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Cost asymmetry and the R&D investment-decision game

Emanuel Namaiwa Barke* Khaireddine Jebsi†

Abstract

This paper studies the R&D investment-decision game in a Cournot duopoly with ex ante asymmetric production costs. Firms first decide whether to participate in process R&D and, conditional on participation, subsequently choose their cost-reducing R&D efforts. Despite cost heterogeneity, investing is a strictly dominant strategy for both firms, so bilateral R&D is the unique participation equilibrium. Cost asymmetry nevertheless changes the nature of this outcome. When firms are sufficiently similar, bilateral R&D may constitute a prisoner's dilemma; as the initial cost gap widens, the dilemma breaks down asymmetrically, with the initially efficient firm preferring bilateral investment while the inefficient firm continues to prefer mutual non-investment. Cost asymmetry also reallocates R&D effort toward the initially efficient firm. The welfare analysis reveals a further wedge between private and social incentives: although bilateral R&D maximizes aggregate R&D and consumer surplus, social surplus may be higher when innovation is concentrated in the initially efficient firm. Extending the model to technological spillovers leaves bilateral participation as the unique equilibrium but changes the private desirability of joint R&D.

Keywords: Process R&D; R&D investment-decision game; Cost asymmetry; Cournot duopoly; Prisoner's dilemma; Technological spillovers.

JEL codes: C72; L13; O31; O32.

1 Introduction

Firms competing in the same product market are rarely identical. They may differ in size, productive efficiency, organizational capabilities, technology, and production costs. Such

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heterogeneity can be substantial even among firms supplying closely related products. In European air transport, for instance, a European Commission report comparing easyJet and bmi reported total operating costs of approximately 4.5 and 6.3 pence per available seat-kilometre, respectively, despite both carriers being primarily active in intra-European markets.[1] Persistent productivity differences are also observed within narrowly defined manufacturing industries. Using plant-level data from the U.S. ready-mixed concrete industry, Syverson (2004) documents substantial productivity dispersion among producers operating in closely related local markets.[2] These examples illustrate a simple but important feature of oligopolistic competition: rivals can enter the same strategic interaction from markedly different cost positions.

Cost asymmetry is particularly relevant when firms can innovate. Process R&D allows a firm to reduce its production cost, but the return to doing so depends on its initial competitive position and on the innovation undertaken by its rival. An initially efficient firm may use R&D to reinforce an existing cost advantage, whereas an initially inefficient firm may invest to narrow the technological gap. Cost heterogeneity may therefore affect not only the *amount* of R&D undertaken by each firm, but also the more basic decision of *whether* engaging in R&D is privately worthwhile.

The strategic process-R&D literature has largely developed from the framework of d'Aspremont and Jacquemin (1988, 1990). In their canonical model, ex ante identical firms first choose cost-reducing R&D efforts and subsequently compete in quantities, with technological spillovers transmitting part of each firm's innovation to its rival. Subsequent contributions have clarified the stability of the non-cooperative R&D equilibrium (Henriques, 1990), extended the analysis to more general oligopoly environments (Suzumura, 1992), and examined research joint ventures and alternative forms of R&D cooperation (Kamien et al., 1992). In this literature, however, participation in R&D is generally taken as given: the strategic margin is how much to invest conditional on engaging in innovative activity.

Bacchiega et al. (2010) introduce an extensive margin by adding a preliminary R&D investment decision to the d'Aspremont–Jacquemin game. With symmetric firms, investing is a dominant strategy and bilateral R&D is the unique equilibrium, although sufficiently efficient R&D and weak spillovers can make mutual non-investment jointly more profitable, generating a prisoner's dilemma. Relatedly, Burr et al. (2013) show that the prisoner's-dilemma outcome in the standard R&D–Cournot framework can coexist with gains for consumers and society, while Ramsza et al. (2021) demonstrate that introducing R&D entry costs can generate alternative social-dilemma structures, including a chicken game. More recently, Buccella et al. (2023b) show that horizontal product dif-

[1] See European Commission, *Airline Business Models*, 2008, Figure 2-3, available here.
[2] See the publisher's page for Syverson (2004), available here.

ferentiation can eliminate the prisoner's dilemma: bilateral R&D may become privately Pareto preferred even in the absence of technological spillovers. Despite these extensions, the investment-decision literature remains largely centered on ex ante symmetric firms.

A separate strand shows that firm heterogeneity can itself shape innovation incentives. Poyago-Theotoky (1996) studies process R&D between firms with different initial production costs and shows that the high-cost firm may invest either more or less than its low-cost rival, depending on the balance between an incentive effect favoring the efficient firm and an effectiveness effect favoring the inefficient firm. In asymmetric Cournot environments, Lahiri and Ono (1999) emphasize the welfare consequences of initial cost advantages, while Ishida et al. (2011) show that low- and high-cost firms can respond differently to competitive pressure. Ishikawa and Shibata (2020) explicitly combine production-cost asymmetry with R&D spillovers, and Saglam (2022) examines process R&D in an asymmetric homogeneous-good duopoly under alternative modes of market competition. These contributions focus primarily on the intensive margin of R&D.

A notable exception is Ferrett (2012), who considers an asymmetric Cournot duopoly in which each firm makes a discrete process-R&D decision. Investment entails a firm-specific fixed cost and moves the firm to an exogenously specified post-innovation unit cost. Thus, while that framework endogenizes whether heterogeneous firms adopt a cost-reducing technology, it does not combine this extensive margin with a subsequent endogenous choice of R&D intensity. This distinction is central here. We allow firms with different initial marginal costs first to decide *whether* to participate in process R&D and, conditional on participation, subsequently to choose *how much* cost-reducing R&D to undertake. This makes it possible to study jointly the endogenous participation pattern, the distribution of R&D effort across heterogeneous firms, and the welfare consequences of allocating innovation to firms with different initial efficiencies.

Against this background, we ask two related questions. First, does ex ante cost asymmetry alter firms' endogenous incentives to participate in process R&D and the private desirability of bilateral innovation? Second, does the privately selected R&D participation pattern coincide with the socially preferred allocation of innovative activity?

To address these questions, we develop a three-stage complete-information Cournot duopoly with homogeneous products and ex ante asymmetric marginal costs. Firm 1 is initially efficient, whereas firm 2 starts with a cost disadvantage. At Stage 0, firms simultaneously decide whether to participate in process R&D, generating the four configurations NN, IN, NI, and II. At Stage 1, each participating firm chooses its cost-reducing R&D effort subject to a quadratic investment cost, and at Stage 2 firms compete in quantities. We first abstract from technological spillovers to isolate the role of initial cost asymmetry and subsequently reintroduce spillovers as a robustness extension.

The analysis delivers four main results. First, cost asymmetry does not overturn the endogenous emergence of bilateral innovation: investing remains a strictly dominant strategy for both firms, so II is the unique participation equilibrium. In this respect, the dominance result of Bacchiega et al. (2010) and Buccella et al. (2023b) extends to initially heterogeneous firms. Asymmetry nevertheless changes the private desirability of this equilibrium. When firms are sufficiently similar, II retains the prisoner's-dilemma structure, but a larger initial cost gap produces a *one-sided breakdown*: the efficient firm comes to prefer II to NN, while the inefficient firm continues to prefer mutual non-investment. Thus, unlike the symmetric transition to a deadlock generated by product differentiation in Buccella et al. (2023b), cost asymmetry changes the firms' rankings in different directions.

Second, cost asymmetry systematically reallocates innovative activity toward the initially efficient firm. Its equilibrium R&D effort increases with the initial cost gap, whereas the inefficient firm's effort decreases. This provides a sharper prediction than the ambiguous ranking identified by Poyago-Theotoky (1996), where the high-cost firm may invest either more or less depending on the relative strength of incentive and effectiveness effects. Moreover, each firm invests more when innovating alone than when its rival also invests, although aggregate R&D remains highest under bilateral innovation.

Third, private participation incentives need not be socially efficient. Although II generates the largest total output and consumer surplus, unilateral R&D by the initially efficient firm can yield higher social surplus because the additional investment undertaken by the inefficient firm entails a resource cost that need not be matched by a sufficiently large productive gain. This contrasts with Burr et al. (2013), for whom the standard R&D prisoner's dilemma benefits both consumers and society. Here, cost heterogeneity makes the *allocation* of innovative activity across firms relevant for welfare: the socially preferred configuration may be IN even though the private participation game selects II.

Fourth, the dominance of R&D participation is robust to technological spillovers. Bilateral R&D remains the unique equilibrium, but spillovers reshape its private desirability. With weak spillovers, the baseline prisoner's dilemma and its one-sided breakdown survive; sufficiently strong spillovers instead make bilateral innovation privately preferable to mutual non-investment for both firms. Hence, extending the symmetric spillover results of Bacchiega et al. (2010), the interaction between spillovers and initial cost heterogeneity determines not only whether the bilateral-investment outcome is jointly desirable, but also whether the two firms evaluate that outcome differently.

The remainder of the paper is organized as follows. Section 2 presents the model and the timing of the R&D participation game. Section 3 derives the equilibrium under the

four participation configurations and characterizes the endogenous participation equilibrium. Section 4 extends the analysis to technological spillovers. Section 5 concludes. Appendix A provides the derivations and proofs of the baseline results, while Appendix B reports the equilibrium outcomes and proofs for the spillover extension.

2 The model

In line with d'Aspremont and Jacquemin (1988, 1990), we consider a non-cooperative cost-reducing R&D game between two quantity-setting firms, indexed by $i \in \{1, 2\}$, producing a homogeneous good. We augment the standard R&D–Cournot framework with a preliminary participation decision, so that firms first decide whether to engage in process innovation, then choose their R&D effort conditional on participation, and finally compete in quantities.

Following the standard linear Cournot specification in Dixit (1979), which corresponds to the perfect-substitutes case of Singh and Vives (1984), inverse market demand is

$$p = a - \gamma(q_i + q_j), \qquad i, j \in \{1, 2\}, \quad i \neq j, \tag{1}$$

where p denotes the market price, q_i denotes the output of firm i, a captures market size, and $\gamma > 0$ measures the slope of inverse demand. For notational tractability, we normalize $\gamma = 1$ throughout. This normalization is without loss of generality, since γ can be absorbed into the units in which quantities are measured and, correspondingly, into the R&D cost parameter.

Unlike the symmetric-cost benchmark commonly considered in the R&D investment-decision literature (d'Aspremont and Jacquemin, 1990; Henriques, 1990; Burr et al., 2013; Buccella et al., 2023a), firms are ex ante asymmetric in production costs. We label firm 1 as the initially efficient firm and firm 2 as the initially inefficient firm. Their pre-innovation marginal costs are $c_1 = c_0$ and $c_2 = c_0 + c$, where $c_0 > 0$ is firm 1's initial marginal cost and $c > 0$ measures firm 2's initial cost disadvantage. For later use, we define $A \equiv a - c_0 > 0$, which measures market size net of firm 1's initial marginal cost.

If firm i engages in process R&D, it chooses an effort $x_i \geq 0$ that reduces its own marginal production cost one-for-one. Total production cost is therefore

$$C_i(q_i, x_i) = (c_i - x_i)q_i, \tag{2}$$

so that R&D undertaken by one firm does not affect the rival's production cost.[3] The cost of R&D effort is

$$X_i(x_i) = \frac{k}{2}x_i^2, \qquad k > 0. \tag{3}$$

Participation in R&D entails no additional fixed cost: an investing firm only incurs the effort cost in Equation (3). The parameter k captures the cost of the available R&D technology: a lower value of k makes a given cost reduction less expensive and therefore corresponds to a more efficient process-innovation technology. We restrict attention to economically meaningful outcomes in which post-innovation marginal costs remain positive, i.e., $c_i - x_i > 0$.[4]

Hence, conditional on R&D effort, firm i's profit is given by

$$\Pi_i = [a - q_i - q_j - c_i + x_i]\, q_i - \frac{k}{2}x_i^2, \tag{4}$$

whenever firm i engages in R&D. A non-investing firm sets $x_i = 0$ and bears no R&D cost.

Since the product is homogeneous and demand is linear, consumer surplus and social surplus are respectively given by [5]

$$CS = \frac{1}{2}Q^2 \qquad \text{and} \qquad SS = CS + \Pi_1 + \Pi_2, \tag{5}$$

where $Q = q_1 + q_2$ denotes total industry output. Because equilibrium profits are defined net of R&D expenditures, Equation (5) already accounts for the resource cost of innovation.

The strategic interaction is modeled as a three-stage non-cooperative game under complete information. At Stage 0, firms simultaneously decide whether to invest in process R&D. The resulting participation profile is publicly observed before Stage 1, at which investing firms choose their R&D efforts. These choices are in turn observed before Stage 2, where firms compete in quantities. We use I to denote participation in R&D and N to denote non-participation; the first (second) letter of a configuration refers to firm 1 (firm 2). Because firms are ex ante asymmetric, the unilateral-investment configurations IN and NI are economically distinct. Figure 1 summarizes the timing of the game.

[3] This no-spillover specification allows us to isolate the strategic consequences of ex ante cost asymmetry. Section 4 relaxes this assumption by allowing a fraction of each firm's R&D effort to reduce the rival's marginal cost.

[4] This is a standard feasibility requirement in models of cost-reducing process R&D; see, for instance, d'Aspremont and Jacquemin (1988), Bacchiega et al. (2010), and Buccella et al. (2023b). It rules out parameter configurations in which R&D would drive the firm's marginal production cost to zero or below.

[5] More generally, under $p(Q) = a - \gamma Q$, consumer surplus is $CS = \int_0^Q (a - \gamma z)\, dz - p(Q)Q = \frac{\gamma}{2}Q^2$. The expression in Equation (5) follows from the normalization $\gamma = 1$.

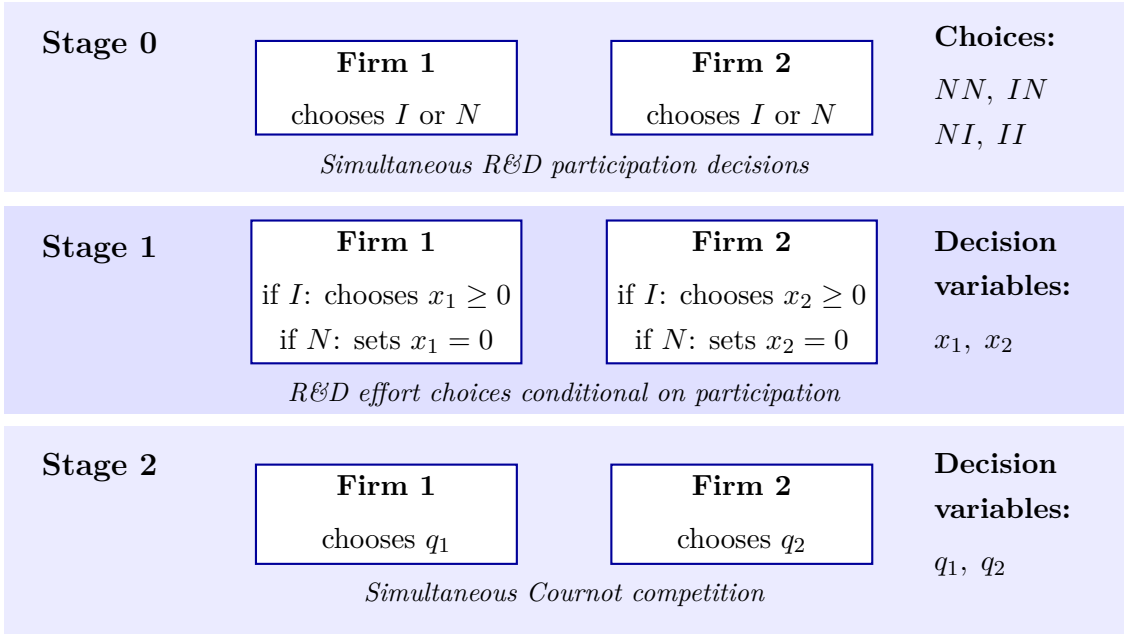


Figure 1: Structure and timing of the R&D investment-decision game.

3 Analysis

The game is solved by backward induction. We first characterize the continuation equilibrium associated with each participation profile selected at Stage 0, namely NN, IN, NI, and II. We then use the resulting equilibrium profits to solve the endogenous R&D participation game at Stage 0.

The following assumption is maintained throughout (see Appendix A for its derivation).

Assumption 1. *We assume that (i)* $A \equiv a - c_0 > 0$*, (ii)* $k > 4/3$*, and (iii)* $\frac{4(A+c)}{9k-8} < c_0$.

Part (i) guarantees a positive net market size for the initially efficient firm. Part (ii) ensures the concavity of the R&D problems and a unique interior equilibrium of the bilateral R&D game. Part (iii) is a sufficient feasibility condition, ensuring that cost-reducing R&D does not drive post-innovation marginal costs below zero in any of the configurations considered below.

3.1 Equilibrium configurations

Throughout this subsection, we derive the equilibrium associated with each participation configuration.

3.1.1 The NN configuration

When neither firm invests in R&D, $x_1 = x_2 = 0$, so that only the market stage remains. The first-order conditions $\partial \Pi_i^{NN}/\partial q_i = 0$ yield

$$\begin{aligned} q_1^{NN} &= \frac{A+c}{3}, \qquad q_2^{NN} = \frac{A-2c}{3}, \qquad Q^{NN} = \frac{2A-c}{3}, \\ \Pi_1^{NN} &= \frac{(A+c)^2}{9}, \qquad \Pi_2^{NN} = \frac{(A-2c)^2}{9}, \\ CS^{NN} &= \frac{(2A-c)^2}{18}, \qquad SS^{NN} = \frac{8A^2-8Ac+11c^2}{18}. \end{aligned} \tag{6}$$

The equilibrium in Equation (6) is interior only if the initially inefficient firm remains active in the market. This yields the following condition.

Lemma 1. *In the NN configuration, the equilibrium is interior if and only if $0 < c < \bar{c}^{NN} \equiv A/2$. At $c = \bar{c}^{NN}$, the initially inefficient firm's equilibrium output falls to zero.*

Proof. Since $q_2^{NN} = (A-2c)/3$, we have $q_2^{NN} > 0$ if and only if $c < A/2$, with $q_2^{NN} = 0$ at $c = A/2$. □

Hence, when the initial cost disadvantage is sufficiently large, the initially inefficient firm exits the market even in the absence of R&D.

Within the interior region $0 < c < \bar{c}^{NN}$, Equation (6) shows that cost asymmetry reallocates production towards the initially efficient firm. Indeed, $q_1^{NN} - q_2^{NN} = c > 0$, $\partial q_1^{NN}/\partial c = 1/3 > 0$, $\partial q_2^{NN}/\partial c = -2/3 < 0$, and $\partial Q^{NN}/\partial c = -1/3 < 0$. Accordingly, $\partial \Pi_1^{NN}/\partial c = 2(A+c)/9 > 0$, $\partial \Pi_2^{NN}/\partial c = -4(A-2c)/9 < 0$, and $\partial CS^{NN}/\partial c = -(2A-c)/9 < 0$. Finally, $\partial SS^{NN}/\partial c = (11c-4A)/9$, so social surplus decreases with cost asymmetry for $c < 4A/11$ and increases for $c > 4A/11$.

3.1.2 The IN configuration

Consider the configuration in which the initially efficient firm invests in R&D, whereas the initially inefficient firm does not. Thus, $x_1 \geq 0$ and $x_2 = 0$. At Stage 2, the first-order conditions $\partial \Pi_i^{IN}/\partial q_i = 0$ yield

$$q_1^{IN}(x_1) = \frac{A+c+2x_1}{3}, \tag{7}$$

and

$$q_2^{IN}(x_1) = \frac{A-2c-x_1}{3}. \tag{8}$$

Substituting Equations (7) and (8) into the profit functions gives the continuation profits

$$\Pi_1^{IN}(x_1) = \frac{(A+c+2x_1)^2}{9} - \frac{k}{2}x_1^2, \tag{9}$$

and

$$\Pi_2^{IN}(x_1) = \frac{(A-2c-x_1)^2}{9}. \tag{10}$$

At Stage 1, firm 1 chooses its R&D effort to maximize $\Pi_1^{IN}(x_1)$. Hence,

$$\frac{\partial \Pi_1^{IN}}{\partial x_1} = \frac{4(A+c+2x_1)}{9} - kx_1. \tag{11}$$

Setting Equation (11) equal to zero gives the following result.

Lemma 2. *Under Assumption 1, the optimal R&D effort of the initially efficient firm in the IN configuration is*

$$x_1^{IN*} = \frac{4(A+c)}{9k-8}.$$

Proof. Straightforward from $\partial \Pi_1^{IN}/\partial x_1 = 0$. Moreover, $\partial^2 \Pi_1^{IN}/\partial x_1^2 = 8/9 - k < 0$ under Assumption 1. □

Lemma 2 shows that the optimal R&D effort of the initially efficient firm increases with the initial cost asymmetry, since $\partial x_1^{IN*}/\partial c = 4/(9k-8) > 0$. By contrast, a higher R&D cost parameter reduces innovation effort, $\partial x_1^{IN*}/\partial k < 0$.

Substituting the optimal R&D effort from Lemma 2 into Equations (7)–(10) yields the equilibrium outcomes under the IN configuration:

$$\begin{aligned}
q_1^{IN*} &= \frac{3k(A+c)}{9k-8}, \qquad q_2^{IN*} = \frac{3k(A-2c)-4(A-c)}{9k-8}, \qquad Q^{IN*} = \frac{3k(2A-c)-4(A-c)}{9k-8}, \\
\Pi_1^{IN*} &= \frac{k(A+c)^2}{9k-8}, \qquad \Pi_2^{IN*} = \left[\frac{3k(A-2c)-4(A-c)}{9k-8}\right]^2, \\
CS^{IN*} &= \frac{1}{2}\left[\frac{3k(2A-c)-4(A-c)}{9k-8}\right]^2, \\
SS^{IN*} &= \frac{(72k^2-112k+48)A^2 + (-72k^2+184k-96)Ac + (99k^2-136k+48)c^2}{2(9k-8)^2}.
\end{aligned} \tag{12}$$

The equilibrium in Equation (12) requires firm 2 to remain active after firm 1's R&D choice. This imposes the following restriction.

Lemma 3. *Under Assumption 1, the IN equilibrium is interior if and only if* $0 < c < \bar{c}^{IN} \equiv A(3k-4)/[2(3k-2)]$. *At* $c = \bar{c}^{IN}$, *firm 2's equilibrium output becomes zero.*

Proof. From $q_2^{IN*} = [3k(A-2c) - 4(A-c)]/(9k-8)$, we have $q_2^{IN*} > 0$ if and only if $c < A(3k-4)/[2(3k-2)]$, with equality at the threshold. □

Since $\bar{c}^{IN} < \bar{c}^{NN} = A/2$, R&D by the initially efficient firm tightens the condition under which its less efficient rival remains active. Thus, firm 1's innovation brings firm 2 to the market-exit boundary at a lower degree of initial cost asymmetry than under NN.

Within the interior region $0 < c < \bar{c}^{IN}$, the effect of the initial cost asymmetry on the equilibrium outcomes is summarized below.

Proposition 1. *Under Assumption 1 and $0 < c < \bar{c}^{IN}$, we have*

$$\frac{\partial x_1^{IN*}}{\partial c} > 0, \qquad \frac{\partial \Pi_1^{IN*}}{\partial c} > 0, \qquad \frac{\partial \Pi_2^{IN*}}{\partial c} < 0, \qquad \frac{\partial CS^{IN*}}{\partial c} < 0.$$

Moreover, if $k \leq (23+\sqrt{97})/18$, then $\partial SS^{IN*}/\partial c > 0$. If $k > (23+\sqrt{97})/18$, then

$$\frac{\partial SS^{IN*}}{\partial c} \gtreqless 0 \quad \Longleftrightarrow \quad c \gtreqless \tilde{c}^{IN}(k),$$

where

$$\tilde{c}^{IN}(k) \equiv \frac{4A(9k^2-23k+12)}{99k^2-136k+48} \in (0, \bar{c}^{IN}).$$

Proof. See Appendix A. □

Proposition 1 shows that a larger initial cost asymmetry strengthens the R&D incentive of the initially efficient firm and raises its equilibrium profit, while reducing the profit of its inefficient rival and consumer surplus. The mechanism is straightforward: as c increases, firm 1's cost advantage becomes more valuable, which induces a larger cost-reducing investment and shifts market activity further towards the efficient firm. At the same time, the weaker competitive position of firm 2 reduces total output, thereby lowering consumer surplus.

The effect on social surplus reflects the balance between these two forces. When $k \leq (23+\sqrt{97})/18$, the productive-efficiency gain generated by the reallocation of output and the stronger R&D effort dominates, so that social surplus increases with c. When R&D is sufficiently costly, $k > (23+\sqrt{97})/18$, this effect is no longer uniform: social surplus decreases for $c < \tilde{c}^{IN}(k)$ and increases for $c > \tilde{c}^{IN}(k)$. Hence, the threshold $\tilde{c}^{IN}(k)$ identifies the level of cost asymmetry at which the productive-efficiency effect overtakes the output-loss effect.

3.1.3 The NI configuration

We now consider the configuration in which the initially inefficient firm invests in R&D, whereas the initially efficient firm does not. Thus, $x_1 = 0$ and $x_2 \geq 0$. At Stage 2, the first-order conditions $\partial \Pi_i^{NI}/\partial q_i = 0$ yield

$$q_1^{NI}(x_2) = \frac{A + c - x_2}{3}, \tag{13}$$

and

$$q_2^{NI}(x_2) = \frac{A - 2c + 2x_2}{3}. \tag{14}$$

Substituting Equations (13) and (14) into the profit functions gives the continuation profits

$$\Pi_1^{NI}(x_2) = \frac{(A + c - x_2)^2}{9}, \tag{15}$$

and

$$\Pi_2^{NI}(x_2) = \frac{(A - 2c + 2x_2)^2}{9} - \frac{k}{2}x_2^2. \tag{16}$$

At Stage 1, firm 2 chooses its R&D effort to maximize $\Pi_2^{NI}(x_2)$. Hence,

$$\frac{\partial \Pi_2^{NI}}{\partial x_2} = \frac{4(A - 2c + 2x_2)}{9} - kx_2. \tag{17}$$

Setting Equation (17) equal to zero yields the following interior solution.

Lemma 4. *Under Assumption 1, the optimal R&D effort of the initially inefficient firm in an interior NI configuration is*

$$x_2^{NI*} = \frac{4(A - 2c)}{9k - 8}.$$

Proof. Straightforward from $\partial \Pi_2^{NI}/\partial x_2 = 0$. Moreover, $\partial^2 \Pi_2^{NI}/\partial x_2^2 = 8/9 - k < 0$ under Assumption 1. □

Lemma 4 shows that cost asymmetry weakens the initially inefficient firm's incentive to innovate, since $\partial x_2^{NI*}/\partial c = -8/(9k - 8) < 0$. Thus, a larger initial cost disadvantage reduces the return to cost-reducing R&D for the initially inefficient firm.

Substituting the optimal R&D effort from Lemma 4 into Equations (13)–(16) yields

the equilibrium outcomes under the NI configuration:

$$\begin{aligned}
q_1^{NI*} &= \frac{3k(A+c)-4A}{9k-8}, \qquad q_2^{NI*} = \frac{3k(A-2c)}{9k-8}, \qquad Q^{NI*} = \frac{3k(2A-c)-4A}{9k-8}, \\
\Pi_1^{NI*} &= \left[\frac{3k(A+c)-4A}{9k-8}\right]^2, \qquad \Pi_2^{NI*} = \frac{k(A-2c)^2}{9k-8}, \\
CS^{NI*} &= \frac{1}{2}\left[\frac{3k(2A-c)-4A}{9k-8}\right]^2, \\
SS^{NI*} &= \frac{(72k^2-112k+48)A^2+(-72k^2+40k)Ac+(99k^2-64k)c^2}{2(9k-8)^2}.
\end{aligned} \tag{18}$$

The NI equilibrium remains interior only while the initially inefficient firm continues to produce after undertaking R&D. This gives the following condition.

Lemma 5. *Under Assumption 1, the NI equilibrium is interior if and only if $0 < c < \bar{c}^{NI} \equiv A/2$. At $c = \bar{c}^{NI}$, firm 2's equilibrium output is zero. Moreover, $\bar{c}^{NI} = \bar{c}^{NN}$.*

Proof. Since $q_2^{NI*} = 3k(A-2c)/(9k-8)$, we have $q_2^{NI*} > 0$ if and only if $c < A/2$, with $q_2^{NI*} = 0$ at $c = A/2$. □

The equality $\bar{c}^{NI} = \bar{c}^{NN}$ follows from the fact that q_2^{NI*} remains proportional to $A-2c$. Thus, although firm 2's own R&D increases its output whenever it is active, it does not shift the cost-asymmetry threshold at which the firm leaves the market.

Within this interior region, the effect of the initial cost asymmetry on the equilibrium outcomes is summarized below.

Proposition 2. *Under Assumption 1 and $0 < c < \bar{c}^{NI}$, we obtain*

$$\frac{\partial x_2^{NI*}}{\partial c} < 0, \qquad \frac{\partial \Pi_1^{NI*}}{\partial c} > 0, \qquad \frac{\partial \Pi_2^{NI*}}{\partial c} < 0, \qquad \frac{\partial CS^{NI*}}{\partial c} < 0.$$

Moreover,

$$\frac{\partial SS^{NI*}}{\partial c} \gtreqless 0 \quad \Longleftrightarrow \quad c \gtreqless \tilde{c}^{NI}(k),$$

where

$$\tilde{c}^{NI}(k) \equiv \frac{4A(9k-5)}{99k-64} \in (0, \bar{c}^{NI}).$$

Proof. See Appendix A. □

Proposition 2 highlights a markedly different effect of cost asymmetry when R&D is undertaken by the initially inefficient firm. As c increases, firm 2 faces both a larger production-cost disadvantage and a weaker incentive to invest in cost reduction, so that

its R&D effort and profit decline. By contrast, firm 1 benefits from the deterioration of its rival's competitive position, while the associated reduction in total output lowers consumer surplus.

The welfare effect is non-monotonic. For $c < \widetilde{c}^{NI}(k)$, the output loss and the decline in the inefficient firm's R&D effort dominate, so social surplus falls with cost asymmetry. Beyond $\widetilde{c}^{NI}(k)$, the reallocation of production towards the initially efficient firm becomes sufficiently strong to reverse this effect, and social surplus increases with c. This contrasts with the IN configuration, where a larger cost asymmetry stimulates the efficient firm's R&D effort and social surplus may therefore increase monotonically when R&D is sufficiently efficient. Hence, the identity of the innovating firm is crucial for the welfare consequences of cost asymmetry.

3.1.4 The II configuration

Finally, consider the configuration in which both firms invest in R&D, so that $x_1 \geq 0$ and $x_2 \geq 0$. At Stage 2, the first-order conditions $\partial \Pi_i^{II}/\partial q_i = 0$ yield

$$q_1^{II}(x_1, x_2) = \frac{A + c + 2x_1 - x_2}{3}, \tag{19}$$

and

$$q_2^{II}(x_1, x_2) = \frac{A - 2c + 2x_2 - x_1}{3}. \tag{20}$$

Substituting Equations (19) and (20) into the profit functions gives the continuation profits

$$\Pi_1^{II}(x_1, x_2) = \frac{(A + c + 2x_1 - x_2)^2}{9} - \frac{k}{2}x_1^2, \tag{21}$$

and

$$\Pi_2^{II}(x_1, x_2) = \frac{(A - 2c + 2x_2 - x_1)^2}{9} - \frac{k}{2}x_2^2. \tag{22}$$

At Stage 1, both firms simultaneously choose their R&D efforts. The first-order conditions are

$$\frac{\partial \Pi_1^{II}}{\partial x_1} = \frac{4(A + c + 2x_1 - x_2)}{9} - kx_1 = 0, \quad \frac{\partial \Pi_2^{II}}{\partial x_2} = \frac{4(A - 2c + 2x_2 - x_1)}{9} - kx_2 = 0. \tag{23}$$

The corresponding R&D reaction functions are

$$x_1 = \frac{4(A + c - x_2)}{9k - 8}, \qquad x_2 = \frac{4(A - 2c - x_1)}{9k - 8}. \tag{24}$$

Under Assumption 1, R&D efforts are strategic substitutes, since $\partial x_i/\partial x_j = -4/(9k -$

8) < 0 for $i \neq j$. Moreover, $k > 4/3$ implies $|\partial x_i/\partial x_j| < 1$, so the two reaction functions admit a unique intersection. Solving Equation (24) yields the following result.

Lemma 6. *Under Assumption 1, the equilibrium R&D efforts in an interior II configuration are*

$$x_1^{II*} = \frac{4\left[3k(A+c) - 4A\right]}{(9k-4)(3k-4)}, \qquad x_2^{II*} = \frac{4\left[3k(A-2c) - 4(A-c)\right]}{(9k-4)(3k-4)}.$$

Proof. Straightforward from solving the system in Equation (24) under Assumption 1. □

Lemma 6 shows that cost asymmetry has opposite effects on the two firms' R&D efforts. In particular, $\partial x_1^{II*}/\partial c = 12k/[(9k-4)(3k-4)] > 0$, whereas $\partial x_2^{II*}/\partial c = 4(4-6k)/[(9k-4)(3k-4)] < 0$. Hence, a larger initial cost asymmetry strengthens the R&D effort of the initially efficient firm while weakening that of the initially inefficient firm. Moreover,

$$\frac{\partial(x_1^{II*} + x_2^{II*})}{\partial c} = -\frac{4}{9k-4} < 0,$$

so cost asymmetry polarizes innovation efforts while reducing aggregate R&D investment.

Substituting the equilibrium R&D efforts from Lemma 6 into Equations (19)–(22) yields the equilibrium outcomes under the II configuration:

$$\begin{aligned}
q_1^{II*} &= \frac{3k\left[A(3k-4) + 3kc\right]}{(9k-4)(3k-4)}, \qquad q_2^{II*} = \frac{3k\left[A(3k-4) - 2c(3k-2)\right]}{(9k-4)(3k-4)}, \\
Q^{II*} &= \frac{3k(2A-c)}{9k-4}, \\
\Pi_1^{II*} &= \frac{k(9k-8)\left[A(3k-4) + 3kc\right]^2}{(9k-4)^2(3k-4)^2}, \qquad \Pi_2^{II*} = \frac{k(9k-8)\left[A(3k-4) - 2c(3k-2)\right]^2}{(9k-4)^2(3k-4)^2}, \\
CS^{II*} &= \frac{9k^2(2A-c)^2}{2(9k-4)^2}, \\
SS^{II*} &= \frac{k\left[8A(A-c)(3k-4)^2 + (99k^2 - 156k + 64)c^2\right]}{2(9k-4)(3k-4)^2}.
\end{aligned} \tag{25}$$

The II equilibrium is interior only as long as the initially inefficient firm remains active when both firms undertake R&D. This leads to the following restriction.

Lemma 7. *Under Assumption 1, the II equilibrium is interior if and only if $0 < c < \bar{c}^{II} \equiv A(3k-4)/[2(3k-2)]$. At $c = \bar{c}^{II}$, firm 2's equilibrium output falls to zero. Moreover, $\bar{c}^{II} = \bar{c}^{IN}$.*

Proof. Since $q_2^{II*} = 3k[A(3k-4) - 2c(3k-2)]/[(9k-4)(3k-4)]$, we have $q_2^{II*} > 0$ if and only if $c < A(3k-4)/[2(3k-2)]$, with equality at the threshold. □

The identity $\bar{c}^{II} = \bar{c}^{IN}$ implies that the market-survival threshold of firm 2 is governed by whether the initially efficient firm invests in R&D. Once firm 1 innovates, additional R&D by firm 2 does not shift this boundary. Thus, bilateral innovation does not offset the competitive pressure created by firm 1's cost-reducing investment.

Within the interior region $0 < c < \bar{c}^{II}$, the effect of the initial cost asymmetry on the equilibrium outcomes is summarized below.

Proposition 3. *Under Assumption 1 and $0 < c < \bar{c}^{II}$, the effects of c are*

$$\frac{\partial x_1^{II*}}{\partial c} > 0, \quad \frac{\partial x_2^{II*}}{\partial c} < 0, \quad \frac{\partial \Pi_1^{II*}}{\partial c} > 0, \quad \frac{\partial \Pi_2^{II*}}{\partial c} < 0, \quad \frac{\partial CS^{II*}}{\partial c} < 0.$$

Moreover,

$$\frac{\partial SS^{II*}}{\partial c} \gtreqless 0 \quad \Longleftrightarrow \quad c \gtreqless \tilde{c}^{II}(k),$$

where

$$\tilde{c}^{II}(k) \equiv \frac{4A(3k-4)^2}{99k^2 - 156k + 64} \in (0, \bar{c}^{II}).$$

Proof. See Appendix A. □

Proposition 3 combines the two asymmetric R&D responses identified in Propositions 1 and 2. As c increases, the initially efficient firm strengthens its R&D effort, whereas the initially inefficient firm reduces its own investment. Hence, cost asymmetry reallocates innovation towards the more efficient firm, while simultaneously weakening the competitive position and profitability of firm 2. Firm 1 therefore benefits from a larger cost advantage, whereas firm 2's profit falls. Despite the increase in x_1^{II*}, total output declines with c, which explains the reduction in consumer surplus.

The welfare effect is again non-monotonic. For $c < \tilde{c}^{II}(k)$, the decline in aggregate output and aggregate R&D dominates the productive-efficiency gain generated by the reallocation of innovation and production towards firm 1, so social surplus decreases with c. Once $c > \tilde{c}^{II}(k)$, the efficiency gain from this reallocation becomes sufficiently strong to dominate, and social surplus increases with cost asymmetry.

This pattern differs from the IN configuration in Proposition 1, where social surplus may increase monotonically when R&D is sufficiently efficient. Instead, it is qualitatively closer to the NI case in Proposition 2, where social surplus also changes from decreasing to increasing at an endogenous cost-asymmetry threshold. The underlying mechanism is nevertheless different: under NI, the reversal is driven by the reallocation of production away from an innovating but increasingly disadvantaged firm, whereas under II it also reflects the endogenous reallocation of R&D effort from the inefficient firm towards the efficient one.

3.2 Comparison

We now compare the equilibrium outcomes across the four R&D configurations. Under Assumption 1, we restrict attention to the common interior region

$$0 < c < \bar{c} \equiv \bar{c}^{IN} = \bar{c}^{II} = \frac{A(3k-4)}{2(3k-2)},$$

which also implies $c < \bar{c}^{NN} = \bar{c}^{NI} = A/2$. Within this region, all four configurations are simultaneously feasible and can therefore be compared on a common basis. The following proposition summarizes the main rankings of equilibrium R&D efforts, firms' profits, consumer surplus, and social surplus.

Proposition 4. *Under Assumption 1 and $0 < c < \bar{c}$, the equilibrium outcomes across the four R&D configurations satisfy the following rankings.*

[1] **R&D efforts.** $x_1^{IN*} > x_1^{II*} > x_2^{II*}$, $x_1^{IN*} > x_2^{NI*} > x_2^{II*}$, *and* $x_1^{II*} + x_2^{II*} > x_1^{IN*} > x_2^{NI*} > 0$.

[2] **Firms' profits.** *For firm 2,* $\Pi_2^{NI*} > \Pi_2^{NN} > \Pi_2^{II*} > \Pi_2^{IN*}$. *For firm 1,* $\Pi_1^{IN*} > \max\{\Pi_1^{NN}, \Pi_1^{II*}\} > \Pi_1^{NI*}$, *with* $\Pi_1^{II*} \gtreqless \Pi_1^{NN}$ *if and only if* $c \gtreqless \hat{c}^{\Pi}(k)$.

[3] **Consumer surplus.** $CS^{II*} > CS^{IN*} > CS^{NI*} > CS^{NN}$.

[4] **Social surplus.** *For all* $0 < c < \bar{c}$, $SS^{IN*} > \max\{SS^{NI*}, SS^{NN}\}$ *and* $SS^{II*} > SS^{NN}$. *If* $4/3 < k < (23 + \sqrt{97})/18$, *then* $SS^{IN*} > SS^{II*}$ *and* $SS^{II*} \gtreqless SS^{NI*}$ *if and only if* $c \gtreqless c_{NI,II}^{SS}(k)$. *If* $k = (23+\sqrt{97})/18$, *then* $SS^{IN*} > SS^{II*} > SS^{NI*}$. *If* $k > (23+\sqrt{97})/18$, *then* $SS^{II*} > SS^{NI*}$ *and* $SS^{II*} \gtreqless SS^{IN*}$ *if and only if* $c \lesseqgtr c_{II,IN}^{SS}(k)$. *Finally,* $SS^{NI*} > SS^{NN}$ *for* $4/3 < k \leq (14 + 2\sqrt{15})/9$, *whereas, for* $k > (14 + 2\sqrt{15})/9$, $SS^{NN} \gtreqless SS^{NI*}$ *if and only if* $c \gtreqless c_{NI,NN}^{SS}(k)$.

where

$$\hat{c}^{\Pi}(k) \equiv \frac{A(3k-4)\left[9k-4-\sqrt{9k(9k-8)}\right]}{3k\sqrt{9k(9k-8)}-(3k-4)(9k-4)},$$

$$c_{NI,II}^{SS}(k) \equiv \frac{2A(3k-4)(23k-9k^2-12)}{k(3k-2)(45k-52)}, \qquad c_{II,IN}^{SS}(k) \equiv \frac{2A(3k-4)(9k^2-23k+12)}{3(63k^3-152k^2+120k-32)},$$

and

$$c_{NI,NN}^{SS}(k) \equiv \frac{2A(9k-5)}{63k-44}.$$

The relevant threshold belongs to $(0, \bar{c})$ over the parameter region in which it applies. Moreover, when $k > (14 + 2\sqrt{15})/9$, $c_{II,IN}^{SS}(k) < c_{NI,NN}^{SS}(k) < \bar{c}$.

Proof. See Appendix A. □

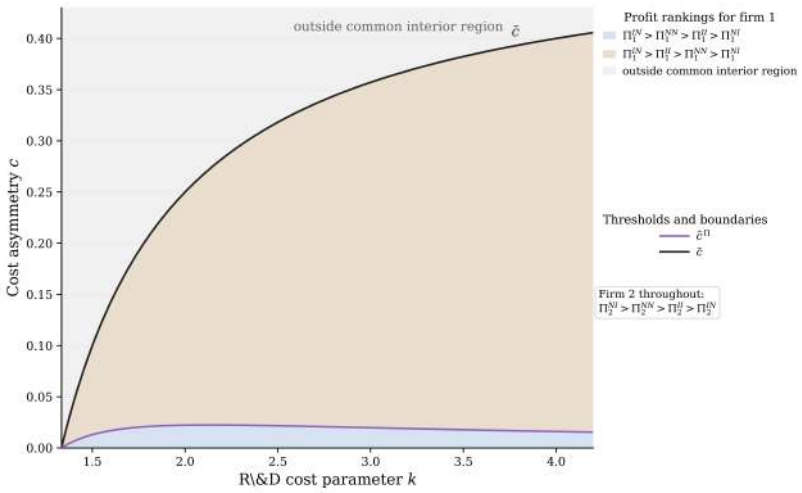
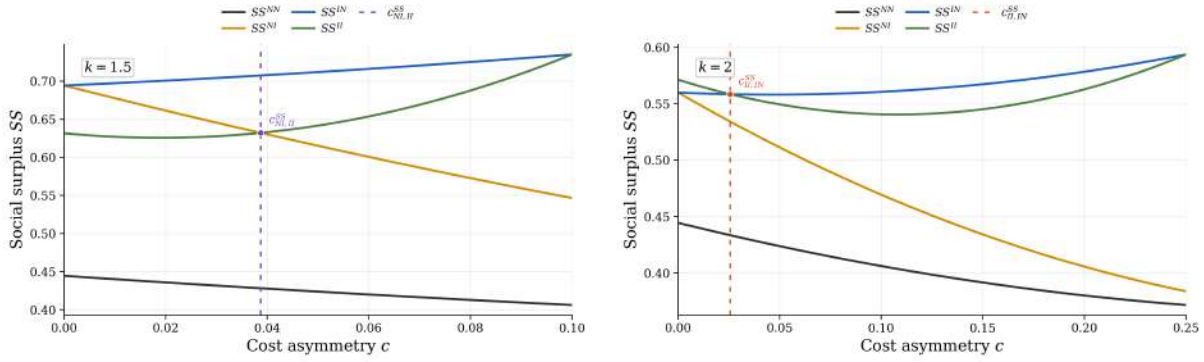
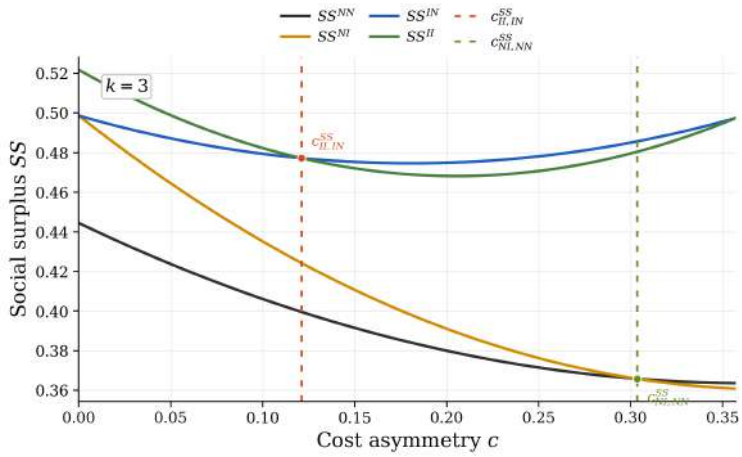


Figure 2: Profit-ranking regions for firm 1 in the (k, c) parameter space. Cost asymmetry is normalized by A.



(a) $k = 1.5$ (b) $k = 2$



(c) $k = 3$

Figure 3: Social surplus across R&D configurations for selected values of the R&D cost parameter. Panel (a) considers $k = 1.5$, Panel (b) $k = 2$, and Panel (c) $k = 3$. The dashed vertical lines identify the relevant social-surplus thresholds at which the ranking between configurations changes. Cost asymmetry and social surplus are normalized by A and A^2, respectively.

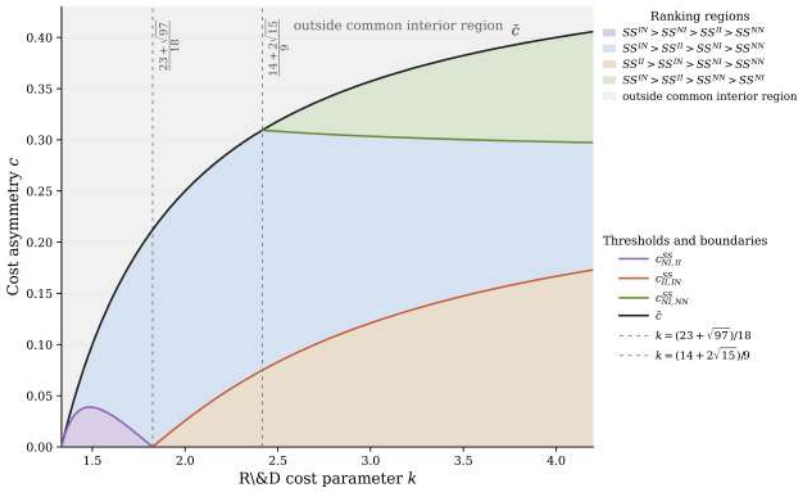


Figure 4: Social-surplus ranking regions in the (k, c) parameter space. Cost asymmetry is normalized by A.

The comparison reveals several economically meaningful patterns. First, the ranking of R&D efforts shows that the identity of the investing firm matters. Recall that firm 1 is initially more efficient than firm 2. When it invests alone, firm 1 chooses the largest individual R&D effort, x_1^{IN*}. Both firms also reduce their individual effort when the rival invests. In particular, $x_1^{IN*} > x_1^{II*}$ and $x_2^{NI*} > x_2^{II*}$. R&D efforts therefore exhibit a strategic-substitution effect. Nevertheless, aggregate R&D is highest under bilateral investment, since $x_1^{II*} + x_2^{II*} > x_1^{IN*}$. Thus, each firm invests less when the rival also innovates, but the presence of two investors more than compensates for this individual reduction. Cost asymmetry further tilts innovative activity toward the initially efficient firm. Firm 1 invests more than firm 2 when each innovates alone, and the same asymmetry persists under bilateral R&D. Hence, a larger initial cost advantage is associated with a stronger concentration of innovative effort in the more efficient firm.

The profit comparison, illustrated in Figure 2, reveals a similarly pronounced asymmetry in private incentives. Each firm earns its highest profit when it is the sole investor. The IN configuration is therefore best for firm 1, whereas NI is best for firm 2. For firm 2, the ranking is unambiguous throughout the common interior region: $\Pi_2^{NI*} > \Pi_2^{NN} > \Pi_2^{II*} > \Pi_2^{IN*}$. In particular, firm 2 prefers mutual non-investment to bilateral R&D. The innovation undertaken by firm 1 strengthens the competitive position of the initially efficient firm. From firm 2's perspective, this competitive loss is large enough to outweigh the benefit of undertaking its own R&D under bilateral investment. The comparison for firm 1 is more subtle. Although IN always yields its highest profit and NI its lowest, the ranking between NN and II changes at $\widehat{c}^{\Pi}(k)$. For $c < \widehat{c}^{\Pi}(k)$, firm 1 prefers mutual non-investment to bilateral R&D. Once $c > \widehat{c}^{\Pi}(k)$, this ranking is reversed. Figure 2 makes this switch particularly transparent. Below

the threshold, the gain from bilateral R&D does not compensate for its investment and strategic costs. As cost asymmetry increases, however, firm 1's initial efficiency advantage becomes more valuable. Bilateral R&D then becomes privately more attractive than mutual non-investment.

Consumer surplus produces a much simpler ordering: $CS^{II*} > CS^{IN*} > CS^{NI*} > CS^{NN}$. This ranking follows directly from total output, which satisfies $Q^{II*} > Q^{IN*} > Q^{NI*} > Q^{NN}$ throughout the common interior region. Since $CS = Q^2/2$, consumers always prefer bilateral R&D. This is followed by unilateral investment by the initially efficient firm, unilateral investment by the initially inefficient firm, and finally mutual non-investment.

The social-surplus comparison is considerably richer. Two rankings hold throughout the common interior region: $SS^{IN*} > \max\{SS^{NI*}, SS^{NN}\}$, $SS^{II*} > SS^{NN}$. Thus, innovation by the initially efficient firm is always socially preferable to either unilateral innovation by the inefficient firm or mutual non-investment. More importantly, the configuration with the largest aggregate R&D effort need not maximize social surplus. Bilateral investment always generates more total R&D than unilateral investment by firm 1, and it also maximizes consumer surplus. However, the additional R&D undertaken by firm 2 entails a real resource cost. When the associated productive gain is too small, concentrating innovation in the initially efficient firm yields higher social surplus. This explains why IN may dominate II, even though bilateral investment generates more aggregate innovation.

Figure 3 illustrates the three qualitatively distinct welfare regimes identified in Proposition 4. The chosen values $k = 1.5$, $k = 2$, and $k = 3$ lie, respectively, below $(23+\sqrt{97})/18$, between $(23+\sqrt{97})/18$ and $(14+2\sqrt{15})/9$, and above $(14+2\sqrt{15})/9$. The dashed vertical lines in each panel identify the relevant social-surplus thresholds at which the ranking between configurations changes.

When R&D is relatively inexpensive, as in Figure 3a, unilateral investment by the initially efficient firm dominates bilateral investment throughout the admissible region. The relevant switch therefore concerns II and NI. Below $c^{SS}_{NI,II}(k)$, unilateral investment by firm 2 yields higher social surplus than bilateral R&D. At the threshold, the two configurations are equivalent. Beyond it, bilateral investment becomes socially preferable. For intermediate R&D costs, as in Figure 3b, bilateral investment already dominates unilateral investment by firm 2. The relevant comparison is therefore between II and IN. When cost asymmetry is small, bilateral investment yields higher social surplus. At $c^{SS}_{II,IN}(k)$, the two configurations coincide. For larger cost asymmetry, $SS^{IN*} > SS^{II*}$.

The economic mechanism is straightforward. When firms are relatively similar, allowing both of them to innovate produces sufficiently large productive gains to justify the two

R&D expenditures. As firm 2's initial cost disadvantage widens, however, its contribution to bilateral innovation becomes less valuable relative to its resource cost. Social surplus then favors concentrating R&D in the initially efficient firm. The high-k case, shown in Figure 3c, adds a further welfare reversal. As in panel (b), the ranking between bilateral investment and unilateral investment by firm 1 changes at $c^{SS}_{II,IN}(k)$. In addition, unilateral R&D by the initially inefficient firm eventually becomes socially worse than no R&D at all.

More precisely, when $k > (14+2\sqrt{15})/9$ and $c > c^{SS}_{NI,NN}(k)$, we obtain $SS^{NN} > SS^{NI*}$. Thus, when R&D is sufficiently costly and the innovating firm is sufficiently inefficient initially, society is better off with mutual non-investment than with innovation undertaken solely by firm 2. This result reinforces the idea that the welfare consequences of R&D depend not only on whether innovation occurs, but also on *which firm undertakes it*. Figure 4 summarizes these welfare switches over the entire (k, c) parameter space. Taken together, the results reveal a clear wedge between private incentives, consumer interests, and social efficiency. Each firm privately prefers to be the sole investor, while consumers always prefer bilateral investment. Social surplus, by contrast, may be maximized either by bilateral R&D or by concentrating innovation in the initially efficient firm, depending on the R&D cost and the degree of initial cost asymmetry.

The broader implication is that more aggregate R&D is not necessarily welfare improving. In asymmetric industries, the allocation of innovative activity across firms can matter as much as the total amount of innovation.

3.3 Endogenous R&D participation

We now return to Stage 0 of the game and endogenize firms' R&D participation decisions. Having solved the continuation game under each of the four possible configurations NN, IN, NI, and II, the initial participation stage can be represented as a simultaneous two-action game in which each firm chooses whether to invest (I) or not (N). The relevant payoffs are the equilibrium profits derived in the preceding subsections. Because firms are ex ante asymmetric, the two unilateral-investment outcomes IN and NI are economically distinct. The resulting R&D participation game is therefore

	Firm 2	
Firm 1	N	I
N	$\left(\Pi_1^{NN*},\, \Pi_2^{NN*}\right)$	$\left(\Pi_1^{NI*},\, \Pi_2^{NI*}\right)$
I	$\left(\Pi_1^{IN*},\, \Pi_2^{IN*}\right)$	$\left(\Pi_1^{II*},\, \Pi_2^{II*}\right)$

(26)

We obtain the following result.

Proposition 5. *Under Assumption 1 and* $0 < c < \bar{c}$*:*

(i) Investing in R&D is a strictly dominant strategy for both firms. Hence, the unique Nash equilibrium of the participation game is (I, I)*.*

(ii) If $c < \hat{c}^{\Pi}(k)$*, the* II *equilibrium is Pareto-dominated by* NN *from the firms' perspective, so the participation game is a prisoner's dilemma. At* $c = \hat{c}^{\Pi}(k)$*, firm 1 is indifferent between* II *and* NN*. If* $c > \hat{c}^{\Pi}(k)$*, the dilemma breaks down asymmetrically: firm 1 prefers* II*, whereas firm 2 still prefers* NN*.*

Proof. See Appendix A. □

Proposition 5 shows that cost asymmetry does not overturn the strong private incentive to engage in process R&D. If the rival does not invest, undertaking R&D provides a unilateral cost advantage; if the rival invests, R&D becomes defensive, as abstaining would leave the firm exposed to the rival's cost reduction. Consequently, I remains strictly dominant for both firms and bilateral R&D emerges endogenously.

This first result is consistent with Bacchiega et al. (2010), who obtain (I, I) as the unique equilibrium of the investment-decision game with symmetric firms and show that it may constitute a prisoner's dilemma. Our result differs, however, in the way this dilemma disappears. Buccella et al. (2023b) show that product differentiation can turn the symmetric prisoner's dilemma into a deadlock, so that both firms prefer bilateral R&D to mutual non-investment. By contrast, in our homogeneous-good setting, ex ante cost asymmetry alone generates a *one-sided breakdown* of the dilemma. For $c < \hat{c}^{\Pi}(k)$, both firms prefer NN to the equilibrium II, reproducing the standard prisoner's-dilemma logic. Once $c > \hat{c}^{\Pi}(k)$, however, the initially efficient firm prefers II to NN, whereas the inefficient firm continues to prefer NN.

The mechanism follows from the endogenous reallocation of R&D incentives. As cost asymmetry rises, firm 1 increases its R&D effort while firm 2 reduces its own, so bilateral R&D becomes increasingly tilted toward the initially efficient firm. The latter therefore captures a larger share of the gains from innovation while facing weaker innovative pressure from its disadvantaged rival. This result complements Saglam (2022), who also studies process R&D under ex ante cost asymmetry but takes R&D participation as given. Here, endogenizing the participation decision reveals that heterogeneity does not change *whether* bilateral R&D emerges, but fundamentally changes *who benefits from it*.

The endogenous participation equilibrium need not, however, coincide with the socially preferred allocation of R&D activity. Since Proposition 4 shows that IN always dominates both NI and NN in terms of social surplus, the relevant welfare comparison reduces to

IN versus the privately selected II configuration. This yields the following immediate implication.

Corollary 1. *Under Assumption 1 and $0 < c < \bar{c}$, the endogenous participation equilibrium is always II, whereas the socially optimal R&D configuration is characterized as follows:*

(i) If $4/3 < k \leq (23 + \sqrt{97})/18$, the socially optimal configuration is IN.

(ii) If $k > (23 + \sqrt{97})/18$, the socially optimal configuration is II for $c < c^{SS}_{II,IN}(k)$, IN for $c > c^{SS}_{II,IN}(k)$.

Proof. See Appendix A. □

Corollary 1 highlights a systematic wedge between private participation incentives and social efficiency. Firms always choose bilateral R&D because investing is individually optimal regardless of the rival's participation decision. The planner, however, also internalizes the resource cost of R&D and therefore need not favor innovation by both firms.

When R&D is relatively inexpensive, $4/3 < k \leq (23 + \sqrt{97})/18$, concentrating innovation in the initially efficient firm is always socially preferable. Although II generates more aggregate R&D and a larger consumer surplus, the additional investment undertaken by firm 2 does not create enough productive gain to compensate for its resource cost. The privately selected II configuration therefore features excessive R&D participation from the social point of view.

When R&D is more costly, the welfare comparison becomes dependent on the degree of initial cost asymmetry. For small c, the firms are sufficiently similar that allowing both of them to innovate remains socially worthwhile, and the private and social participation outcomes coincide at II. As c increases, however, firm 2 becomes progressively less efficient and its contribution to bilateral innovation becomes less valuable. Beyond $c^{SS}_{II,IN}(k)$, social surplus is higher when R&D is concentrated in firm 1 alone. Hence, cost asymmetry can endogenously create a divergence between the privately selected extent of R&D participation and the socially efficient allocation of innovative activity.

4 Extension: R&D spillovers

We now relax the no-spillover assumption and examine whether the endogenous R&D participation result is robust to technological spillovers. The purpose of this extension is deliberately limited: rather than reproducing the full set of comparisons derived in

Section 3, we focus on whether spillovers alter firms' incentives to participate in R&D and, consequently, the equilibrium of the Stage 0 investment-decision game.

Following the standard d'Aspremont–Jacquemin specification, R&D undertaken by one firm may also reduce the rival's marginal production cost. Firm i's production cost is now given by

$$C_i(q_i, x_i, x_j) = (c_i - x_i - \beta x_j)\, q_i, \qquad \beta \in [0, 1], \tag{27}$$

where β measures the intensity of the technological spillover. The case $\beta = 0$ reproduces the baseline model, whereas larger values of β imply that a greater fraction of the rival's R&D effort benefits firm i.

Accordingly, firm i's profit becomes

$$\Pi_i = [a - q_i - q_j - c_i + x_i + \beta x_j]\, q_i - \frac{k}{2} x_i^2. \tag{28}$$

As in the baseline model, a non-investing firm sets $x_i = 0$, although it may still benefit from the rival's R&D through the spillover term βx_j. We restrict attention to economically meaningful outcomes with positive post-innovation marginal costs, $c_i - x_i - \beta x_j > 0$. Under $k > 4/3$, requiring both firms to remain active in all four continuation configurations yields the common interior region $0 < c < \bar{c}(k, \beta)$, where

$$\bar{c}(k, \beta) = \begin{cases} \dfrac{A[9k - 6(2 - \beta)(1 - \beta)]}{6[3k - (2 - \beta)]}, & 0 \le \beta < 1/2, \\ \dfrac{A}{2}, & 1/2 \le \beta \le 1. \end{cases} \tag{29}$$

At $\beta = 0$, $\bar{c}(k, 0) = \frac{A(3k-4)}{2(3k-2)} = \bar{c}$, so the baseline common interior region is recovered.

The equilibrium R&D efforts and continuation profits for the spillover extension are reported in Tables 1 and 2 in Appendix B.

We now characterize the endogenous participation game in this extension.

Proposition 6. *Suppose $k > 4/3$, $\beta \in [0, 1]$, and $0 < c < \bar{c}(k, \beta)$.*

[1] **Endogenous participation**. *Investing in R&D is a strictly dominant strategy for both firms. Hence, the unique Stage 0 equilibrium, and therefore the unique SPNE, induces the II configuration.*

[2] **Private ranking of II versus NN.** *Let $\widehat{\beta}(k) \in (0, 1/2)$ be the unique solution to $2\widehat{\beta}^3(k) + (27k - 6)\widehat{\beta}(k) - 4 = 0$.*

- *If $0 \le \beta < \widehat{\beta}(k)$, then $\Pi_1^{NN} \gtrless \Pi_1^{II*}$ if and only if $c \lessgtr \widehat{c}_1^{\Pi}(k, \beta)$, while $\Pi_2^{NN} > \Pi_2^{II*}$. Thus, II is a prisoner's-dilemma equilibrium for $c < \widehat{c}_1^{\Pi}(k, \beta)$, whereas for $c > \widehat{c}_1^{\Pi}(k, \beta)$ only firm 2 prefers NN.*

– *If* $\beta = \widehat{\beta}(k)$*, then for every admissible* $c > 0$*,* $\Pi_1^{II*} > \Pi_1^{NN}$ *and* $\Pi_2^{NN} > \Pi_2^{II*}$*.*

– *If* $\widehat{\beta}(k) < \beta < 1/2$*, then* $\Pi_2^{II*} \gtreqless \Pi_2^{NN}$ *if and only if* $c \lesseqgtr \widehat{c}_2^{\Pi}(k,\beta)$*, while* $\Pi_1^{II*} > \Pi_1^{NN}$*. Hence, both firms prefer* II *for* $c < \widehat{c}_2^{\Pi}(k,\beta)$*, whereas only firm 2 prefers* NN *for* $c > \widehat{c}_2^{\Pi}(k,\beta)$*.*

– *If* $1/2 \leq \beta \leq 1$*, then* $\Pi_1^{II*} > \Pi_1^{NN}$ *and* $\Pi_2^{II*} > \Pi_2^{NN}$*,*

where

$$\widehat{c}_1^{\Pi}(k,\beta) \equiv \frac{A[9k - 6(2-\beta)(1-\beta)]\left\{9k - 2(2-\beta)(1+\beta) - 3\sqrt{k[9k - 2(2-\beta)^2]}\right\}}{\begin{array}{r} 3\sqrt{k[9k - 2(2-\beta)^2]}[9k - 6\beta(2-\beta)] \\ - [9k - 6(2-\beta)(1-\beta)][9k - 2(2-\beta)(1+\beta)] \end{array}},$$

and

$$\widehat{c}_2^{\Pi}(k,\beta) \equiv \frac{A[9k - 6(2-\beta)(1-\beta)]\left\{3\sqrt{k[9k - 2(2-\beta)^2]} - [9k - 2(2-\beta)(1+\beta)]\right\}}{\begin{array}{r} 18\sqrt{k[9k - 2(2-\beta)^2]}[3k - (2-\beta)] \\ - 2[9k - 6(2-\beta)(1-\beta)][9k - 2(2-\beta)(1+\beta)] \end{array}}.$$

Moreover, $\widehat{c}_1^{\Pi}(k,0) = \widehat{c}^{\Pi}(k)$*.*

Proof. See Appendix B. □

Proposition 6 shows first that the endogenous emergence of bilateral R&D is robust to technological spillovers. Although a non-investing firm can now free-ride on its rival's innovation, choosing to invest still yields a strictly higher continuation profit than not investing, regardless of the rival's participation decision. Hence, II remains the unique equilibrium of the participation game for all $\beta \in [0,1]$. Spillovers therefore affect the *private desirability* of bilateral R&D rather than its endogenous emergence.

The main new result concerns the interaction between spillovers and ex ante cost asymmetry. When spillovers are weak, $0 \leq \beta < \widehat{\beta}(k)$, the qualitative structure of the baseline model survives. For sufficiently small c, both firms prefer NN to the equilibrium II, so the participation game remains a prisoner's dilemma. As cost asymmetry increases beyond $\widehat{c}_1^{\Pi}(k,\beta)$, however, the initially efficient firm switches to preferring II, while the inefficient firm continues to prefer NN. The no-spillover result is nested in this case, since $\widehat{c}_1^{\Pi}(k,0) = \widehat{c}^{\Pi}(k)$. More interestingly, when $\widehat{\beta}(k) < \beta < 1/2$, bilateral R&D is initially preferred by both firms, but sufficiently large cost asymmetry reverses the ranking only for firm 2. Thus, asymmetry can generate a one-sided conflict even when spillovers are strong enough to eliminate the symmetric prisoner's-dilemma outcome. Once $\beta \geq 1/2$,

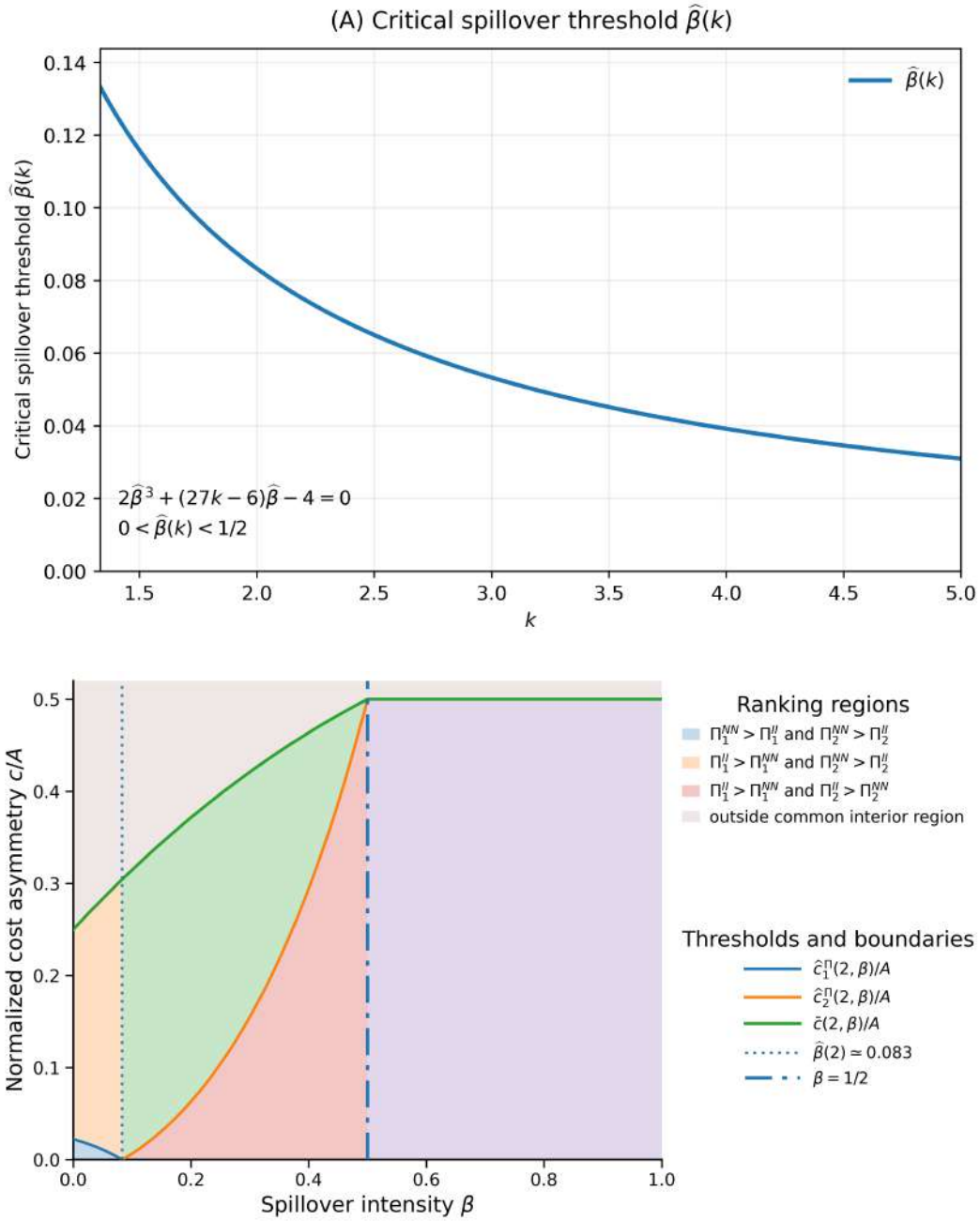


Figure 5: Effects of technological spillovers on the endogenous R&D participation game. Panel (A) reports the critical spillover threshold $\widehat{\beta}(k)$, defined by $2\widehat{\beta}^3+(27k-6)\widehat{\beta}-4=0$. Panel (B) depicts the private-ranking regions in the $(\beta, c/A)$ space for $k = 2$, together with the relevant profit thresholds and the boundary of the common interior region.

the spillover benefit dominates this competitive effect and both firms prefer II throughout the common interior region.

Figure 5 summarizes these mechanisms. Panel (A) shows that the critical spillover $\widehat{\beta}(k)$ decreases with the R&D cost parameter k, while Panel (B) illustrates, for $k = 2$, how the prisoner's-dilemma region, the one-sided-conflict region, and the region in which both firms prefer II are separated by the endogenous thresholds. The result is consistent with Bacchiega et al. (2010), who show in a symmetric investment-decision game that spillovers affect whether bilateral R&D has a prisoner's-dilemma structure, and complements Buccella et al. (2023b), who obtain the disappearance of the symmetric dilemma through product differentiation. Here, by contrast, the relevant margin is ex ante cost heterogeneity: spillovers may remove the symmetric conflict, yet cost asymmetry can recreate it for the initially inefficient firm alone.

5 Conclusion

This paper has examined how ex ante production-cost asymmetry affects endogenous participation in process R&D in a Cournot duopoly. By adding a preliminary participation stage to the standard cost-reducing R&D game, the analysis distinguishes between the decision to innovate and the subsequent choice of innovation effort. This distinction is particularly important when firms begin from different cost positions, since unilateral innovation by the initially efficient firm and unilateral innovation by the initially inefficient firm have markedly different strategic and welfare consequences.

Three main results emerge. First, cost asymmetry does not eliminate firms' private incentives to participate in R&D: investing is a strictly dominant strategy for both firms, so bilateral R&D arises as the unique endogenous participation equilibrium. Asymmetry nevertheless changes the nature of this equilibrium. When firms are sufficiently similar, bilateral investment may constitute a prisoner's dilemma, whereas greater cost asymmetry induces a one-sided breakdown of this conflict: the initially efficient firm comes to prefer bilateral R&D to mutual non-investment, while the initially inefficient firm continues to prefer the latter. Second, the identity of the innovating firm matters for both market performance and welfare. Although joint investment generates the largest total output and consumer surplus, more aggregate R&D is not necessarily socially desirable. In particular, social surplus may be higher when innovation is concentrated in the initially efficient firm. The privately selected allocation of R&D activity may therefore differ systematically from the socially preferred one. Third, the main participation result is robust to technological spillovers. Spillovers modify the private desirability of bilateral R&D and can eliminate the symmetric prisoner's-dilemma region, but they do not overturn

the endogenous emergence of bilateral investment within the interior region considered.

Two extensions appear especially promising. First, introducing horizontal product differentiation would allow us to examine how product substitutability interacts with cost asymmetry in shaping endogenous R&D participation. Second, incorporating network effects would add a demand-side strategic channel through which innovation can affect both production costs and market expansion. These extensions could further clarify how market structure influences the private and social allocation of innovative activity.

Appendix A

Derivation of Assumption 1. Part (i) follows from $A \equiv a - c_0$: $A > 0$ is equivalent to $a > c_0$ and guarantees a positive net market size for firm 1.

For the R&D stage, the unilateral-investment problems satisfy $\partial^2 \Pi_i / \partial x_i^2 = 8/9 - k$, so concavity requires $k > 8/9$. Under bilateral R&D, the reaction functions have slope $-4/(9k-8)$. A unique stable intersection requires $|4/(9k-8)| < 1$, which, together with $k > 8/9$, is equivalent to $k > 4/3$. Hence, $k > 4/3$ provides a single sufficient regularity condition for all R&D configurations.

Finally, positive post-innovation marginal costs require $x_1 < c_0$ and $x_2 < c_0 + c$. Since $x_1^{IN*} = 4(A+c)/(9k-8)$, $x_1^{IN*} > x_1^{II*}$, $x_1^{IN*} > x_2^{NI*} > x_2^{II*}$, and $c_0 + c > c_0$, the single sufficient restriction $4(A+c)/(9k-8) < c_0$ guarantees positive post-innovation marginal costs in all configurations.

Proof of Proposition 1. Under Assumption 1, $k > 4/3$ and $9k-8 > 0$. Differentiation gives

$$\frac{\partial x_1^{IN*}}{\partial c} = \frac{4}{9k-8} > 0, \qquad \frac{\partial \Pi_1^{IN*}}{\partial c} = \frac{2k(A+c)}{9k-8} > 0,$$

and

$$\frac{\partial \Pi_2^{IN*}}{\partial c} = 2q_2^{IN*}\frac{4-6k}{9k-8} < 0, \qquad \frac{\partial CS^{IN*}}{\partial c} = Q^{IN*}\frac{4-3k}{9k-8} < 0,$$

where $q_2^{IN*} > 0$ and $Q^{IN*} > 0$ in the interior region.

For social surplus,

$$\frac{\partial SS^{IN*}}{\partial c} = \frac{(99k^2 - 136k + 48)c - 4A(9k^2 - 23k + 12)}{(9k-8)^2}.$$

Since $99k^2 - 136k + 48 > 0$, the derivative is increasing in c. Moreover, $9k^2 - 23k + 12 \leq 0$ for $4/3 < k \leq (23 + \sqrt{97})/18$, implying $\partial SS^{IN*}/\partial c > 0$. For $k > (23 + \sqrt{97})/18$,

$\partial SS^{IN*}/\partial c = 0$ at

$$\widetilde{c}^{IN}(k) = \frac{4A(9k^2 - 23k + 12)}{99k^2 - 136k + 48} > 0,$$

and therefore

$$\frac{\partial SS^{IN*}}{\partial c} \gtreqless 0 \quad \Longleftrightarrow \quad c \gtreqless \widetilde{c}^{IN}(k).$$

Finally,

$$\bar{c}^{IN} - \widetilde{c}^{IN}(k) = \frac{Ak(9k-8)(9k-4)}{2(3k-2)(99k^2 - 136k + 48)} > 0,$$

so $\widetilde{c}^{IN}(k) \in (0, \bar{c}^{IN})$. □

Proof of Proposition 2. Under Assumption 1, $k > 4/3$ and $9k-8 > 0$. Differentiation gives

$$\frac{\partial x_2^{NI*}}{\partial c} = -\frac{8}{9k-8} < 0, \qquad \frac{\partial \Pi_1^{NI*}}{\partial c} = \frac{6k[A(3k-4) + 3kc]}{(9k-8)^2} > 0,$$

and

$$\frac{\partial \Pi_2^{NI*}}{\partial c} = -\frac{4k(A-2c)}{9k-8} < 0, \qquad \frac{\partial CS^{NI*}}{\partial c} = \frac{3k[A(4-6k) + 3kc]}{(9k-8)^2} < 0,$$

where the last two inequalities follow from $0 < c < \bar{c}^{NI} = A/2$.

For social surplus,

$$\frac{\partial SS^{NI*}}{\partial c} = \frac{k[(99k-64)c - 4A(9k-5)]}{(9k-8)^2}.$$

Since $99k - 64 > 0$ under Assumption 1,

$$\frac{\partial SS^{NI*}}{\partial c} \gtreqless 0 \quad \Longleftrightarrow \quad c \gtreqless \widetilde{c}^{NI}(k) \equiv \frac{4A(9k-5)}{99k-64}.$$

Moreover, $\widetilde{c}^{NI}(k) > 0$ and

$$\bar{c}^{NI} - \widetilde{c}^{NI}(k) = \frac{3A(9k-8)}{2(99k-64)} > 0,$$

so $\widetilde{c}^{NI}(k) \in (0, \bar{c}^{NI})$. □

Proof of Proposition 3. Under Assumption 1, $k > 4/3$, so all relevant denominators are positive. Differentiation gives

$$\frac{\partial x_1^{II*}}{\partial c} = \frac{12k}{(9k-4)(3k-4)} > 0, \qquad \frac{\partial x_2^{II*}}{\partial c} = -\frac{8(3k-2)}{(9k-4)(3k-4)} < 0.$$

For profits,

$$\frac{\partial \Pi_1^{II*}}{\partial c} = \frac{6k^2(9k-8)[A(3k-4)+3kc]}{(3k-4)^2(9k-4)^2} > 0,$$

and

$$\frac{\partial \Pi_2^{II*}}{\partial c} = -\frac{4k(3k-2)(9k-8)[A(3k-4)-2c(3k-2)]}{(3k-4)^2(9k-4)^2} < 0,$$

where the latter inequality follows from $0 < c < \bar{c}^{II} = A(3k-4)/[2(3k-2)]$. Moreover,

$$\frac{\partial CS^{II*}}{\partial c} = -\frac{9k^2(2A-c)}{(9k-4)^2} < 0.$$

For social surplus,

$$\frac{\partial SS^{II*}}{\partial c} = \frac{k[(99k^2-156k+64)c - 4A(3k-4)^2]}{(3k-4)^2(9k-4)}.$$

Since $99k^2 - 156k + 64 > 0$ under Assumption 1,

$$\frac{\partial SS^{II*}}{\partial c} \gtreqless 0 \quad \Longleftrightarrow \quad c \gtreqless \widetilde{c}^{II}(k) \equiv \frac{4A(3k-4)^2}{99k^2-156k+64}.$$

Moreover, $\widetilde{c}^{II}(k) > 0$ and

$$\bar{c}^{II} - \widetilde{c}^{II}(k) = \frac{3Ak(3k-4)(9k-4)}{2(3k-2)(99k^2-156k+64)} > 0,$$

so $\widetilde{c}^{II}(k) \in (0, \bar{c}^{II})$. □

Proof of Proposition 4. Under Assumption 1, $k > 4/3$. Moreover, $0 < c < \bar{c}$ implies $A - 2c > 0$ and

$$\Delta \equiv A(3k-4) - 2c(3k-2) > 0.$$

For R&D efforts, $x_1^{IN*} - x_2^{NI*} = 12c/(9k-8) > 0$ and $x_1^{II*} - x_2^{II*} = 4c/(3k-4) > 0$. Moreover,

$$x_1^{IN*} - x_1^{II*} = \frac{16\Delta}{(3k-4)(9k-8)(9k-4)} > 0, \qquad x_2^{NI*} - x_2^{II*} = \frac{16[A(3k-4)+3kc]}{(3k-4)(9k-8)(9k-4)} > 0.$$

Finally, $x_1^{II*} + x_2^{II*} - x_1^{IN*} = 12\Delta/[(9k-8)(9k-4)] > 0$, which establishes part [1].

For profits, $\Pi_1^{IN*} - \Pi_1^{NN} = 8(A+c)^2/[9(9k-8)] > 0$ and $\Pi_2^{NI*} - \Pi_2^{NN} = 8(A-2c)^2/[9(9k-8)] > 0$. Furthermore, a rival's R&D effort lowers a firm's continuation payoff, since $\partial \Pi_1^{II}/\partial x_2 = -2(A+c+2x_1-x_2)/9 < 0$ and $\partial \Pi_2^{II}/\partial x_1 = -2(A-2c+2x_2-x_1)/9 < 0$ in the interior region. Hence, $\Pi_1^{IN*} > \Pi_1^{II*}$ and $\Pi_2^{NI*} > \Pi_2^{II*}$.

In addition,

$$\frac{\Pi_1^{II*}}{\Pi_1^{NI*}} = \frac{\Pi_2^{II*}}{\Pi_2^{IN*}} = \frac{k(9k-8)^3}{(9k-4)^2(3k-4)^2} > 1,$$

because $k(9k-8)^3 - (9k-4)^2(3k-4)^2 = 8(81k^3 - 180k^2 + 128k - 32) > 0$; indeed, the polynomial equals $32/3$ at $k = 4/3$ and its derivative is $(9k-8)(27k-16) > 0$. Thus, $\Pi_1^{II*} > \Pi_1^{NI*}$ and $\Pi_2^{II*} > \Pi_2^{IN*}$. Also,

$$\Pi_1^{NN} - \Pi_1^{NI*} = \frac{8(A-2c)[A(9k-10) + c(9k-4)]}{9(9k-8)^2} > 0.$$

For firm 2, $\Delta < (3k-4)(A-2c)$, so

$$\Pi_2^{II*} < \frac{k(9k-8)(A-2c)^2}{(9k-4)^2} < \frac{(A-2c)^2}{9} = \Pi_2^{NN},$$

where the second inequality follows from $9k(9k-8) < (9k-4)^2$.

It remains to compare Π_1^{II*} and Π_1^{NN}. Let $s \equiv \sqrt{9k(9k-8)}$. Since all terms are positive,

$$\Pi_1^{II*} \gtreqless \Pi_1^{NN} \quad \Longleftrightarrow \quad s[A(3k-4) + 3kc] \gtreqless (9k-4)(3k-4)(A+c),$$

which is equivalent to $c \gtreqless \widehat{c}^{\Pi}(k)$. Under Assumption 1, $9k-8 < s < 9k-4$, so $\widehat{c}^{\Pi}(k) > 0$, and

$$\bar{c} - \widehat{c}^{\Pi}(k) = \frac{A(3k-4)(9k-4)[s-(9k-8)]}{2(3k-2)[3ks - (3k-4)(9k-4)]} > 0.$$

This proves part [2].

For consumer surplus, $CS = Q^2/2$, and $Q^{II*} - Q^{IN*} = 4\Delta/[(9k-8)(9k-4)] > 0$, $Q^{IN*} - Q^{NI*} = 4c/(9k-8) > 0$, and $Q^{NI*} - Q^{NN} = 4(A-2c)/[3(9k-8)] > 0$. Hence, $CS^{II*} > CS^{IN*} > CS^{NI*} > CS^{NN}$, proving part [3].

For social surplus, direct comparison gives

$$\begin{aligned}
SS^{IN*} - SS^{NI*} &= \frac{12c(2A-c)(3k-2)}{(9k-8)^2} > 0, \\
SS^{IN*} - SS^{NN} &= \frac{4(A+c)[A(18k-10) + c(45k-34)]}{9(9k-8)^2} > 0, \\
SS^{II*} - SS^{NN} &= \frac{4\{4A(A-c)(3k-4)^2 + c^2(171k^2 - 258k + 88)\}}{9(3k-4)^2(9k-4)} > 0.
\end{aligned}$$

The last inequality follows from $c < \bar{c} < A/2$ and $171k^2 - 258k + 88 > 0$ for $k > 4/3$.

To establish the remaining rankings, define $R(k) \equiv 9k^2 - 23k + 12$ and $P(k) \equiv 63k^3 -$

$152k^2 + 120k - 32$. Under Assumption 1, $P(k) > 0$ and $45k - 52 > 0$. We have

$$SS^{IN*} - SS^{II*} = \frac{4\Delta\left[3cP(k) - 2A(3k-4)R(k)\right]}{(3k-4)^2(9k-8)^2(9k-4)},$$

and

$$SS^{II*} - SS^{NI*} = \frac{4[A(3k-4)+3kc]\left[2A(3k-4)R(k) + ck(3k-2)(45k-52)\right]}{(3k-4)^2(9k-8)^2(9k-4)}.$$

Since $R(k) \lesseqgtr 0$ if and only if $k \lesseqgtr (23+\sqrt{97})/18$, these expressions generate the three cases reported in part [4]. For $4/3 < k < (23+\sqrt{97})/18$, $SS^{IN*} > SS^{II*}$, while $SS^{II*} - SS^{NI*}$ changes sign at $c^{SS}_{NI,II}(k)$. At equality, $SS^{IN*} > SS^{II*} > SS^{NI*}$. For $k > (23+\sqrt{97})/18$, $SS^{II*} > SS^{NI*}$, while $SS^{IN*} - SS^{II*}$ changes sign at $c^{SS}_{II,IN}(k)$. The relevant thresholds lie inside the common interior region since

$$\bar{c} - c^{SS}_{NI,II}(k) = \frac{3A(3k-4)^2(9k-4)}{2k(3k-2)(45k-52)} > 0, \qquad \bar{c} - c^{SS}_{II,IN}(k) = \frac{Ak(3k-4)(9k-8)(9k-4)}{6(3k-2)P(k)} > 0.$$

Finally,

$$SS^{NI*} - SS^{NN} = \frac{4(2c-A)[-2A(9k-5) + c(63k-44)]}{9(9k-8)^2}.$$

Since $2c - A < 0$, the sign changes at $c^{SS}_{NI,NN}(k)$. Moreover,

$$\bar{c} - c^{SS}_{NI,NN}(k) = \frac{A(81k^2 - 252k + 136)}{2(3k-2)(63k-44)}.$$

The numerator is non-positive for $4/3 < k \leq (14+2\sqrt{15})/9$, so $c^{SS}_{NI,NN}(k) \geq \bar{c}$ and $SS^{NI*} > SS^{NN}$ throughout the common interior region. For $k > (14+2\sqrt{15})/9$, $c^{SS}_{NI,NN}(k) \in (0, \bar{c})$, yielding $SS^{NN} \gtreqless SS^{NI*}$ if and only if $c \gtreqless c^{SS}_{NI,NN}(k)$. This completes the proof. □

Proof of Proposition 5. From Proposition 4, $\Pi_1^{IN*} > \Pi_1^{NN}$ and $\Pi_1^{II*} > \Pi_1^{NI*}$, so I strictly dominates N for firm 1. Likewise, $\Pi_2^{NI*} > \Pi_2^{NN}$ and $\Pi_2^{II*} > \Pi_2^{IN*}$, so I strictly dominates N for firm 2. Therefore, the Stage 0 participation game has the unique Nash equilibrium (I, I). Since the continuation equilibrium associated with each participation profile is unique under Assumption 1, the full three-stage game has a unique SPNE inducing the II configuration.

It remains to characterize the nature of this equilibrium. Proposition 4 gives $\Pi_2^{NN} > \Pi_2^{II*}$ throughout the common interior region, whereas $\Pi_1^{II*} \gtreqless \Pi_1^{NN}$ if and only if $c \gtreqless \hat{c}^{\Pi}(k)$. Hence, for $0 < c < \hat{c}^{\Pi}(k)$, both firms strictly prefer NN to the equilibrium II, yielding a prisoner's dilemma. At $c = \hat{c}^{\Pi}(k)$, firm 1 is indifferent while firm 2 still strictly

prefers NN. For $c > \hat{c}^{\Pi}(k)$, firm 1 prefers II whereas firm 2 continues to prefer NN, establishing the asymmetric breakdown of the dilemma. □

Proof of Corollary 1. Proposition 5 establishes that the unique endogenous participation equilibrium is II. From Proposition 4, $SS^{IN*} > \max\{SS^{NI*}, SS^{NN}\}$ and $SS^{II*} > SS^{NN}$; hence only IN and II can be socially optimal.

For $4/3 < k \leq (23 + \sqrt{97})/18$, Proposition 4 gives $SS^{IN*} > SS^{II*}$. For $k > (23 + \sqrt{97})/18$, $SS^{II*} \gtreqless SS^{IN*}$ if and only if $c \lesseqgtr c^{SS}_{II,IN}(k)$. The stated welfare ranking follows immediately. □

Appendix B

Equilibrium continuation outcomes with R&D spillovers. Tables 1 and 2 collect, respectively, the equilibrium R&D efforts and continuation profits under the four possible participation configurations.

Table 1: Equilibrium R&D efforts with technological spillovers

Configuration	x_1^*	x_2^*
NN	0	0
IN	$\frac{2(2-\beta)(A+c)}{9k-2(2-\beta)^2}$	0
NI	0	$\frac{2(2-\beta)(A-2c)}{9k-2(2-\beta)^2}$
II	$\frac{2(2-\beta)\{AF+c[9k-6\beta(2-\beta)]\}}{F[9k-2(2-\beta)(1+\beta)]}$	$\frac{2(2-\beta)\{AF-6c[3k-(2-\beta)]\}}{F[9k-2(2-\beta)(1+\beta)]}$

Note: $F \equiv 9k - 6(2-\beta)(1-\beta)$.

Table 2: Equilibrium continuation profits with technological spillovers

Configuration	Π_1^*	Π_2^*
NN	$\frac{(A+c)^2}{9}$	$\frac{(A-2c)^2}{9}$
IN	$\frac{k(A+c)^2}{9k-2(2-\beta)^2}$	$\frac{\{AF-6c[3k-(2-\beta)]\}^2}{9[9k-2(2-\beta)^2]^2}$
NI	$\frac{\{AF+c[9k-6\beta(2-\beta)]\}^2}{9[9k-2(2-\beta)^2]^2}$	$\frac{k(A-2c)^2}{9k-2(2-\beta)^2}$
II	$\frac{k[9k-2(2-\beta)^2]\left\{\begin{matrix}AF+\\c[9k-6\beta(2-\beta)]\end{matrix}\right\}^2}{F^2[9k-2(2-\beta)(1+\beta)]^2}$	$\frac{k[9k-2(2-\beta)^2]\left\{\begin{matrix}AF-\\6c[3k-(2-\beta)]\end{matrix}\right\}^2}{F^2[9k-2(2-\beta)(1+\beta)]^2}$

Note: $F \equiv 9k - 6(2-\beta)(1-\beta)$.

Proof of Proposition 6. From Table 2,

$$\frac{\Pi_1^{IN*}}{\Pi_1^{NN}} = \frac{\Pi_2^{NI*}}{\Pi_2^{NN}} = \frac{9k}{9k-2(2-\beta)^2} > 1.$$

Moreover,

$$\frac{\Pi_1^{II*}}{\Pi_1^{NI*}} = \frac{\Pi_2^{II*}}{\Pi_2^{IN*}} = \frac{9k[9k-2(2-\beta)^2]^3}{[9k-6(2-\beta)(1-\beta)]^2[9k-2(2-\beta)(1+\beta)]^2} > 1.$$

Indeed,

$$\begin{aligned}&[9k-6(2-\beta)(1-\beta)][9k-2(2-\beta)(1+\beta)]\\ &\quad = [9k-2(2-\beta)^2]^2 - 4(2-\beta)^2(2\beta-1)^2 \leq [9k-2(2-\beta)^2]^2,\end{aligned}$$

while $3\sqrt{k[9k-2(2-\beta)^2]} > 9k-2(2-\beta)^2$. Thus I strictly dominates N for both firms, proving part [1].

For part [2], positivity in the interior region allows the profit comparisons to be written as

$$\begin{aligned}\Pi_1^{II*} \gtreqless \Pi_1^{NN} \quad \Longleftrightarrow \quad & 3\sqrt{k[9k-2(2-\beta)^2]}\left\{\begin{matrix}A[9k-6(2-\beta)(1-\beta)]\\+\,c[9k-6\beta(2-\beta)]\end{matrix}\right\}\\ & \gtreqless [9k-6(2-\beta)(1-\beta)][9k-2(2-\beta)(1+\beta)](A+c),\end{aligned}$$

and

$$\Pi_2^{II*} \gtreqless \Pi_2^{NN} \quad \Longleftrightarrow \quad 3\sqrt{k[9k-2(2-\beta)^2]} \left\{ \begin{matrix} A[9k-6(2-\beta)(1-\beta)] \\ -6c[3k-(2-\beta)] \end{matrix} \right\} \gtreqless [9k-6(2-\beta)(1-\beta)][9k-2(2-\beta)(1+\beta)](A-2c).$$

At $c=0$, both comparisons reduce to

$$3\sqrt{k[9k-2(2-\beta)^2]} \gtreqless 9k-2(2-\beta)(1+\beta).$$

Furthermore,

$$\begin{aligned}&[9k-2(2-\beta)(1+\beta)]^2 - 9k[9k-2(2-\beta)^2] \\ &\quad = 2(\beta-2)\left[2\beta^3+(27k-6)\beta-4\right].\end{aligned}$$

Since the polynomial on the right satisfies

$$-4<0 \quad \text{at } \beta=0, \qquad \frac{27}{4}(2k-1)>0 \quad \text{at } \beta=\frac{1}{2},$$

and has derivative $6\beta^2+27k-6>0$, it admits a unique root $\widehat{\beta}(k)\in(0,1/2)$.

For $\beta<1/2$, the two relevant terms vary with cost asymmetry according to

$$\frac{\partial}{\partial c}\frac{A[9k-6(2-\beta)(1-\beta)]+c[9k-6\beta(2-\beta)]}{A+c} = \frac{6A(2-\beta)(1-2\beta)}{(A+c)^2} > 0,$$

whereas

$$\frac{\partial}{\partial c}\frac{A[9k-6(2-\beta)(1-\beta)]-6c[3k-(2-\beta)]}{A-2c} = \frac{6A(2-\beta)(2\beta-1)}{(A-2c)^2} < 0.$$

Thus cost asymmetry makes II relatively more attractive to firm 1 and less attractive to firm 2.

If $0 \le \beta < \widehat{\beta}(k)$, both firms prefer NN at $c=0$. Firm 1's comparison is strictly increasing in c and changes sign exactly once, while firm 2's comparison is strictly decreasing and therefore remains in favor of NN. Solving firm 1's indifference condition gives $\widehat{c}_1^{\Pi}(k,\beta)$.

If $\beta=\widehat{\beta}(k)$, both firms are indifferent at $c=0$; the preceding derivatives imply $\Pi_1^{II*}>\Pi_1^{NN}$ and $\Pi_2^{NN}>\Pi_2^{II*}$ for every $c>0$. If $\widehat{\beta}(k)<\beta<1/2$, both firms prefer II at $c=0$. Firm 1 continues to prefer II, whereas firm 2's comparison decreases and crosses zero exactly once. Solving its indifference condition yields $\widehat{c}_2^{\Pi}(k,\beta)$.

Finally, for $\beta \ge 1/2$, both firms prefer II at $c=0$. Firm 2's relative payoff from II is non-decreasing in c. For firm 1, the comparison is non-increasing, but at the upper

quantity-interiority boundary $c = A/2$,

$$\frac{A[9k - 6(2-\beta)(1-\beta)] + \frac{A}{2}[9k - 6\beta(2-\beta)]}{A + \frac{A}{2}} = 9k - 2(2-\beta)^2.$$

Together with

$$3\sqrt{k[9k - 2(2-\beta)^2]} > 9k - 2(2-\beta)^2$$

and

$$\begin{aligned}[9k - 6(2-\beta)(1-\beta)][9k - 2(2-\beta)(1+\beta)] \\ \leq [9k - 2(2-\beta)^2]^2,\end{aligned}$$

this implies $\Pi_1^{II*} > \Pi_1^{NN}$ throughout the interior region. Hence both firms prefer II for $\beta \geq 1/2$, completing the proof. □

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