

and

$$q_2^{S*} = \frac{(a-c)\begin{pmatrix} - b^4n^2 + 16b^4 - 8b^3n^2 - 32b^3n + 32b^3 \\ + 48b^2n^2 + 8b^2n - 96b^2 \\ - 72bn^2 + 144bn - 64b \\ + 32n^2 - 128n + 128 \end{pmatrix}}{\mathcal{D}_S(b,n)}. \tag{25}$$

The leader's equilibrium profit is

$$\Pi_1^{S*} = \frac{2(a-c)^2 \begin{pmatrix} \left[-n^4+8n^3+48n^2-512n+1024\right] b^8 \\ +\left[-24n^4+8n^3+1520n^2-5056n+1536\right] b^7 \\ +\left[-44n^4-1352n^3+4904n^2+7104n-9088\right] b^6 \\ +\left[920n^4-704n^3-25888n^2+35840n-9216\right] b^5 \\ +\left[-1832n^4+19040n^3-8576n^2-38272n+30208\right] b^4 \\ +\left[-1792n^4-26752n^3+90368n^2-83200n+18432\right] b^3 \\ +\left[9184n^4-16512n^3-33024n^2+84992n-44544\right] b^2 \\ +\left[-9984n^4+50688n^3-90112n^2+63488n-12288\right] b \\ +3584n^4-24576n^3+61440n^2-65536n+24576 \end{pmatrix}}{\mathcal{D}_S(b,n)^2}. \tag{26}$$

The follower's equilibrium profit is

$$\Pi_2^{S*} = \frac{(a-c)^2 \begin{pmatrix} \left[n^4-128n^2+768n-1280\right] b^8 \\ +\left[16n^4+256n^3-2112n^2+1408n+7680\right] b^7 \\ +\left[-128n^4+1440n^3+7584n^2-32512n-512\right] b^6 \\ +\left[-544n^4-13056n^3+27968n^2+58368n-43008\right] b^5 \\ +\left[6432n^4+17216n^3-132288n^2+75776n+29696\right] b^4 \\ +\left[-20800n^4+45184n^3+109568n^2-232960n+79872\right] b^3 \\ +\left[31936n^4-145408n^3+155904n^2+37888n-71680\right] b^2 \\ +\left[-24064n^4+143360n^3-288768n^2+221184n-49152\right] b \\ +7168n^4-49152n^3+122880n^2-131072n+49152 \end{pmatrix}}{\mathcal{D}_S(b,n)^2}. \tag{27}$$

Proof. Straightforward. □

We now examine how the strength of the network externality affects the equilibrium outsourcing prices and the Stackelberg outcomes characterized in Lemma 4.

Proposition 2. *Under Assumption 2, the network externality affects the Stackelberg equilibrium outcomes as follows:*

$$\frac{\partial r_1^{S*}}{\partial n} \leq 0, \qquad \frac{\partial q_1^{S*}}{\partial n} > 0, \qquad \frac{\partial q_2^{S*}}{\partial n} > 0, \qquad \frac{\partial \Pi_1^{S*}}{\partial n} > 0, \qquad \frac{\partial \Pi_2^{S*}}{\partial n} > 0.$$

For the follower's equilibrium outsourcing price:

(i) If $0 < b < \bar{b}_S$*, then*

$$\frac{\partial r_2^{S*}}{\partial n} \gtreqless 0 \quad \Longleftrightarrow \quad n \lesseqgtr \widehat{n}_S(b),$$

where $\bar{b}_S \simeq 0.9620$ *is the unique root in* $(0,1)$ *of the polynomial defined in* (34)*, and* $\widehat{n}_S(b) \in (0,1)$ *is the unique root with respect to* n *of the polynomial defined in* (31)*. Numerically,*

$$\widehat{n}_S(0.2) \simeq 0.1581, \qquad \widehat{n}_S(0.5) \simeq 0.4259, \qquad \widehat{n}_S(0.8) \simeq 0.7451.$$

(ii) If $\bar{b}_S \leq b < 1$*, then*

$$\frac{\partial r_2^{S*}}{\partial n} > 0 \qquad \textit{for all admissible } n.$$

Proof. See Appendix D. □

Proposition 2 shows that a stronger network externality expands output and raises the profits of both firms. Under fulfilled expectations, an increase in n raises consumers' willingness to pay through the larger realized network. Although the firms adjust their outsourcing prices strategically, these adjustments do not offset the resulting demand expansion. Consequently, both the leader and the follower increase their equilibrium quantities and benefit from the stronger network effect.

The responses of the outsourcing prices are nevertheless asymmetric. The leader's outsourcing price is non-increasing in n. By reducing the price charged to the follower, the leader limits the contraction of the follower's output and thereby preserves the common network from which its own product also benefits. The follower's outsourcing price exhibits a less uniform response. When $0 < b < \bar{b}_S$, a stronger network effect initially allows the follower to increase r_2^{S*}, but this effect is reversed once n exceeds $\widehat{n}_S(b)$. Beyond this threshold, maintaining the leader's output, and hence the size of the common network, becomes relatively more important than extracting a higher outsourcing payment. The numerical values reported in Proposition 2 indicate that this reversal occurs at a higher value of n when products are more substitutable. For sufficiently high substitutability, $b \geq \bar{b}_S$, the threshold lies outside the admissible interval and the follower's outsourcing price increases with n throughout.

3.3 Comparison

In this subsection, we compare the equilibrium outcomes under Cournot and Stackelberg competition. We begin with the equilibrium outsourcing prices summarized in the following proposition.

Proposition 3. *Consider the common interior parameter region satisfying Assumptions 1 and 2. There exist two thresholds satisfying*

$$0 < \widehat{n}_1(b) < \widehat{n}_2(b) < 1$$

such that

$$r_1^{S*} \gtreqless r_i^{C*} \quad \Longleftrightarrow \quad n \lesseqgtr \widehat{n}_1(b),$$
$$r_1^{S*} \gtreqless r_2^{S*} \quad \Longleftrightarrow \quad n \lesseqgtr \widehat{n}_2(b),$$

and

$$r_i^{C*} > r_2^{S*} \qquad \text{for all admissible } (b, n).$$

The threshold $\widehat{n}_1(b)$ is the unique admissible root with respect to n of the polynomial defined in (40). The second threshold is given by

$$\widehat{n}_2(b) = \frac{2b^4 + 23b^3 - 42b^2 - 32b + 56 - \sqrt{\Delta_2(b)}}{5b^3 + 32b^2 - 112b + 80},$$

where

$$\Delta_2(b) = 4b^8 + 12b^7 - 111b^6 + 228b^5 - 204b^4 - 144b^3 + 928b^2 - 1280b + 576.$$

Proof. See Appendix E. □

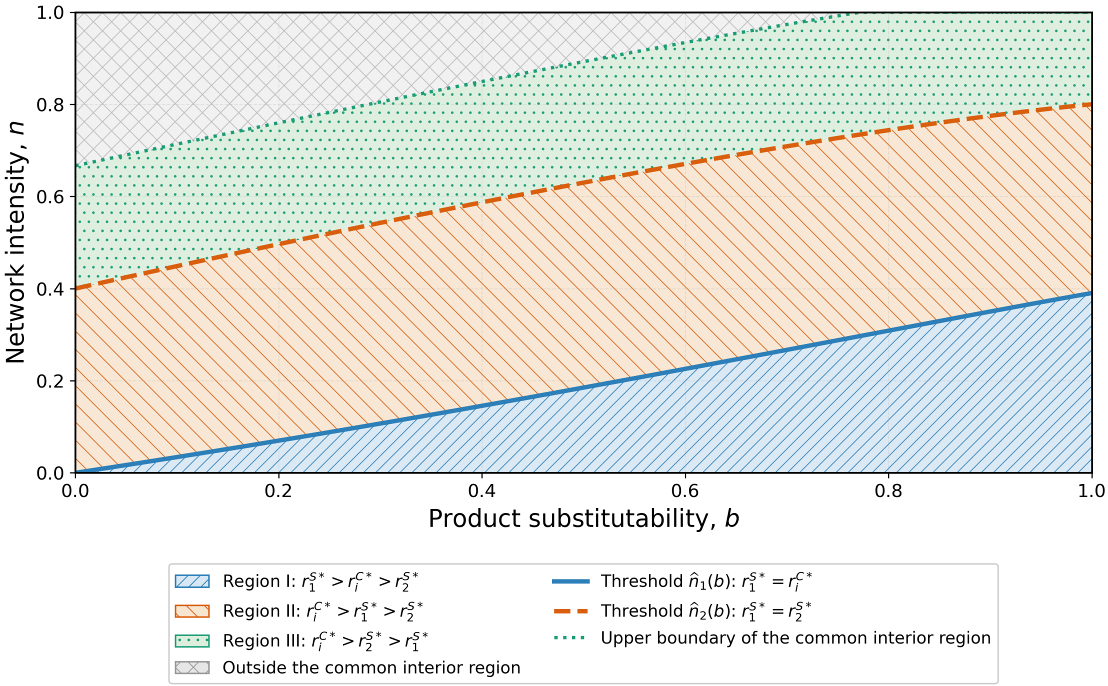
Proposition 3 shows that the timing of quantity decisions changes the strategic role of outsourcing prices. The follower's outsourcing price is always lower than the symmetric Cournot outsourcing price: $r_2^{S*} < r_i^{C*}$. Under Stackelberg competition, firm 2 chooses the input price paid by the quantity leader. A higher r_2 directly raises the leader's marginal cost and reduces its output. Since the leader commits before the follower, this contraction is transmitted to the follower through both the usual strategic-substitution channel and the reduction in the realized network size. The follower therefore has a stronger incentive to moderate its outsourcing price than a firm operating under simultaneous Cournot competition.

The comparison involving the leader is instead non-monotonic. When $n < \widehat{n}_1(b)$, the leader charges a higher outsourcing price than under Cournot: $r_1^{S*} > r_i^{C*}$. For relatively weak network externalities, the leader can exploit its strategic position by raising the input cost of the follower. The resulting reduction in the follower's quantity reallocates demand toward the leader, while the loss in the common network remains limited. Consequently, the cost-raising motive dominates the network-expansion motive.

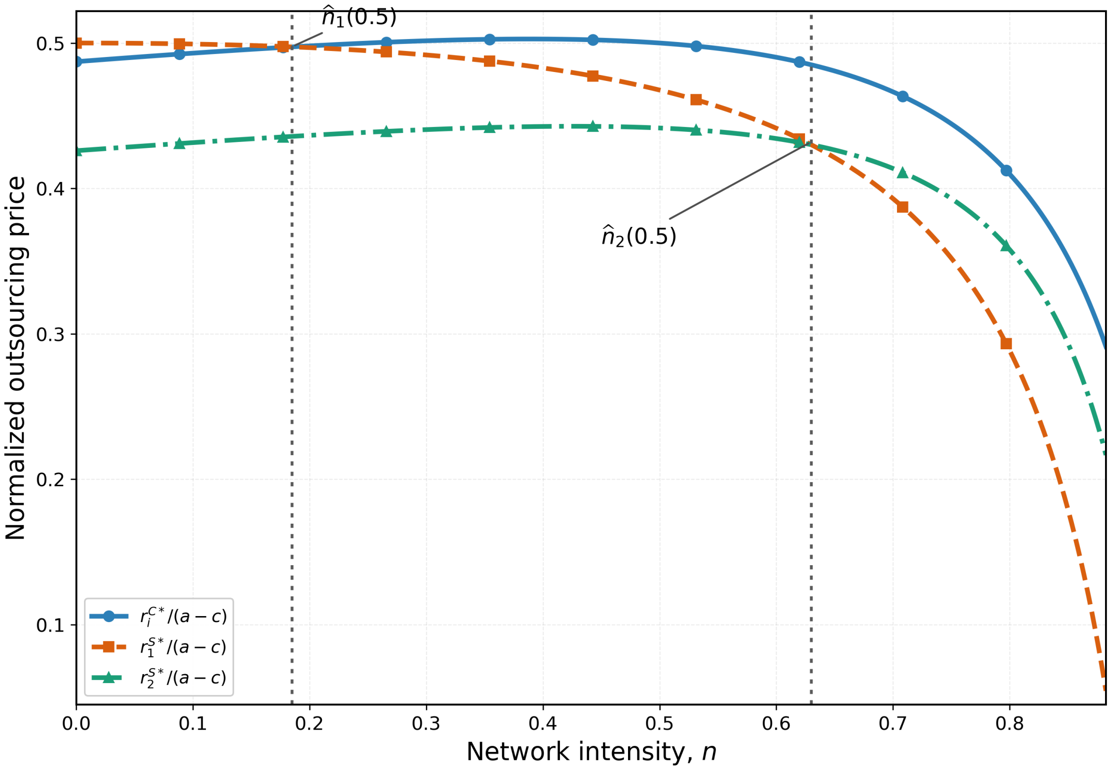
Once the network externality exceeds $\widehat{n}_1(b)$, this ranking is reversed: $r_1^{S*} < r_i^{C*}$. A higher r_1 then reduces the follower's output sufficiently strongly to contract the common installed base. Because the leader commits its quantity before observing the follower's final choice, it internalizes more directly the effect of the follower's contraction on the market environment that supports its own demand. The leader therefore moderates its outsourcing price relative to the Cournot benchmark.

The second threshold, $\widehat{n}_2(b)$, governs the ranking of the two Stackelberg outsourcing prices. For $n < \widehat{n}_2(b)$, the leader charges more than the follower: $r_1^{S*} > r_2^{S*}$. In this region, the leader's incentive to raise the follower's marginal cost remains stronger than the follower's incentive to tax the leader's committed output. By contrast, when $n > \widehat{n}_2(b)$, the ranking becomes $r_2^{S*} > r_1^{S*}$. The leader then places greater weight on preserving the follower's output and the associated network size, whereas the follower can still extract outsourcing revenue from the leader's previously committed production. This difference in commitment positions explains why the two Stackelberg prices respond asymmetrically as the network externality becomes stronger.

These rankings are illustrated in Figure 1. Panel a shows how the two thresholds partition the parameter space into three regions. Below $\widehat{n}_1(b)$, the leader's Stackelberg price is the highest; between $\widehat{n}_1(b)$ and $\widehat{n}_2(b)$, the Cournot price becomes the highest while the leader still charges more than the follower; above $\widehat{n}_2(b)$, the Cournot price remains the highest, but the follower's price exceeds that of the leader. Panel b illustrates the same two reversals for $b = 0.5$.



(a) Thresholds and outsourcing-price rankings.



(b) Normalized outsourcing prices for $b = 0.5$.

Figure 1: Comparison of equilibrium outsourcing prices under Cournot and Stackelberg competition.

We next compare the equilibrium profits generated by the two modes of competition.

Proposition 4. *Consider the common interior parameter region satisfying Assumptions 1 and 2. There exist two thresholds satisfying*

$$0 < \widehat{n}_3(b) < \widehat{n}_4(b)$$

such that

$$\Pi_1^{S*} > \Pi_i^{C*} \qquad \text{for all admissible } (b, n),$$

$$\Pi_2^{S*} \gtreqless \Pi_i^{C*} \quad \Longleftrightarrow \quad n \gtreqless \widehat{n}_3(b),$$

and

$$\Pi_1^{S*} \gtreqless \Pi_2^{S*} \quad \Longleftrightarrow \quad n \lesseqgtr \widehat{n}_4(b).$$

The threshold $\widehat{n}_3(b)$ is the unique admissible root with respect to n of the polynomial defined in (53), whereas $\widehat{n}_4(b)$ is the unique admissible root with respect to n of the polynomial defined in (57).

Proof. See Appendix F. □

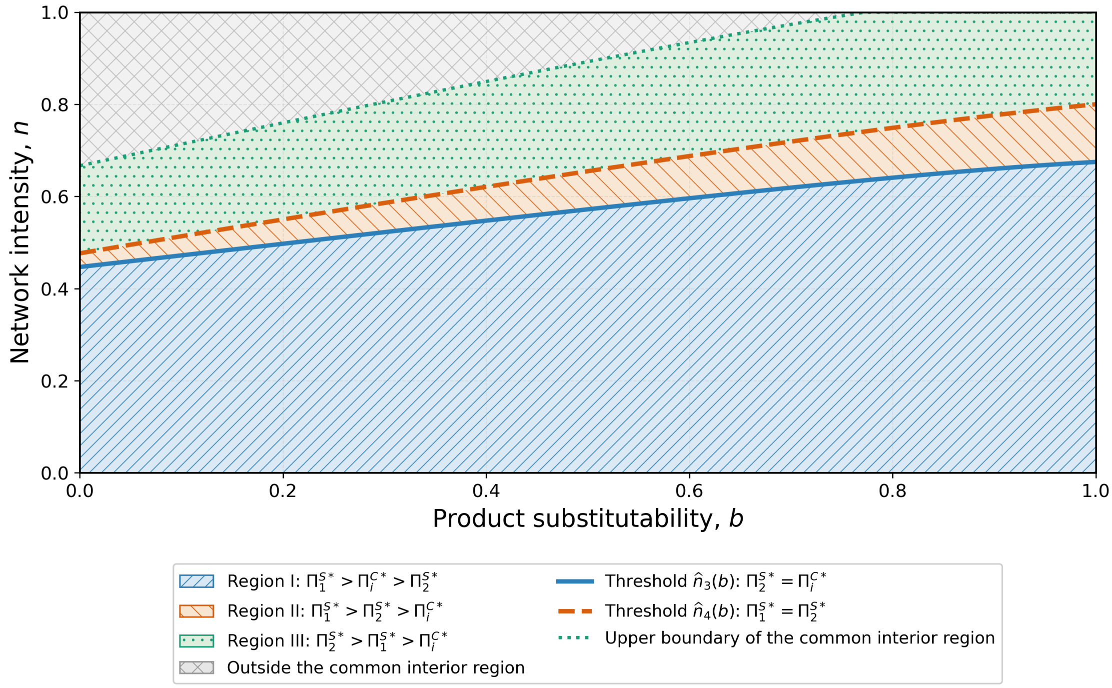
Proposition 4 shows that the timing of quantity decisions has an asymmetric effect on firms' equilibrium profitability. The Stackelberg leader always earns a higher profit than a firm under Cournot competition: $\Pi_1^{S*} > \Pi_i^{C*}$. The leader benefits from its ability to commit to output before the follower moves. This commitment allows it to anticipate the follower's reaction and to choose its quantity accordingly. Although the outsourcing prices are themselves endogenous and may be lower under Stackelberg for some values of n, this adjustment does not overturn the strategic advantage associated with quantity leadership.

The follower's comparison with Cournot is instead governed by the first threshold, $\widehat{n}_3(b)$. When $n < \widehat{n}_3(b)$, the follower earns less than under Cournot: $\Pi_2^{S*} < \Pi_i^{C*}$. For relatively weak network externalities, the follower mainly bears the disadvantage associated with moving after the leader. The leader's prior quantity commitment restricts the follower's residual market opportunities, and the network benefit generated by the leader's installed base is not yet strong enough to compensate for this strategic disadvantage.

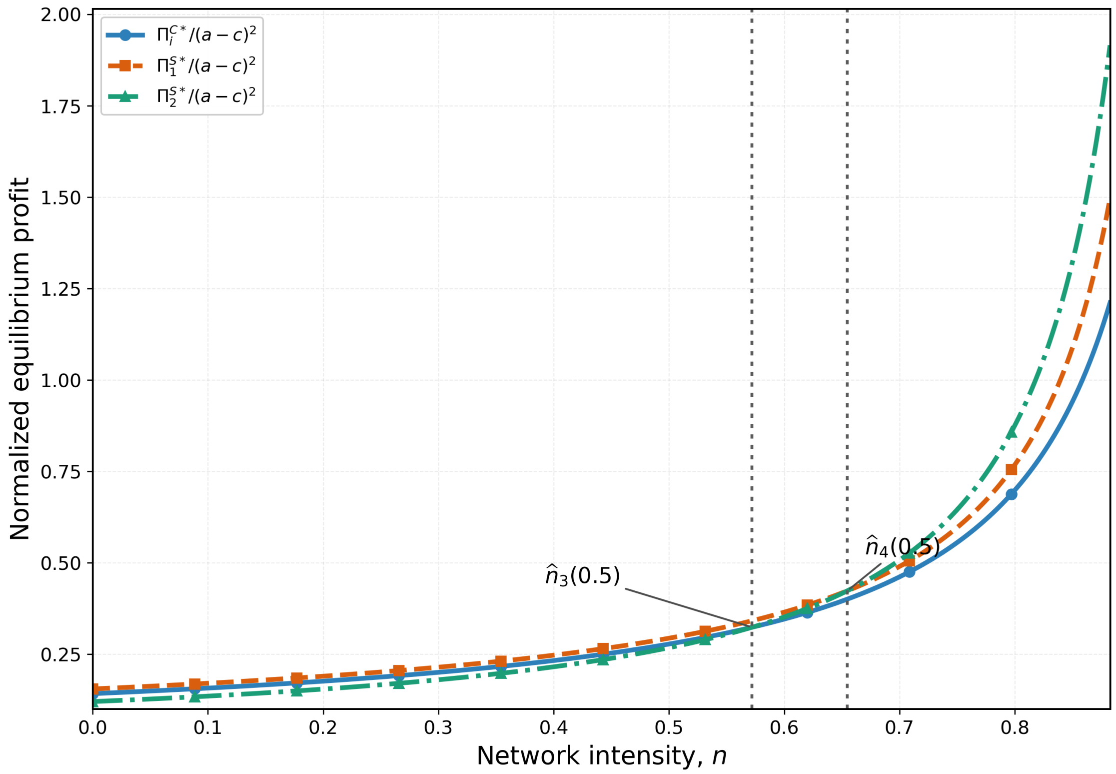
Once the network externality exceeds $\widehat{n}_3(b)$, the ranking is reversed: $\Pi_2^{S*} > \Pi_i^{C*}$. A stronger network effect increases the value of the aggregate installed base generated by both firms. The follower is then able to respond to the leader's committed quantity while benefiting from the larger network that this commitment helps create. In addition, the follower receives outsourcing revenue from the leader's production. Beyond $\widehat{n}_3(b)$, these effects become sufficiently strong for the follower's Stackelberg profit to exceed the Cournot benchmark.

The second threshold, $\widehat{n}_4(b)$, determines whether quantity leadership itself generates a profit advantage within the Stackelberg regime. For $n < \widehat{n}_4(b)$, the conventional ranking holds: $\Pi_1^{S*} > \Pi_2^{S*}$. The leader's commitment advantage dominates the follower's ability to adjust after observing the leader's quantity. However, when $n > \widehat{n}_4(b)$, the ranking becomes $\Pi_2^{S*} > \Pi_1^{S*}$. For sufficiently strong network externalities, the follower benefits more from the leader's prior market expansion. Since the leader's quantity is already committed, the follower can adapt its own output to the enlarged network while still collecting outsourcing revenue from the leader. The network effect therefore weakens, and eventually reverses, the usual first-mover advantage. Because $\widehat{n}_3(b) < \widehat{n}_4(b)$, there is an intermediate region in which both Stackelberg firms earn more than under Cournot, while the leader still remains more profitable than the follower: $\Pi_1^{S*} > \Pi_2^{S*} > \Pi_i^{C*}$. For stronger network effects, the follower becomes the most profitable firm, but the leader continues to earn more than under Cournot. Hence, the network externality does not eliminate the profitability gain associated with leadership relative to simultaneous competition; rather, it changes how this gain is distributed between the leader and the follower.

Figure 2 illustrates these rankings. Panel a shows how $\widehat{n}_3(b)$ and $\widehat{n}_4(b)$ partition the common interior parameter region into the three profit-ordering regimes identified in the proposition. Panel b illustrates the same mechanism for $b = 0.5$. In particular, the follower's profit first crosses the Cournot profit at $\widehat{n}_3(0.5)$, while the second crossing at $\widehat{n}_4(0.5)$ marks the point at which the follower's profit overtakes that of the Stackelberg leader.



(a) Profit rankings in the common interior parameter region.



(b) Normalized equilibrium profits for $b = 0.5$.

Figure 2: Comparison of equilibrium profits under Cournot and Stackelberg competition.

4 Conclusion

This paper has examined mutual outsourcing in a differentiated network duopoly and has shown that positive network externalities substantially modify the strategic role of outsourcing prices. By comparing Cournot and Stackelberg quantity competition, the analysis highlights how input pricing, network expansion, and the timing of market decisions interact. Under both competitive regimes, stronger network effects increase equilibrium quantities and profits, while their effect on outsourcing prices is more nuanced. In particular, when network effects become sufficiently strong, firms may find it optimal to moderate the prices charged for inputs supplied to rivals in order to preserve the size of the common network. Sequential competition further generates asymmetric incentives: the Stackelberg follower always sets a lower outsourcing price than under Cournot competition, whereas the leader's relative pricing position depends on network intensity.

The profit comparison reinforces the importance of network effects for the value of strategic timing. The Stackelberg leader always earns more than under Cournot competition, while the follower benefits from sequential competition only once network effects are sufficiently strong. Beyond a further threshold, the follower can even become more profitable than the leader, reversing the conventional first-mover profit ranking. Thus, network externalities do not merely strengthen demand; they also reshape the trade-off between raising a rival's cost and preserving the installed base from which both firms benefit.

These findings also have a direct managerial interpretation. In technology-intensive ecosystems, such as the relationship between Google and Samsung, firms may compete downstream while remaining mutually dependent on software, components, manufacturing capabilities, or other critical inputs. Our results suggest that outsourcing prices should therefore not be set solely to maximize input-market margins or weaken a rival. When network effects are strong, excessively aggressive input pricing may reduce the rival's activity and, in turn, erode the value of the network supporting both firms. Moreover, the appropriate outsourcing strategy depends on whether firms move simultaneously or occupy leader–follower positions in the market, implying that sourcing decisions and market-timing strategies should be considered jointly.

The analysis deliberately adopts a parsimonious framework with symmetric production costs and quantity competition under passive, fulfilled expectations. Future research could consider asymmetric costs or input technologies, endogenous make-or-buy decisions, richer expectation formation, and alternative competitive regimes such as Bertrand competition and price leadership. It would also be useful to examine endogenous product compatibility or nonlinear outsourcing contracts. These extensions would help assess the robustness of the mechanisms identified here across a broader range of network industries.

Appendix

Appendix A: Proof of Proposition 1

Proof. Differentiating the equilibrium outsourcing price in (9) with respect to n gives

$$\frac{\partial r_i^{C*}}{\partial n} = \frac{(a-c)\left[4(3-b)n^2 + 4\left(b^2-2b-4\right)n + b\left(12-b^2\right)\right]}{\left(8+4b-b^2+bn+2n^2-12n\right)^2}.$$

The two roots of the numerator are

$$\widehat{n}_C(b) = \frac{4+2b-b^2-(2-b)\sqrt{4-b}}{2(3-b)}$$

and

$$\widetilde{n}_C(b) = \frac{4+2b-b^2+(2-b)\sqrt{4-b}}{2(3-b)}.$$

For $b \in (0,1)$,

$$0 < \widehat{n}_C(b) < \bar{n}_C(b), \qquad \widetilde{n}_C(b) > \min\{1, \bar{n}_C(b)\}.$$

Since the coefficient of n^2 is positive, it follows that

$$\frac{\partial r_i^{C*}}{\partial n} \gtreqless 0 \quad \Longleftrightarrow \quad n \lesseqgtr \widehat{n}_C(b)$$

over the admissible parameter region.

Differentiating (10) yields

$$\frac{\partial q_i^{C*}}{\partial n} = \frac{(a-c)\left(b^2-6b+2n^2-8n+16\right)}{\left(8+4b-b^2+bn+2n^2-12n\right)^2}.$$

Moreover,

$$b^2-6b+2n^2-8n+16 = (1-b)^2 + 4(1-b) + 2(1-n)^2 + 4(1-n) + 5 > 0,$$

and hence $\frac{\partial q_i^{C*}}{\partial n} > 0$.

Finally, differentiating (11) gives

$$\frac{\partial \Pi_i^{C*}}{\partial n} = \frac{(a-c)^2 \begin{bmatrix} -b^4 + 3b^3 n + 6b^3 - 6b^2 n^2 - 4b^2 n - 20b^2 \\ +8bn^3 - 12bn^2 + 36bn + 8b \\ -28n^3 + 120n^2 - 224n + 128 \end{bmatrix}}{(8 + 4b - b^2 + bn + 2n^2 - 12n)^3}.$$

The polynomial in square brackets is strictly decreasing in n on $b \in (0,1)$ and $n \in [0,1)$, since the negative of its derivative with respect to n can be written as

$$\begin{aligned} &3(1-b)^3 + 7(1-b)^2 + (1-b)\left[13 - 12(1-b)(1-n)\right] \\ &\quad + 24(1-b)(1-n)^2 + 60(1-n)^2 + 84(1-n) + 45, \end{aligned}$$

which is strictly positive.

If $\bar{n}_C(b) \leq 1$, the polynomial is therefore bounded below by its value at $n = \bar{n}_C(b)$, namely

$$-\frac{(b-4)^3(b-2)^2(2b-5)}{2(b-3)^3} > 0.$$

If $\bar{n}_C(b) > 1$, then $b > 2 - \sqrt{2}$, and the polynomial is bounded below by its value at $n = 1$. This value is increasing in b on $(0,1)$ and, at $b = 2 - \sqrt{2}$, equals $8 + 2\sqrt{2} > 0$. Thus, under Assumption 1, $\frac{\partial \Pi_i^{C*}}{\partial n} > 0$. □

Appendix B: Proof of Corollary 2

Proof. Differentiating the threshold defined in Proposition 1 gives

$$\frac{\partial \widehat{n}_C(b)}{\partial b} = \frac{(2b^2 - 12b + 20)\sqrt{4-b} + b^2 - 7b + 14}{4(3-b)^2\sqrt{4-b}}.$$

For $b \in (0,1)$, the denominator is strictly positive. Moreover, $2b^2 - 12b + 20 > 0, \quad b^2 - 7b + 14 > 0$, so that $\frac{\partial \widehat{n}_C(b)}{\partial b} > 0$. Finally, evaluating the threshold at the boundaries of the admissible interval for b yields

$$\lim_{b \to 0^+} \widehat{n}_C(b) = 0, \qquad \lim_{b \to 1^-} \widehat{n}_C(b) = \frac{5 - \sqrt{3}}{4}.$$

□

Appendix C: Proof of Corollary 3

Proof. Differentiating the equilibrium quantities reported in Lemma 3 yields

$$\frac{\partial q_1^S}{\partial r_1} = -\frac{(2-b)n}{8-4b^2+b^2n+4bn-8n} \leq 0,$$

and

$$\frac{\partial q_1^S}{\partial r_2} = -\frac{2(2-n)}{8-4b^2+b^2n+4bn-8n} < 0.$$

For the follower,

$$\frac{\partial q_2^S}{\partial r_1} = -\frac{4-2b^2+bn-2n}{8-4b^2+b^2n+4bn-8n} < 0$$

Moreover,

$$\frac{\partial q_2^S}{\partial r_2} = \frac{2(b-n)}{8-4b^2+b^2n+4bn-8n},$$

and therefore

$$\frac{\partial q_2^S}{\partial r_2} \gtreqless 0 \quad \Longleftrightarrow \quad b \gtreqless n.$$

Finally, summing the relevant derivatives gives

$$\frac{\partial \left(q_1^S + q_2^S\right)}{\partial r_1} = -\frac{2(2-b^2)}{8-4b^2+b^2n+4bn-8n} < 0$$

and

$$\frac{\partial \left(q_1^S + q_2^S\right)}{\partial r_2} = -\frac{2(2-b)}{8-4b^2+b^2n+4bn-8n} < 0.$$

□

Appendix D: Proof of Proposition 2

Proof. Differentiating (22) with respect to n yields

$$\frac{\partial r_1^{S*}}{\partial n} = -\frac{2(a-c)n(2-b)^2 P_1^S(b,n)}{\mathcal{D}_S(b,n)^2}, \tag{28}$$

where

$$
\begin{aligned}
P_1^S(b,n) = {}& b^6n^3 - 28b^6n^2 + 176b^6n - 320b^6 \\
&+ 34b^5n^3 - 512b^5n^2 + 2072b^5n - 1600b^5 \\
&+ 240b^4n^3 - 1328b^4n^2 - 1712b^4n + 3712b^4 \\
&- 552b^3n^3 + 7168b^3n^2 - 12000b^3n + 5760b^3 \\
&- 1024b^2n^3 - 2048b^2n^2 + 10496b^2n - 10240b^2 \\
&+ 2816bn^3 - 11264bn^2 + 14848bn - 5120b \\
&- 1536n^3 + 8192n^2 - 14336n + 8192.
\end{aligned} \tag{29}
$$

The exact Sturm sequence of $P_1^S(b,\cdot)$ has no root on $n \in [0,1]$ for every $b \in (0,1)$. Since $P_1^S(b,0) > 0$, we have $P_1^S(b,n) > 0$ on $(0,1) \times [0,1)$. Hence,

$$
\frac{\partial r_1^{S*}}{\partial n} \leq 0.
$$

Differentiating (23) gives

$$
\frac{\partial r_2^{S*}}{\partial n} = -\frac{4(a-c)P_2^S(b,n)}{\mathcal{D}_S(b,n)^2}, \tag{30}
$$

where

$$
\begin{aligned}
P_2^S(b,n) = {}& 4b^9n^3 - 24b^9n^2 + 128b^9 \\
&- 2b^8n^4 + 64b^8n^3 - 64b^8n^2 - 704b^8n - 1088b^8 \\
&- 21b^7n^4 - 156b^7n^3 + 2576b^7n^2 + 4608b^7n - 256b^7 \\
&+ 132b^6n^4 - 2744b^6n^3 - 11624b^6n^2 - 1856b^6n + 7936b^6 \\
&+ 400b^5n^4 + 14752b^5n^3 + 11008b^5n^2 - 24832b^5n - 3584b^5 \\
&- 4240b^4n^4 - 25632b^4n^3 + 30784b^4n^2 + 28288b^4n - 18688b^4 \\
&+ 12048b^3n^4 + 4288b^3n^3 - 65280b^3n^2 + 32256b^3n + 13312b^3 \\
&- 16512b^2n^4 + 39168b^2n^3 + 11904b^2n^2 - 51712b^2n + 14336b^2 \\
&+ 11264bn^4 - 46080bn^3 + 49152bn^2 - 2048bn - 12288b \\
&- 3072n^4 + 16384n^3 - 28672n^2 + 16384n.
\end{aligned} \tag{31}
$$

For $0 < b < \bar{b}_S$, the exact Sturm sequence of $P_2^S(b,\cdot)$ shows that

$$
P_2^S(b,n) = 0 \tag{32}
$$

has a unique root $\widehat{n}_S(b) \in (0,1)$, with

$$P_2^S(b,n) < 0 \quad \text{for} \quad n < \widehat{n}_S(b), \qquad P_2^S(b,n) > 0 \quad \text{for} \quad n > \widehat{n}_S(b).$$

Therefore,

$$\frac{\partial r_2^{S*}}{\partial n} \gtreqless 0 \quad \Longleftrightarrow \quad n \lesseqgtr \widehat{n}_S(b).$$

The boundary $\bar{b}_S$ is determined by $\widehat{n}_S(b) = 1$. In particular,

$$P_2^S(b,1) = (3b-4)B_S(b), \tag{33}$$

where

$$\begin{aligned} B_S(b) = 36b^8 - 550b^7 + 1517b^6 - 696b^5 \\ - 1680b^4 + 1264b^3 + 560b^2 - 192b - 256. \end{aligned} \tag{34}$$

The polynomial $B_S(b)$ has a unique root in $(0,1)$, numerically $\bar{b}_S \simeq 0.9620$. Since $3b-4<0$, and the Sturm sequence of $P_2^S(b,\cdot)$ gives $P_2^S(b,n)<0$ for $\bar{b}_S \le b < 1$ and $n \in [0,1)$, it follows that

$$\frac{\partial r_2^{S*}}{\partial n} > 0 \qquad \text{for all } n \in [0,1).$$

Differentiating (24) gives

$$\begin{aligned} \frac{\partial q_1^{S*}}{\partial n} = {} & \frac{4(a-c)(4-3b)}{\mathcal{D}_S(b,n)^2} \\ & \times \Bigg\{ \Big[b^2 - 2bn + 4b + 4n - 8\Big]\mathcal{D}_S(b,n) \\ & \quad - \Big[b^2n - 4b^2 - bn^2 + 4bn + 2n^2 - 8n + 8\Big]\frac{\partial \mathcal{D}_S(b,n)}{\partial n} \Bigg\}, \end{aligned} \tag{35}$$

while differentiating (25) yields

$$
\begin{aligned}
\frac{\partial q_2^{S*}}{\partial n} = \frac{a-c}{\mathcal{D}_S(b,n)^2}\Bigg\{\Big[&- 2b^4n - 16b^3n - 32b^3 \\
&+ 96b^2n + 8b^2 - 144bn + 144b + 64n - 128\Big]\mathcal{D}_S(b,n) \\
&- \Big[- b^4n^2 + 16b^4 - 8b^3n^2 - 32b^3n + 32b^3 \\
&\quad + 48b^2n^2 + 8b^2n - 96b^2 - 72bn^2 + 144bn - 64b \\
&\quad + 32n^2 - 128n + 128\Big]\frac{\partial \mathcal{D}_S(b,n)}{\partial n}\Bigg\}.
\end{aligned} \tag{36}
$$

Here

$$
\begin{aligned}
\frac{\partial \mathcal{D}_S(b,n)}{\partial n} &= 3b^4n^2 - 56b^4 + 24b^3n^2 + 168b^3n - 352b^3 \\
&- 180b^2n^2 + 128b^2n + 672b^2 \\
&+ 336bn^2 - 1248bn + 640b \\
&- 192n^2 + 1024n - 1024.
\end{aligned} \tag{37}
$$

The exact Sturm sequences of the numerators in (35) and (36) show that both are strictly positive for $b \in (0,1)$ and $n \in [0,1)$. Hence,

$$
\frac{\partial q_1^{S*}}{\partial n} > 0, \qquad \frac{\partial q_2^{S*}}{\partial n} > 0.
$$

Finally, differentiating (26) and (27) gives

$$
\frac{\partial \Pi_i^{S*}}{\partial n} = \frac{(a-c)^2\left[\mathcal{D}_S(b,n)\frac{\partial \mathcal{P}_i^S(b,n)}{\partial n} - 2\mathcal{P}_i^S(b,n)\frac{\partial \mathcal{D}_S(b,n)}{\partial n}\right]}{\mathcal{D}_S(b,n)^3}, \qquad i = 1,2,
$$

where $\mathcal{P}_i^S(b,n)$ denotes the complete numerator of the corresponding profit expression. The exact Sturm sequences imply

$$
\mathcal{D}_S(b,n)\frac{\partial \mathcal{P}_i^S(b,n)}{\partial n} - 2\mathcal{P}_i^S(b,n)\frac{\partial \mathcal{D}_S(b,n)}{\partial n} > 0, \qquad i = 1,2,
$$

throughout the admissible interior region. Therefore,

$$
\frac{\partial \Pi_1^{S*}}{\partial n} > 0, \qquad \frac{\partial \Pi_2^{S*}}{\partial n} > 0.
$$

□

Appendix E: Proof of Proposition 3

Proof. For notational convenience, define

$$\mathcal{D}_C(b,n) = 8 + 4b - b^2 + bn + 2n^2 - 12n. \tag{38}$$

In the common interior region, $a - c > 0$, $b > 0$, $\mathcal{D}_C(b,n) > 0$, and $\mathcal{D}_S(b,n) > 0$. Hence, the signs of the price differences are determined by their numerator polynomials.

Subtracting (9) from (22) gives

$$r_1^{S*} - r_i^{C*} = \frac{(a-c)bP_{1C}(b,n)}{\mathcal{D}_C(b,n)\mathcal{D}_S(b,n)}, \tag{39}$$

where

$$\begin{aligned} P_{1C}(b,n) = {} & b^5n^3 - 2b^5n^2 - 28b^5n + 80b^5 \\ & - 2b^4n^4 + 8b^4n^3 + 132b^4n^2 - 416b^4n \\ & - 6b^3n^4 - 284b^3n^3 + 792b^3n^2 + 336b^3n - 288b^3 \\ & + 256b^2n^4 - 320b^2n^3 - 1600b^2n^2 + 1184b^2n \\ & - 616bn^4 + 2240bn^3 - 1184bn^2 - 512bn + 256b \\ & + 384n^4 - 1760n^3 + 2112n^2 - 768n. \end{aligned} \tag{40}$$

For every $b \in (0,1)$, the Sturm sequence of $P_{1C}(b,\cdot)$ shows that $P_{1C}(b,n) = 0$ has a unique root in the common admissible interval, denoted by $\widehat{n}_1(b)$, with

$$P_{1C}(b,n) \gtreqless 0 \quad \Longleftrightarrow \quad n \lesseqgtr \widehat{n}_1(b).$$

Therefore,

$$r_1^{S*} \gtreqless r_i^{C*} \quad \Longleftrightarrow \quad n \lesseqgtr \widehat{n}_1(b). \tag{41}$$

Similarly,

$$r_2^{S*} - r_i^{C*} = \frac{(a-c)bP_{2C}(b,n)}{\mathcal{D}_C(b,n)\mathcal{D}_S(b,n)}, \tag{42}$$

where

$$\begin{aligned} P_{2C}(b,n) = {} & -8b^6n + 32b^6 + b^5n^3 + 16b^5n^2 - 120b^5n - 64b^5 \\ & - 2b^4n^4 + 14b^4n^3 + 88b^4n^2 + 584b^4n - 288b^4 \\ & - 26b^3n^4 - 44b^3n^3 - 1008b^3n^2 + 432b^3n + 256b^3 \\ & + 128b^2n^4 + 336b^2n^3 + 1024b^2n^2 - 2080b^2n + 704b^2 \\ & - 168bn^4 - 864bn^3 + 1664bn^2 - 320bn - 256b \\ & + 64n^4 + 608n^3 - 1984n^2 + 1792n - 512. \end{aligned} \tag{43}$$

The Sturm sequence of $P_{2C}(b,\cdot)$ has no root in the common admissible interval. Since $P_{2C}(b,0)<0$ for all $b\in(0,1)$, we have $P_{2C}(b,n)<0$ throughout that region. Hence,

$$r_2^{S*} < r_i^{C*}. \tag{44}$$

Finally,

$$r_1^{S*} - r_2^{S*} = \frac{2(a-c)bP_{12}(b,n)}{\mathcal{D}_S(b,n)}, \tag{45}$$

where

$$\begin{aligned} P_{12}(b,n) = {} & \Big(5b^3+32b^2-112b+80\Big)\,n^2 \\ & + \Big(-4b^4-46b^3+84b^2+64b-112\Big)\,n \\ & + 16b^4-8b^3-48b^2+16b+32. \end{aligned} \tag{46}$$

Since $5b^3+32b^2-112b+80>0$ for $b\in(0,1)$, the discriminant is $4\Delta_2(b)$, where

$$\Delta_2(b) = 4b^8+12b^7-111b^6+228b^5-204b^4-144b^3+928b^2-1280b+576.$$

The two roots are

$$\frac{2b^4+23b^3-42b^2-32b+56\mp\sqrt{\Delta_2(b)}}{5b^3+32b^2-112b+80},$$

and only the smaller one belongs to the admissible interval. Thus,

$$\widehat{n}_2(b) = \frac{2b^4+23b^3-42b^2-32b+56-\sqrt{\Delta_2(b)}}{5b^3+32b^2-112b+80},$$

with

$$P_{12}(b,n) \gtreqless 0 \quad\Longleftrightarrow\quad n \lesseqgtr \widehat{n}_2(b).$$

Therefore,

$$r_1^{S*} \gtreqless r_2^{S*} \quad\Longleftrightarrow\quad n \lesseqgtr \widehat{n}_2(b). \tag{47}$$

Finally, the Sturm sequence of $P_{1C}\Big(b,\widehat{n}_2(b)\Big)$, or equivalently the resultant of $P_{1C}(b,n)$ and $P_{12}(b,n)$ with respect to n, establishes

$$0<\widehat{n}_1(b)<\widehat{n}_2(b)<1$$

throughout the common interior region. Combining (41), (44), and (47) proves Proposition 3.

$\square$

Appendix F: Proof of Proposition 4

Proof. In the common interior region, $a-c>0$, $b>0$, $\mathcal{D}_C(b,n)>0$, and $\mathcal{D}_S(b,n)>0$, where $\mathcal{D}_C(b,n)$ is defined in (38). Hence, the signs of all profit differences below are determined by their numerator polynomials.

Subtracting (11) from (26) gives

$$\Pi_1^{S*}-\Pi_i^{C*}=\frac{(a-c)^2 b\, G_{1C}(b,n)}{\mathcal{D}_C(b,n)^2\mathcal{D}_S(b,n)^2}, \tag{48}$$

where

$$\begin{aligned}
G_{1C}(b,n) = \; & -2b^{11}n^4+16b^{11}n^3+96b^{11}n^2-1024b^{11}n+2048b^{11}\\
& +4b^{10}n^5-64b^{10}n^4-304b^{10}n^3+4320b^{10}n^2-6016b^{10}n-13312b^{10}\\
& -b^9n^7+8b^9n^6+96b^9n^5-56b^9n^4-5264b^9n^3-9008b^9n^2\\
& \quad +93056b^9n+8448b^9\\
& +2b^8n^8-26b^8n^7-140b^8n^6+1808b^8n^5+2800b^8n^4+14560b^8n^3\\
& \quad -100032b^8n^2-341504b^8n+155648b^8\\
& +17b^7n^8+348b^7n^7-4436b^7n^6-10208b^7n^5+114528b^7n^4\\
& \quad -409408b^7n^3+1548544b^7n^2-625408b^7n-140288b^7\\
& -416b^6n^8+4952b^6n^7+26768b^6n^6-293344b^6n^5+907776b^6n^4\\
& \quad -1715456b^6n^3-1412096b^6n^2+2684416b^6n-675840b^6\\
& -1432b^5n^8-35056b^5n^7+244736b^5n^6-147456b^5n^5-1826304b^5n^4\\
& \quad +8191488b^5n^3-8242688b^5n^2+1525760b^5n+476160b^5\\
& +23040b^4n^8-42912b^4n^7-818624b^4n^6+3918720b^4n^5-7730176b^4n^4\\
& \quad +1289728b^4n^3+9451520b^4n^2-7354368b^4n+1392640b^4\\
& -80432b^3n^8+585920b^3n^7-735872b^3n^6-3650048b^3n^5+17054208b^3n^4\\
& \quad -25735168b^3n^3+14513152b^3n^2-1507328b^3n-655360b^3\\
& +127360b^2n^8-1309312b^2n^7+4973056b^2n^6-9047040b^2n^5+3621888b^2n^4\\
& \quad +12275712b^2n^3-17784832b^2n^2+8634368b^2n-1376256b^2\\
& -96768bn^8+1190400bn^7-5937920bn^6+16468992bn^5-26070016bn^4\\
& \quad +21954560bn^3-8323072bn^2+458752bn+327680b\\
& +28672n^8-395264n^7+2260992n^6-7282688n^5+14041088n^4\\
& \quad -16056320n^3+10551296n^2-3670016n+524288.
\end{aligned} \tag{49}$$

The Sturm sequence of $G_{1C}(b,\cdot)$ has no zero in the common interior region for any $b \in (0,1)$. Moreover,

$$\begin{aligned} G_{1C}(b,0) = 256(2-b^2)^2\Big(8b^7 - 52b^6 + 65b^5 + 400b^4 \\ - 320b^3 - 832b^2 + 320b + 512\Big) > 0, \end{aligned} \tag{50}$$

so $G_{1C}(b,n) > 0$ throughout the common interior region. Hence,

$$\Pi_1^{S*} > \Pi_i^{C*}. \tag{51}$$

Similarly,

$$\Pi_2^{S*} - \Pi_i^{C*} = \frac{(a-c)^2 b\, G_{2C}(b,n)}{\mathcal{D}_C(b,n)^2 \mathcal{D}_S(b,n)^2}, \tag{52}$$

where

$$
\begin{aligned}
G_{2C}(b,n) = {} & b^{11}n^4 - 128b^{11}n^2 + 768b^{11}n - 1280b^{11} \\
& - 2b^{10}n^5 + 8b^{10}n^4 + 512b^{10}n^3 - 2624b^{10}n^2 - 2176b^{10}n + 17920b^{10} \\
& - b^9n^7 - b^9n^6 + 112b^9n^5 - 928b^9n^4 - 5280b^9n^3 + 74272b^9n^2 \\
& \quad - 161536b^9n - 10752b^9 \\
& + 2b^8n^8 - 14b^8n^7 - 420b^8n^6 + 2208b^8n^5 + 26016b^8n^4 - 235968b^8n^3 \\
& \quad + 314688b^8n^2 + 512000b^8n - 223232b^8 \\
& + 29b^7n^8 + 396b^7n^7 - 3468b^7n^6 - 32272b^7n^5 + 242976b^7n^4 \\
& \quad + 443328b^7n^3 - 2958272b^7n^2 + 1370112b^7n + 107520b^7 \\
& - 160b^6n^8 + 2680b^6n^7 + 29952b^6n^6 - 70400b^6n^5 - 1746880b^6n^4 \\
& \quad + 5070592b^6n^3 + 504320b^6n^2 - 3876352b^6n + 1038336b^6 \\
& - 1592b^5n^8 - 26096b^5n^7 - 27920b^5n^6 + 1532416b^5n^5 - 982976b^5n^4 \\
& \quad - 12204544b^5n^3 + 15950080b^5n^2 - 4410368b^5n - 292864b^5 \\
& + 13504b^4n^8 + 65312b^4n^7 - 472704b^4n^6 - 3529984b^4n^5 + 16258560b^4n^4 \\
& \quad - 10976256b^4n^3 - 10401792b^4n^2 + 11210752b^4n - 2310144b^4 \\
& - 40048b^3n^8 - 51008b^3n^7 + 2073728b^3n^6 - 2042112b^3n^5 - 18999040b^3n^4 \\
& \quad + 43412480b^3n^3 - 30681088b^3n^2 + 6291456b^3n + 327680b^3 \\
& + 58496b^2n^8 - 33920b^2n^7 - 3624448b^2n^6 + 15221760b^2n^5 - 16362496b^2n^4 \\
& \quad - 9101312b^2n^3 + 25731072b^2n^2 - 14516224b^2n + 2490368b^2 \\
& - 42496bn^8 + 73216bn^7 + 2932992bn^6 - 17540096bn^5 + 39474176bn^4 \\
& \quad - 41418752bn^3 + 20021248bn^2 - 3342336bn - 131072b \\
& + 12288n^8 - 30720n^7 - 909312n^6 + 6479872n^5 - 18006016n^4 \\
& \quad + 25231360n^3 - 18808832n^2 + 7077888n - 1048576.
\end{aligned}
\tag{53}
$$

For every $b \in (0,1)$, the Sturm sequence of $G_{2C}(b,\cdot)$ shows that $G_{2C}(b,n) = 0$ has a unique root in the common interior region, denoted by $\widehat{n}_3(b)$. Since

$$
\begin{aligned}
G_{2C}(b,0) = {} & -256(2-b^2)^2(5b^2-8) \\
& \times \left(b^5 - 14b^4 + 14b^3 + 96b^2 - 16b - 128\right) < 0,
\end{aligned}
\tag{54}
$$

the sign changes from negative to positive at $\widehat{n}_3(b)$. Thus,

$$
\Pi_2^{S*} \gtreqless \Pi_i^{C*} \quad \Longleftrightarrow \quad n \gtreqless \widehat{n}_3(b). \tag{55}
$$

The difference between the Stackelberg profits is

$$\Pi_1^{S*} - \Pi_2^{S*} = \frac{(a-c)^2 b\, G_{12}(b,n)}{\mathcal{D}_S(b,n)^2}, \tag{56}$$

where

$$\begin{aligned} G_{12}(b,n) = \; & -3b^7n^4 + 16b^7n^3 + 224b^7n^2 - 1792b^7n + 3328b^7 \\ & - 64b^6n^4 - 240b^6n^3 + 5152b^6n^2 - 11520b^6n - 4608b^6 \\ & + 40b^5n^4 - 4144b^5n^3 + 2224b^5n^2 + 46720b^5n - 17664b^5 \\ & + 2384b^4n^4 + 11648b^4n^3 - 79744b^4n^2 + 13312b^4n + 24576b^4 \\ & - 10096b^3n^4 + 20864b^3n^3 + 115136b^3n^2 - 152320b^3n + 30720b^3 \\ & + 17216b^2n^4 - 98688b^2n^3 + 71168b^2n^2 + 66560b^2n - 43008b^2 \\ & - 13568bn^4 + 112384bn^3 - 221952bn^2 + 132096bn - 17408b \\ & + 4096n^4 - 41984n^3 + 108544n^2 - 94208n + 24576. \end{aligned} \tag{57}$$

The Sturm sequence of $G_{12}(b,\cdot)$ shows that $G_{12}(b,n)=0$ has a unique root in the common interior region, denoted by $\widehat{n}_4(b)$. Moreover,

$$G_{12}(b,0) = 256(2-b^2)^2\left(13b^3 - 18b^2 - 17b + 24\right) > 0, \tag{58}$$

since $13b^3 - 18b^2 - 17b + 24 > 0$ for $b \in (0,1)$. Hence,

$$G_{12}(b,n) \gtreqless 0 \quad \Longleftrightarrow \quad n \lesseqgtr \widehat{n}_4(b),$$

and therefore

$$\Pi_1^{S*} \gtreqless \Pi_2^{S*} \quad \Longleftrightarrow \quad n \lesseqgtr \widehat{n}_4(b). \tag{59}$$

Finally, the numerator polynomials satisfy

$$G_{1C}(b,n) - G_{2C}(b,n) = \mathcal{D}_C(b,n)^2 G_{12}(b,n). \tag{60}$$

At $n = \widehat{n}_3(b)$, where $G_{2C}(b, \widehat{n}_3(b)) = 0$,

$$G_{12}\Big(b, \widehat{n}_3(b)\Big) = \frac{G_{1C}\Big(b, \widehat{n}_3(b)\Big)}{\mathcal{D}_C\Big(b, \widehat{n}_3(b)\Big)^2} > 0.$$

Since $G_{12}(b,n)$ has a unique admissible root and is positive below it, $\widehat{n}_3(b) < \widehat{n}_4(b)$. Together with (51), (55), and (59), this proves Proposition 4.

□

References

Ai, Y. and Lu, T. Y. (2019). On the rationality of bundled rebate program in modem chip industry: An analysis on qualcomm's case. *Journal of Industry, Competition and Trade*, 19(4):641–660.

Arai, Y. and Matsushima, N. (2023). The impacts of suppliers and mutual outsourcing on organizational forms. *Canadian Journal of Economics/Revue canadienne d'économique*, 56(1):114–132.

Arya, A., Mittendorf, B., and Sappington, D. E. (2008a). The make-or-buy decision in the presence of a rival: Strategic outsourcing to a common supplier. *Management Science*, 54(10):1747–1758.

Arya, A., Mittendorf, B., and Sappington, D. E. (2008b). Outsourcing, vertical integration, and price vs. quantity competition. *International Journal of Industrial Organization*, 26(1):1–16.

Buccella, D., Fanti, L., and Gori, L. (2023). Network externalities, product compatibility and process innovation. *Economics of Innovation and New Technology*, 32(8):1156–1189.

Buccella, D., Fanti, L., and Gori, L. (2025a). Cross-ownership in network industries: when less competition implies less profits or more social welfare. *Journal of Economics*, 144(3):203–230.

Buccella, D., Fanti, L., and Gori, L. (2025b). Endogenous partial cross-ownership in network industries: D. buccella et al. *Review of Economic Design*, pages 1–25.

Buccella, D., Fanti, L., and Gori, L. (2025c). Is optimal interlocking cross-ownership for the network industry? *Managerial and Decision Economics*, 46(6):3520–3526.

Chang, R.-Y., Hu, J.-L., and Lin, Y.-S. (2018). The choice of prices versus quantities under outsourcing. *The BE Journal of Theoretical Economics*, 18(2):20160195.

Chen, Y. (2010). Strategic outsourcing between rivals. *Annals of Economics and Finance*, 11:301–311.

Chirco, A. and Scrimitore, M. (2013). Choosing price or quantity? the role of delegation and network externalities. *Economics Letters*, 121(3):482–486.

Choi, K. and Lee, K.-D. (2025). Mutual outsourcing between downstream firms on endogenous competition. *The Manchester School*, 93(5):434–448.

Fanti, L. and Buccella, D. (2018). Profitability of corporate social responsibility in network industries. *International Review of Economics*, 65:271–289.

Hoernig, S. (2012). Strategic delegation under price competition and network effects. *Economics Letters*, 117(2):487–489.

Katz, M. L. and Shapiro, C. (1985). Network externalities, competition, and compatibility. *The American Economic Review*, 75(3):424–440.

Leswing, K. (2019). Every high-end android phone will support 5g next year, qualcomm says. CNBC. https://www.cnbc.com/2019/12/04/qualcomm-announces-snapdragon-865-and-765-chipset-for-android-phones.html.

Milliou, C. and Serfes, K. (2025). Complementary inputs, mutual outsourcing, and supplier encroachment. *The Journal of Industrial Economics*, 73(3):458–479.

Namaiwa Barke, E. (2026). Strategic CSR in asymmetric network duopoly. *Journal of Economics*, 147:8.

Pun, H. (2015). The more the better? optimal degree of supply-chain cooperation between competitors. *Journal of the Operational Research Society*, 66(12):2092–2101.

Srivastava, S. (2017). Qualcomm leads smartphone soc market; fueled from adoption by fast growing chinese brands. Counterpoint Research. https://counterpointresearch.com/en/insights/qualcomm-leads-smartphone-soc-market-fueled-adoption-fast-growing-chinese-brands.

Wang, Y., Deng, T., and Cheng, H. (2024). A study of production outsourcing strategies of dual oligopoly manufacturers based on quality investments. *Frontiers in Physics*, 12:1334698.

Xu, L., Fan, X., and Lee, S.-H. (2026). Mutual outsourcing in a vertically related market with strategic environmental corporate social responsibility. *The Manchester School*, pages 1–18.