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Abstract

We study the incentives to merge for energy producers in the presence of distributed renewable energy producers. Utilizing a Cournot model, we explore how uncertainty surrounding the cost of grid integration influences the profitability of mergers, where uncertainty comes in the form of an industry-wide shock (or common) and firm-specific errors (private shock). We find that the effect of these uncertainties on merger profitability depends on average energy grid integration costs, the size of the merger, and quality of private information. Overall, results suggest that mergers are more likely to be profitable when firms can effectively absorb private shocks due to the scale of the merger, unless average grid integration costs become too high. The incentives to merge are less clear-cut in the presence of an industry-wide shock, unless the quality of private information is high enough.

JEL Classification: Q4, G34, Q2

Keywords: M&As; green electricity; grid integration cost; signaling games; Cournot competition; uncertainties.
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1 Introduction

The energy landscape is undergoing a significant transformation with the coexistence of traditional centralized energy producers (firms) with the emergence of distributed renewable energy producers (DREPs) (Alanne and Saari, 2006). The impact of DREPs on the operation of large-scale power generators depends on several factors such as the scale of renewable generation (Begovic et al., 2001; Iweh et al., 2021), its variability, existing policy frameworks such as feed-in-tariffs and net metering provisions (Passey et al., 2011), as well as the specific characteristics of power generators' infrastructure that enables grid integration and balanced power flow (e.g., two-way power flow, advanced metering infrastructure or AMI, digitization, etc.) (da Silva et al., 2019). Rehmani et al. (2018) highlights, for instance, challenges related with the relatively low generation capacity of DREPs compared to conventional fossil-fuel-based power plants and the need to have advanced technology and a smart grid to efficiently integrate more renewable energy into the grid. Renewable energy integration continues, with the ongoing deployment of technological advances in the production of green energy, to be at the center of the energy policy debate in different countries around the world.

The growth of DREPs poses several challenges for traditional centralized power generators. One of the key challenges lies in persistent and perpetual uncertainties regarding the integration of variable and intermittent renewable energy into the existing grid. This uncertainty stems from a combination of technical and economic factors that is either faced by all firms equally on average (industry-wide shocks), or firm-specific forecasting errors (private information). Economic sources of uncertainty include potentially fluctuating cost of procuring renewable energy and managing/integrating thereafter, while technical sources are mostly due to the variability and intermittency of renewable energy generation which requires centralized generators to adjust production volumes accordingly. Climate change is also a source of uncertainties and is affecting the energy sector from technological and economic point of view (e.g., renewable production costs highly depend on weather conditions).

Consequently, centralized power generators could be impacted by uncertainty that affects them uniformly (e.g., fluctuation in renewable generation). For example, when DREPs generate more or less energy than anticipated due to weather conditions, each power plant observes this equally and has to adjust its production operation in a similar way (e.g., publicly available weather forecasts, publicly available renewable generation forecasting models, etc.). Another example is potential uncertainty regarding changes in market forces or policy-induced costs which is observed by all generators in the industry (e.g., changes in renewable compensation rates, either by the government or market forces affect all generators and can be observed/predicted by all).

In addition to these commonly shared sources of DREP related uncertainties, each power plant may face a residual uncertainty specific to its infrastructure or technology which is not necessarily observed by its competitors (asymmetric information) and which affects its own cost of integrating renewable energy (e.g., proprietary models or data to predict variation in renewable energy is private information, strategic plans on expansion or new projects that affects grid integration could be private, etc.). For example, a firm may have specific infrastructure or technology that is unique to it and hence affects its cost of integrating renewable energy. These are idiosyncratic cost errors specific to one firm's technology. Due to both industry-wide shocks and private errors that affect costs, each power generator must consider both items when choosing their production strategies in the presence of DREPs.

A growing number of recent studies show that power generators consider mergers and acquisitions (M&As) as a strategic response to the challenges and uncertainty posed by integrating a growing number of DREPs (Fikru and Gautier, 2021). This is because by merging with or acquiring other companies, power generators can pool resources, rationalize costs, share knowledge and technology, and better manage the integration of DREPs. For example, power plants could merge to strengthen their market position and enhance their capabilities in integrating DREPs into their portfolio (e.g., optimizing production based on availability of renewables and deploying advanced monitoring and control systems efficiently). The study by Pereira et al. (2022) finds that electric utilities engage in strategic business decision-making such as M&As to efficiently integrate more sustainable activities into their traditional operations. Based on a global dataset of energy M&As that took place between 1995-2020, AndriuÅ¡keviÄius and Å treimikienÄ (2021) show that since the 2010s, energy mergers have been pursued with the objective of ensuring supply for the increasing demand for green electricity and creating growth opportunities. Similarly, the study by Niemczyk et al. (2022) finds that an increasing number of mergers in the electricity sector to be motivated by green energy initiatives. Hurduzeu and Popescu (2014) document the rising trend in mergers in the energy sector in terms of number and value of transaction and highlights merger motives related to increasing the reliability of the electricity network.

Under conditions of uncertainties associated with renewable energy generation due to its variability and unpredictability, M&As may help to identify and exploit potential synergies within and between players in the energy sector to achieve a clean, affordable and secure highly-regulated energy system (e.g. economies of scope/scale, lowering investment in extra-capacity and transaction costs between players). Power generators have incentives to merge as a strategic response to manage risks, deal with security issues and stabilize operations. Previous studies indicate that firms facing high renewable integration uncertainties often consolidate to diversify energy portfolios, achieve economies of scale, and improve market power. For example, Joskow (2019) and Bloom et al. (2016) highlight how market consolidation helps firms handle renewable energy challenges, while Mulder and Scholtens (2013) show that regulatory uncertainty may increase M&A activity as firms seek to secure assets and market positions. Empirical evidence from Bresler and Olkkonen (2021) shows that mergers could help firms mitigate risks associated with renewable energy variability, ensuring stable operations and investment outlooks.

We contribute to the growing literature on energy mergers by modeling the profitability of a merger when renewable integration costs are affected by both industry-wide and firm-specific private cost shocks. Although previous studies show that merger incentives are affected by the presence/absence of greener electricity from distributed generation, none have modelled the relative role of private errors versus industry-wide shocks and how the precision of private information is likely to affect merger profitability. Thus far energy merger models have presented a deterministic model where distributed renewable energy affects energy mergers (Fikru and Gautier, 2021; Pereira et al., 2022) with no stochastic models to evaluate uncertainty arising from renewable integration costs.

Hence, to account for energy producers' costs asymmetric information and to analyze the impact of the uncertainty of green energy integration on M&As, we adopt a linear-quadratic framework, which yields closed-form solutions in Bayesian games. By focusing on linear equilibria, we capture the impact of players' actions on each others' targets. In addition, our signaling game allows us to analyze how a given player values its rivals' information and how this information can be used to infer what the others know. This captures the role of uncertainty, information asymmetry, and the precision of private information on the incentives for mergers among energy producers in the presence of renewable integration. We also explore how these factors affect the profitability of mergers and the resulting market structure, which yields new economic insights. More specifically, by considering an information structure with common and private values related to integrating DREPs into the electric grid, our framework allows us to examine the impact of cost uncertainties and quality of information on the profitability of a proposed merger between a given number of power generators in the presence of DREPs.

The model considers a Cournot market of n energy producers (firms), and an exogenous volume of renewable energy procured by each firm (q^r) to address the following research question: *How does the presence of uncertainty about grid integration costs affect the incentives for energy producers to merge?* We assume a merger of m number of firms where $m < n$ and characterize the market equilibrium before and after the merger takes place and then derive conditions under which the proposed m plant merger can be more/less profitable when uncertainty is present.

The findings provide valuable insights for policymakers, energy producers, and other stakeholders in the energy sector. We find that a merger among energy producers facilitates market power that could potentially allow the combined entity to better manage both industry-wide and private shocks and benefit from sharing private information. In the presence of variable and intermittent renewable generation, mergers can serve as a strategic avenue for firms to gain access to each other's private information related to renewable integration. For instance, during the due diligence process, a firm might uncover the other's cost structure or technological capabilities, which were previously undisclosed. This could include the cost of integrating renewable energy into the grid or proprietary technology that enhances energy production efficiency. For example, a plant specializing in integrating solar energy might gain insights from its merger partner (e.g., the partner may have a different integration technology or specializes in integrating wind energy from distributed generation); a firm might also discover details about the other's supplier contracts, customer relationships, or even strategic plans.

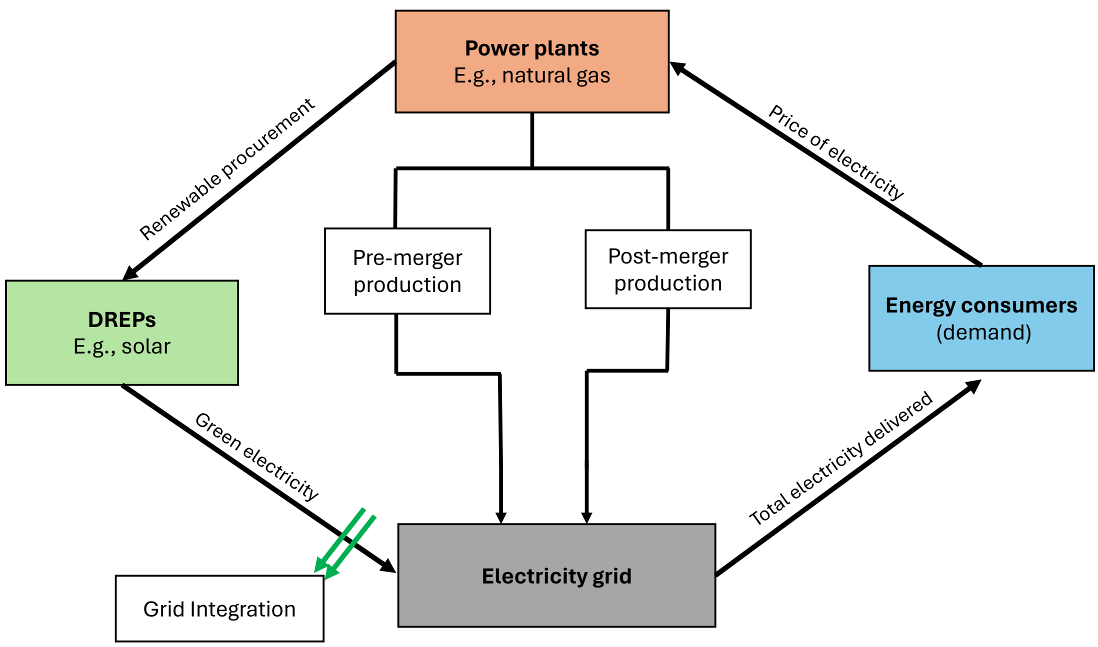
Our analysis is, for example, of relevance for the European wholesale electricity markets where deployment of renewable energy leads to economic and environmental benefits. The integration of green electricity into the power grid and the resulting policy regulation, however, have impacts on the traditional energy sector production costs (e.g. higher operational and capital expenditures in transmission and distribution network systems). Over the last two decades, weather shocks caused the electricity price to drop below zero because renewables have priority in the power grid and come first in the merit order. Weather patterns of 2022 and 2023 summers that reached unprecedented limits combined with low demand led to negative power prices in different European countries (e.g. France, Germany, Netherlands). In the day-ahead market, hours with low and negative prices have increased these decades and are expected to continue with the ongoing climate change concerns. The decline in prices was mainly driven by the abundance of green energy generated by weather-dependant sources (such as wind-based generation technologies, solar or hydro sources), combined with the relatively low energy demand. To eliminate the surplus of electricity from the grid and to avoid supporting extra production and high start-up costs and overloading the system energy, producers have no choice but offering negative prices.

Section 2 presents the basic model and characterizes the equilibrium without uncertainty. Section 3 introduces uncertainty in affecting grid integration costs, and characterizes equilibrium pre and post merger as well as the function used to evaluate the profitability of the proposed merger. Section 4 presents comparative static results. Section 5 concludes with a discussion on policy implications and questions for future work.

2 Model Set-Up and Assumptions

We consider a generic (unregulated) energy market with imperfect competition. There are n number of energy producers (or firms) that primarily use nonrenewable assets or fossil fuel sources (for example, natural gas) to generate and sell x_i amounts of electricity where $i = 1, \dots, n$. There are also h number of distributed renewable energy producers (DREPs) that generate renewable energy using technologies like rooftop solar panels or small-scale wind turbines. We assume that both DREPs and firms are connected to the electricity grid. DREPs sell all the generated renewable energy to the firms which in turn sell it to final energy consumers. Thus, the electricity grid has both renewable (green) and non-renewable energy. We refer to the h producers as *DREPs* and the n producers as *firms*. We consider a policy regime where firms are required to procure all the energy produced by DREPs via regulatory frameworks such as feed-in-tariffs or net metering provisions (Fikru, 2022). Figure 1 presents the conceptual framework of the model.

Figure 1: Conceptual framework



2.1 Cost of firms

DREPs sell the green energy they produce to firms which in turn sell it to the ultimate electricity consumer. This imposes two types of cost for firms: the cost of procuring renewable energy (e.g., such as a feed-in-tariff or a compensation rate either determined by the market or regulator) and the cost of integrating the renewable energy into the electric grid (Fikru and Gautier, 2021). Brown and Sappington (2017) argue that energy producers (firms) incur the cost of integrating variable and intermittent renewable energy into the electricity grid and such costs include ramping up and down costs. For instance, as renewable energy declines due to weather conditions (e.g., cloudy day or at night), firms will need to ramp up fossil-fuel-based production and vice-versa. Joos and Staffell (2018) find evidence for growing congestion management costs associated with the deployment of more variable renewable energy sources. Thus, we capture the cost of integrating renewables into the electricity grid as a significant cost affecting power generators.

Each firm $i = 1, \ldots, n$ exhibits costs $C_i(x_i, q^r)$ and emits pollution or carbon emissions proportional to non-renewable production. Total emissions per firm are given by ϕx_i, where ϕ represents the carbon intensity of the production process. Each firm faces an identical per-unit emission tax, t, for its net emissions, $e_i(x_i, q^r) = \phi x_i - q^r$. This implies that each firm gets credit for allowing, on average, q^r units of renewable generation in the energy mix in the form of tax savings.

For each firm, the total cost of operation includes producing energy from fossil-fuel-based sources, the cost of procuring renewable energy from DREPs, and the cost of integrating variable renewable energy with production from the fossil assets. We consider the case where each firm procures the same units of renewable energy, q^r, which affects its cost and net emissions. We consider the cost of integrating the two technologies (fossil, renewable) through the parameter $\theta_i > 0$. More formally, each firm's cost function is given by:

$$C_i(x_i, q^r) = c_{f,i}x_i + (c_{r,i} + \theta_i x_i)q^r \tag{1}$$

The parameter $c_f > 0$ is the average cost of producing non-renewable energy, and $c_r > 0$ is the average cost of procuring renewable energy from DREPs. We assume that firms are required to procure all renewable energy generated by DREPs. The cost of integrating renewable generation with the grid is $\theta x q^r$, where θq^r captures how renewable energy raises each firm's marginal production costs. This function is based on the literature which shows that the marginal cost of power plants depends on the level of renewable energy that enters the electric grid (Zhang et al., 2018; Fikru and Gautier, 2021).

2.2 Demand for energy

Following the literature, we use a differentiated Cournot oligopoly model to represent the output competition among the n firms (Borenstein et al., 2000; Oren, 1997). For instance, Willems (2002), Yao et al. (2008), and Milstein and Tishler (2015) use a linear energy demand function where (p) is the price of energy (e.g., dollars per kilowatt hour). Since there are two ways of producing energy (using fossil and renewable assets), we account for the possibility of product differentiation. Empirical evidence suggests that consumers are increasingly becoming more aware of the source of their electricity, i.e., from renewable versus non-renewable sources and from centralized versus distributed sources (Agarwal et al., 2024).

Thus, the inverse electricity demand function each firm faces is given by $p_i = a - \sum_{i=1}^{n} x_i - \gamma \sum_{i=1}^{n} q_i^r$ where q_i^r is the volume of renewable energy procured by each firm, and $\gamma \in [0, 1]$ denotes the degree of product differentiation. If $\gamma = 0$ then consumers view renewable and fossil-based electricity as totally different. If $\gamma = 1$ consumers view renewable and fossil-based electricity as homogeneous like in Milstein and Tishler (2015). The parameter $a > 0$ is assumed to be positive constant capturing the energy market size.

We assume that each firm procures q_i^r units of renewable energy from DREPs per period while the total renewable procured by all firms is $\sum_{i=1}^{n} q_i^r$, which in turn is equal to total renewable production by the h DERPs. This assumption allows each DREP to produce different volumes while maintaining that each firm procures the same volume. To reduce the complexities of the model, we assume that each firm, on average, procures the same volume of renewable energy such that $q_1^r = q_2^r = ...q_n^r = q^r$ holds.

2.3 Energy mergers with full information

In this subsection, we present the full information case (without uncertainty) as a benchmark to illustrate how uncertainty affects the model in subsequent sections. We present the pre-merger market solution followed by the post-merger solutions and examine the profitability of an energy merger. Following previous merger models, we characterize solutions under the assumption of a symmetric equilibrium (Choi et al., 2022).

2.3.1 Equilibrium in the pre-merger market with full information

Under symmetry, pre-merger output and profits are calculated by maximizing individual profits in a Cournot-Nash fashion; that is, each firm solves:

$$\max_{x_i} \pi_i = p_i x_i - (c_{f_i} x_i + c_{r_i} q^r + \theta_i x_i q^r) - t(\phi_i x_i - q^r)$$

from which the first-order conditions, $\partial \pi_i / \partial x_i = 0$, yield under symmetry ($c_{f_1} = \ldots = c_{f_n} = c_f$; $c_{r_1} = \ldots = c_{r_n} = c_r$; $\theta_1 = \ldots = \theta_n = \theta$; $\phi_1 = \ldots = \phi_n = \phi$; $x_1 = \ldots = x_n = x$) the equilibrium level of energy production and profits for each firm. Closed-form solutions are presented as follows:

$$x = \frac{a - (n\gamma + \theta)q^r - t\phi - c_f}{(n+1)} \tag{2}$$

$$\pi = x^2 + q^r(t - c_r) \tag{3}$$

These equations show that as more renewable energy is procured by each firm, each firm reduces fossil-based production and vice-versa ($\partial x / \partial q^r < 0$). This is because of the need to ramp down the fossil plant when the renewable source is generating and the need to ramp up when renewable is not generating. This holds because green energy has priority in the power grid and comes first in the merit order, which means that firms are required to purchase and integrate all renewable generation and hence need to respond to renewable generation accordingly. The solutions also illustrate that buying greener electricity improves the firms profit due to tax savings net of the cost of renewable procurement.

We also find that $\partial x / \partial \theta < 0$ illustrating the output-reducing effect of higher grid integration costs. This is because of the added costs the energy procured entails suggesting that the more expensive it is to integrate a given unit of renewable energy, the higher the total costs for each power plant and thus the lower the production volume of fossil-fuel-based energy.

2.3.2 Equilibrium in the post-merger market with full information

We consider the case where $n > m \geq 2$ number of firms merge. In the post-merger market we have a merged entity operating m number of power plants which are referred to as insiders as in Salant et al. (1983). The rest $n - m$ firms are outsiders. The outsiders maximize profits independently ($\max \pi_i^o$, for $i = 1, \ldots, n - m$), while the insiders maximize joint profits, $\max \sum_{j=1}^{m} \pi_j^m$ for $j = 1, 2, \ldots, m$. In the post-merger market, under symmetry each insider's output and profit are given by:

$$x^m = \frac{a - (n\gamma + \theta)q^r - t\phi - c_f}{m(n - m + 2)} \quad (4)$$

$$\pi^m = m(x^m)^2 + q^r(t - c_r) \quad (5)$$

Let $B = a - (n\gamma + \theta)q^r - t\phi - c_f > 0$ to ensure positive output. Additionally, each outsider's production level and profits are given by $x^o = B/(n - m + 2)$ and $\pi^o = (x^o)^2 + q^r(t - c_r)$, respectively.

2.3.3 Merger profitability and implications

Following previous studies (Salant et al., 1983; Fikru and Gautier 2016; Choi et al., 2022), the merger profitability is defined as Δ which is the difference between post-merger profit of the merged entity (made of m plants) and the pre-merger profits of the independent firms before they merge. That is,

$$\Delta = m\pi^m - m\pi = m^2(x^m)^2 - m(x)^2$$

$$\Delta = \frac{B^2}{(n+1)^2(n - m + 2)^2}[(n+1)^2 - m(n - m + 2)^2] \quad (6)$$

This suggests that the merger is profitable only if $n + 1 > \sqrt{m}(n - m + 2)$, but not otherwise. This condition implies that the size of the merger (in terms of the number of plants that consolidate or m) is high enough relative to n and the post-merger market is concentrated to give the merged firm a higher market power to be profitable. For example, consider a market with $n = 10$ firms. A given merger is profitable only if $m = 9, 10$, consolidating at least 90% of the firms (i.e., $n = 10, m = 9$ gives $\Delta > 0$). This finding suggests that the proposed merger is profitable only if it creates market concentration by consolidating a significant majority of market participants.

The term in square brackets in Equation (6) represents the combined effect of the total number of firms post-merger $(n-m+1)$, pre-merger market participants (n), as well as the size of the merger (m) on the profitability of the merger. Our findings are consistent with previous literature that model merger between firms that have a constant marginal cost. For example, each insider produces less post-than pre-merger $(x^m < x)$ as in Salant et al. (1983) and this is what causes the merger to be unprofitable if m is not large enough (that is, the term in squared bracket is negative). Furthermore, we find that each outsider responds to the merger by producing more than each insider's output $(x^o > x^m)$ and also producing more than before $x^o > x$.

Salant et al. (1983) discusses losses from small sized horizontal mergers when firms have constant marginal costs due to the possibility that the increase in outsiders' production (following the merger) will reduce insiders' profits by more than the increase in profits that could have occurred if outsiders had produced the same as before in the pre-merger market. For the electricity and oil industry in OECD member countries, Ten Brug and Sahib (2018) empirically analyze over five thousand M&A deals over the years 1997-2012 and find that close to 12% of these deals were abandoned after being announced, highlighting the challenges of merger deals in the industry. Similarly, the study by Federico (2011) highlights regulatory challenges that may make electricity mergers less profitable (e.g., the need for remedial actions to gain approval). Likewise, the study by Fikru and Gautier (2021) shows unprofitable mergers in the energy sector for a similar market structure (which includes the consolidation of fewer energy firms relative to the total number of firms) with exogenous renewable generation.

Despite the strict conditions for the profitability of the proposed merger ($\Delta > 0$ only when a majority of firms consolidate), we find that an increase in grid integration cost θ reduces pre-merger output (and hence pre-merger profits) relatively more than post-merger output and profits, and thus reduces the unprofitable of the merger if $\Delta < 0$ and increases the profitability of a merger if $\Delta > 0$. That is, $d\Delta/d\theta > 0$, for all Δ. This means the merger allows the merged entity to reduce the rate by which θ (grid integration cost) reduces its joint profit.

For the case where $\Delta < 0$, this result suggests that pre-merger firms are more sensitive to changes in the cost of integrating renewable energy compared to the merged entity. This is because the merged entity uses joint profit maximization (i.e., optimize output and allocate production to maximize gross and not individual profits) to derive optimal production level x^m which allows each insider to better manage the cost of integrating renewable and maintain production (or decrease only by a small amount) compared to the case of individual profit maximization in the pre-merger market. This joint optimization can lead to a more efficient allocation of resources, which can help the merged entity manage the cost of integrating renewable energy and reduce its response to renewable generation (i.e., cost rationalization).

This is a key advantage of a merger because it allows the firm to consider the overall profitability of the entity rather than the profitability of each individual plant. In addition, the merger provides additional market power (i.e., less competition post-merger) for the merged entity which allows each insider's allocated production volume to be lower (than before) because production decision is now made considering all plants and not just a single one.

We also find that an increase in θ decreases each outsider's profit by a larger extent. This is because of the individual profit maximization each outsider faces and a higher sensitivity to grid integration costs. In summary, $|\partial x^o/\partial\theta| > |\partial x/\partial\theta| > |\partial x_m/\partial\theta|$. Overall, the results from the full information case suggest that the proposed merger is unprofitable ($\Delta < 0$) unless it consolidate a majority of the firms (i.e., monopolization), and that an increase in grid integration cost renders unprofitable mergers to be less unprofitable.

3 Renewable Integration with Uncertainty

3.1 The role of uncertainty in affecting cost

To examine M&As in the presence of uncertainty regarding renewable energy integration costs, we consider a signaling game in line with Elnaboulsi et al. (2023), Myatt and Wallas (2015) and Vives (2011). Before uncertainty is realized, all players face ex ante the same prospects. The cost function depends on θs, an unobserved and unknown state of the world generated by shocks. Each firm receives a signal about the cost of integrating green energy into the grid composed of a common shock represented by s plus some error term, ε_i. Thus, $\theta_i = s + \varepsilon_i$ is a random variable determined by, respectively, industry-wide shocks s (e.g., difficulty in managing variable renewable energy, intermittent renewable production which affects all firms equally, uncertainty regarding where in the grid renewable will be pumped more and at what rate, weather related uncertainties, etc.), and ε_i, a private noisy term representing the remaining firm-level uncertainty over its own costs. It can also be viewed as firm-specific forecast errors about economic, political or weather conditions (e.g, a firm will not have complete information about how variable renewable generation affects its own costs). Without loss of generality, the signals received by players in the marketplace are assumed to be unbiased estimators of their priors. Note that this information structure encompasses the cases of common and private values.

The s signals are positive and drawn randomly and observed by all firms in the industry (i.e., publicly disclosed information or common knowledge to all). The idiosyncratic estimations ε_i are also drawn randomly, but only observed by firm i and not the other firms (i.e., private information only known to a firm but not others). Electric generators may face (uncertain) integration costs that are specific to their own infrastructure or operational context, which are not available to other competitor firms. This could involve proprietary technologies or solutions such as advanced sensors, tailored grid management strategies involving customized algorithms, real-time monitoring or automated response. Other examples include proprietary risk assessment technologies which may be confidential. Overall, firms may face uncertainties in customizing and optimizing grid integration solutions tailored to their unique operational requirements.

We assume s is distributed according to some prior density function such that $s \sim (\mu_s, \sigma_s^2)$, where a lower variance (σ_s^2) means all firms are more informed about the common shock (less uncertainty or in other words better or more precise information about the common shock), and a higher variance (σ_s^2) means all firms are equally less informed (more uncertainty) about the magnitude of the true cost imposed by the common shock. Similarly, we assume that ε_i is distributed according to some prior density function such that $\varepsilon_i \sim (\mu_{\varepsilon_i}, \sigma_{\varepsilon_i}^2)$ where a lower variance indicates more precise private information. We have the overall mean given by $\mu_i = \mu_s + \mu_{\varepsilon_i}$ and the overall variance given by σ^2. We also assume that s and ε_i are uncorrelated, and that individual shocks are also not correlated, that is , $cov[\varepsilon_i, \varepsilon_j] = 0$. Finally, while the vast majority of the related literature assume variables are normally distributed, in our case we allow θ_i to be of any distribution with the given mean and variance without specifying the probability law.

It follows that with a given or fixed σ_s^2 we measure the extent of *relative* certainty of the private shocks as a proportion of all shocks (common and private) as $z_i = \dfrac{\sigma_s^2}{\sigma_s^2 + \sigma_{\varepsilon_i}^2}$. An increase in z_i implies that firm i has higher quality information about its private shocks that affect renewable integration. Since $\sigma_{\varepsilon_i}^2$ ranges from zero (known private shock) to infinity (high uncertainty, no information), the parameter z_i ranges from one to zero.

Integrating a large volume of renewable energy could be risky (e.g., grid stability), which is an industry-wide risk that is known to all firms. For example, it is well known that weather parameters impose risk on all firms and the level of risk is known to all firms and this risk affects all in the same way (e.g., unexpected cloud covers reduce solar generation in the entire area). When renewable production goes down unexpectedly, each firm would need to fill the shortfall from their fossil assets. Likewise, when renewable generation peaks, fossil-fuel-based plants need to produce below capacity or ramp down production levels. We assume that the n firms are within the same geographical location where weather conditions are similar. Another type of industry level shock is fuel prices which affect all firms in the same way (e.g., when fuel prices fall, procuring renewable becomes unprofitable) or a change in climate policies or other regulatory changes that affect the cost of producing energy as renewable volume changes.

The firm-specific integration cost has to do with the specific technology the firm owns, and how it interacts with its existing (e.g., aging) fossil-fuel infrastructure. It can also be explained by patents on new technologies (e.g., advanced infrastructure, digitization, artificial intelligence, etc.) that enable the firm to integrate renewable more/less efficiency. It may also be related to patented battery chemistry, when a firm uses energy storage to offset some of the uncertainties that come along with integrating renewable energy (e.g., battery may reduce the cost of integration on net or increase it). This is private information. The firm itself does not have complete information about how this uncertainty will unfold, and so the firm can only form expectations based on its internal knowledge of the proprietary algorithm's capabilities and the evolving landscape of renewable energy integration.

3.2 Equilibrium in the pre-merger market

Equilibrium is determined by solving for the output decision of each firm given emission taxes as exogenous. Initially, uncertainty from both sources (common and private shock) is revealed followed by production decisions.

We solve for output levels by maximizing profits conditional on θ. A similar timing of game is adopted in previous studies such as Colombo et al. (2012), Vives (2017), and Elnaboulsi et al. (2023).

In the pre-merger market, each firm i maximizes expected profits contingent upon its own marginal energy integration cost, θ_i, by choosing the level of output, x_i in a Cournot-Nash fashion:

$$Max_{x_i}E[\pi_i|\theta_i] = Max_{x_i}E[p_ix_i - (c_{f_i}x_i + c_{r_i}q^r + \theta_ix_iq^r) - t(\phi_ix_i - q^r)|\theta_i] \quad (7)$$

To make the analysis tractable and obtain closed-form solutions, we impose the following assumptions as in previous studies that model uncertainity (Colombo et al., 2012; Vives, 2017; Elnaboulsi et al., 2023).

Assumption 3.1. *(1)* $E[\theta_i|\theta_j] = z_j\theta_j + \lambda_j$ *where* $z_i = \frac{\sigma_s^2}{\sigma_s^2+\sigma_{\varepsilon_i}^2}$ *and* $\lambda_j = \mu_i - z_j\mu_j$*, (2)* $c_{f_i} = c_{f_j} = c_f$ $\forall i \neq j$*, (3)* $\phi_i = \phi_j = \phi$ $\forall i \neq j$*, (4)* $z_i = z_j = z$ $\forall i \neq j$*, (5)* $\mu_i = \mu_j = \mu$ $\forall i \neq j$*.*

Assumption 1 implies that firm i's expected cost conditional on firm j's is linear and depends on firm j's cost and relative information precision (Vives, 2002). That is, if firm j is well informed about its private costs with a higher z_j, then firm i's expected cost is higher. This is because with higher z_j firm i becomes relatively less well informed. Similarly, if firm j has a higher θ_j, then firm i will also have a higher expected cost, $E[\theta_i|\theta_j]$.

The economic intuition behind this assumption is that any firm can form expectations about its costs based on its competitor's costs. Assumption 1 also suggests that conditional expectation is linear (if a firm realizes its marginal cost specific to grid integration, then it can say what it means for its competitor's marginal cost). For example, if weather condition is predicted and it affects one firm's marginal cost in a certain way, that firm can expect that the same weather condition affects other firms in a similar way (this may not be known exactly so the firm can make expectations on average). λ is a constant or intercept term which is affected by the mean of θ for the firm and its competitor (Elnaboulsi et al., 2023).

Assumptions 2 and 3 imply that firms have identical average cost of producing energy using fossil assets, and the carbon intensity of production is identical across firms, respectively. We introduce two other assumptions, Assumptions 4 and 5, to obtain a symmetric equilibrium as in Vives (1988) and Vives (2011). Combining all assumptions we get $\lambda_i = \lambda_j = \lambda = \mu(1-z)$.

Based on Assumption 3.1, the pre-merger Bayesian Nash Equilibrium is unique and is characterized in the following Lemma.

Lemma 3.2. $x_i(\theta_i) = \alpha_i\theta_i + \eta_i$, $\forall i = 1, ...n$.

The lemma implies that energy production from fossil assets is a linear combination of the variable grid integration cost. This allows us to focus the analysis on linear strategies which makes the equilibrium more tractable.

From Equation (7) each firm maximizes expected profit and solves for production volume, x_i. In the pre-merger market (initial market with no mergers), each firm's equilibrium output is obtained by maximizing profits based on Assumption 3.1 and Lemma 3.2. In particular, for all $i, j = 1, ..., n$

$$\alpha_i = \alpha_j = \alpha = \frac{-q^r}{(n-1)z+2} < 0; \qquad \eta_i = \eta_j = \eta = \frac{A-(n-1)\alpha\lambda}{n+1} > 0 \tag{8}$$

where $A = a - n\gamma q^r - c_f - t\phi > 0$ to ensure positive production volume. Hence, using Lemma 3.2 equilibrium output for each firm is given by $\tilde{x}_i(\tilde{\theta}_i) = \alpha_i\tilde{\theta}_i + \eta_i$; that is,

$$\tilde{x}_i(\tilde{\theta}_i) = \frac{-q^r(n+1)\tilde{\theta}_i + A[2+(n-1)z] + q^r\mu(n-1)(1-z)}{(n+1)[2+(n-1)z]} \tag{9}$$

where $\tilde{x}_i$ is a random variable conditional on $\tilde{\theta}_i$. This characterizes the pre-merger equilibrium with uncertainty. Taking expectations in Equation (9) we find the mean of the equilibrium output of each firm, which represents the average or expected volume of fossil-fuel-based generation, where $E[\theta_i] = \mu$ and $\partial\overline{x}/\partial\mu < 0$ and $\partial\overline{x}/\partial q^r < 0$. That is, an increase in the average grid integration cost (μ) raises costs associated to integrating more renewable energy and leads to lower production volumes on average. Similarly, when q^r is low (e.g., night time), the firm has to ramp up and increase production volume on average to compensation for reduced green energy available to the grid, and vice-versa.

Using the definition of profits from Equation (8) and Lemma 3.2 we obtain the expression for expected profits for each firm pre-merger (see Appendix for a detailed derivation). We use the unconditional expectation given below to make comparisons with the expected profits post-merger, and thus analyze expected profitability of a merger under uncertainty. In particular,

$$\begin{aligned} E[\tilde{\pi}_i] &= E[\tilde{p}_i\tilde{x}_i] - E[\tilde{\theta}_i\tilde{x}_i]q^r - (c_f + t\phi)E[\tilde{x}_i] + (t - c_r)q^r \\ &= A\bar{x}_i - \mu\bar{x}_i q^r - \alpha^2\sigma^2\left(1 + (n-1)z\right) - \alpha\sigma^2 q^r - n\bar{x}_i^2 \\ &\quad + (t - c_r)q^r \end{aligned} \tag{10}$$

3.3 Equilibrium in the post-merger market

We consider the case where $m < n$ firms decide to merge. Merging allows firms to observe each partner's costs such that, $\tilde{\theta}_i = \tilde{\theta}_j = \tilde{\theta}$. The merged entity, now composed of $i = 1, 2, \ldots, m$, jointly operating plants maximize joint expected profit given the signal $\tilde{\theta}$. Additionally, the merged entity now competes with $k = m + 1, ..., n$ outsider firms (those firms which did not join the merged entity) which compete by maximizing individual profits. Since our goal is to examine the effects of the shocks on merger profitability, we focus the exposition of the model on the insiders and relegate the characterization of solution of the outsiders to the Appendix.

The profit maximizing problem for the m plant merged entity ($i = 1, 2, \ldots, m$) is given by:

$$max_{x_1,\ldots x_m} E\left[\sum_{i=1}^{m} \pi_i^m | \tilde{\theta}\right] \tag{11}$$

where

$$E\left[\sum_{i=1}^{m} \pi_i^m | \tilde{\theta}\right] = E\left[\sum p_i x_i - (c_{f_i} \sum x_i + c_{r_i} m q^r + \theta q^r \sum x_i) - t(\sum \phi_i x_i - m q^r) | \tilde{\theta}\right] \tag{12}$$

As before $A = a - n\gamma q^r - c_f - t\phi > 0$. Following the same steps and imposing symmetry assumptions as in the pre-merger case along with Lemma 3.2, the equilibrium output for each insider is given by:

$$x_i^m(\tilde{\theta}) = \frac{-q^r \tilde{\theta} n + A\,(2 + (n-m)z) + q^r \mu (n-m)(1-z)}{mn(2 + (n-m)z)} \tag{13}$$

Under symmetry we have $\alpha^m = -q^r/m\,((n-m)z + 2)$, $\lambda^m = (1-z)\mu$, $\eta^m = (Am - (n-m)\alpha^m \lambda)/(nm)$, for for all i. Additionally, we find that each plant owned by the merged entity ($i = 1, 2, \ldots, m$), denoted by x^m, produces on average $\bar{x}^m$ amount of fossil-fuel-based energy, where as before $E[\tilde{\theta}] = \mu$ and $\partial \bar{x}^m / \partial \mu < 0$, $\partial \bar{x}^m / q^r \mu < 0$.

As in the full information case, each insider produces less than each outsider on average and less than the pre-merger market, on average. Additionally, each outsider produces more than before on average. Next, consider the expected unconditional post-merger profit for each insider, (using Lemma 3.2):

$$
\begin{aligned}
E[\tilde{\pi}_i^m] &= A\bar{x}_i^m - \mu\bar{x}_i^m q^r - m(\alpha^m)^2\sigma^2[1+(n-m)z] - m(n-m+1)(\bar{x}_i^m)^2 \\
&\quad +(t-c_r)q^r - \alpha^m\sigma^2 q^r \qquad (14)
\end{aligned}
$$

3.4 Merger profitability with uncertainty

The goal of this study is to understand the impact of uncertainty caused by the private and common shocks, σ_ϵ^2 and σ_s^2, on merger profitability. To achieve this goal we compare the change in expected profits pre- and post-merger based on the model presented in the previous sections. We follow the standard definition of merger profitability in the literature, namely, the difference between the sum of the expected profits of insiders firms, $\tilde{\pi}_i^m$, which comprise the merger in the post-merger market and the sum of the expected profits of those firms, $\tilde{\pi}_i$, had they remained as individual firms in the pre-merger market. Under the symmetry assumption ($\tilde{\pi}_i^m = \tilde{\pi}^m$,$\tilde{\pi}_i = \tilde{\pi}$,$\forall i$) the definition of expected profitability of the proposed m plant merger is given by:

$$
\Delta^u = mE[\tilde{\pi}^m] - mE[\tilde{\pi}] \qquad (15)
$$

We examine the change in the expected merger profitability, $d\Delta^u$, by analyzing the effects of the common and private shocks on the pre-merger market ($dE[\tilde{\pi}]$) and, separately, on the post-merger market ($dE[\tilde{\pi}^m]$). We then compare these results to say something about the change in the expected profitability of a merger.

We formally define "quality of private information" through the term $\sigma_\epsilon^2/\sigma_s^2$. This term denotes the relative variance of the private shock. That is, this term captures the degree to which the uncertainty coming from the private shock is high (or low) relative to the uncertainty coming from the common shock. A higher ratio implies higher uncertainty due to the private shock (poor private information quality) and a lower ratio implies lower uncertainty due to the private shock (high quality private information).

If this term is relatively high, then the relative uncertainty from the private shock is high meaning the firm enjoys lower quality private information. The relatively high uncertainty on the private shock is captured by a relatively high variance on the private shock (the term $\sigma_\epsilon^2/\sigma_s^2$ is relatively high) and this relative high variance reflects as lower quality information. That is, the firm does not have too much information on the private shock relative to the information it has on the common shock (or the relative little information on the private shock it has is not good enough). This reflects as a relative high variance ($\sigma_\epsilon^2/\sigma_s^2$). Note that when the term $\sigma_\epsilon^2/\sigma_s^2$ is high z (defined in Section 3.1) is low. A low z is associated with low quality of private information.

By the same token if the term $\sigma_\epsilon^2/\sigma_s^2$ is relatively low (because the numerator is low or because the denominator is high), then the uncertainty from the private shock is low relative to the uncertainty from the common shock. In this case we can say the quality of private information is higher because the firm has better quality information on the private shock relative to the information it has on the common shock (captured by low relative variance, $\sigma_\epsilon^2/\sigma_s^2$). In this case z is higher, which is associated with higher quality private information. We summarize the notion of quality of private information in the following remark.

Remark 3.3. *A high (low) relative variance, $\sigma_\epsilon^2/\sigma_s^2$ or equivalently a low (high) value for z, denotes relatively low (high) quality of private informa-*

tion available to the firm.

4 Impact of Uncertainty on Profits

4.1 Uncertainty affects pre-merger profits

In the pre-merger market analysis we consider the change in expected profits pre-merger of each individual firm. This is the second term in Equation (15), which comes from Equation (10). Differentiation gives the change in expected profits for each firm pre-merger with respect to the private and common shock (see Appendix):

$$dE[\tilde{\pi}] = [\mu^2 a_\epsilon^{pre} + \mu b_\epsilon^{pre} + c_\epsilon^{pre}] d\sigma_\epsilon^2 + [\mu^2 a_s^{pre} + \mu b_s^{pre} + c_s^{pre}] d\sigma_s^2 \quad (16)$$

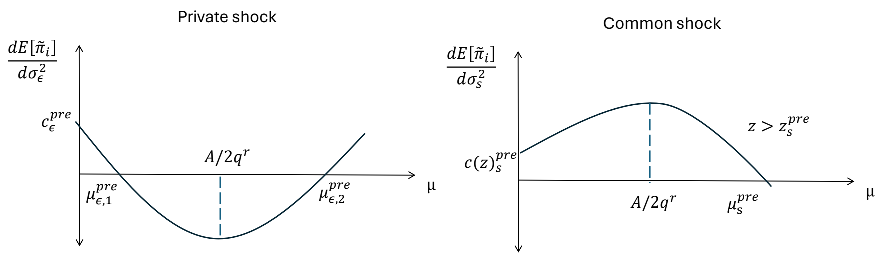
where $a_\epsilon^{pre} > 0$, $b_\epsilon^{pre} < 0$ and $c_\epsilon^{pre} > 0$ and $a_s^{pre} < 0$, $b_s^{pre} > 0$, but the sign of c_s^{pre} is ambiguous and discussed below. These are items that are functions of q^r, z and n but not μ.

Private Shock

We first discuss the role of the private shock, $d\sigma_\epsilon^2$, using Figure 2 (left panel). $d\sigma_\epsilon^2 > 0$ means that each firm experiences higher uncertainty associated with private information when it comes to integration costs and therefore lower quality of private information (Remark 3.3). The horizontal axis in the figure denotes the average shock (average integration costs), μ, and the vertical axis represents changes in expected profits of each firm pre-merger with respect to changes in the private shock, i.e., $dE[\tilde{\pi}]/d\sigma_\epsilon^2$.

For each firm pre-merger (left panel, Figure 2), an increase in the private shock does not decrease expected profits for initially small levels of average shock, μ. This average shock can be interpreted as the average

Figure 2: Pre-merger private and common shocks



grid integration cost each firm i faces. When this average shock is small enough, then firm i's expected profits do not decrease as a result of an increase in the private shock, σ_ϵ^2. That is, the firm can handle (absorb) an increase in the private shock when the average integration cost is small enough. But as the average integration cost becomes higher firm i is not able to handle an increase in private shock, i.e., expected profits decrease. In this case integration costs are just too high for firm i so that an increase in the private shock decreases its expected profits.

But after a certain threshold level of average integration cost, firm i is better able to handle an increase in the private shock. This is because of the oligopolistic interdependence in the pre-merger market. As other pre-merger firms also experience higher average integration costs, they are not able to handle private shocks as effectively, which forces them to lower their output on average. As a result, firm i reacts strategically by increasing on average its production. This results in an increase in the expected profits for the firm i. This is captured through the upward-sloping segment in the figure, above the x-axis. As we move up along this upward segment, the oligopolistic interdependence effect becomes stronger.

Common Shock

Next, we examine the effects of the common shock pre-merger using the right panel of Figure 2. The key difference between the private and common shock analysis in the pre-merger market is that in the latter the relative quality of the private information plays a role.

With this in mind, we examine Figure 2 (right panel). If z is large enough ($z > z_s^{pre}$) the quality of private information is relatively high so each firm is in a better position to absorb a common shock for a longer range of average integration costs, μ. That is, expected profits increase as a result of an increase in the common shock for a longer range of μ. This is captured by the term c_s^{pre} which is increasing in z. Intuitively, with better private information each firm can handle the common shock more effectively up to the point where the level of average integration cost is just too high (i.e., expected profits fall for $\mu > \mu_s^{pre}$). As other firms in the market also experience an increase in the common shock and higher average integration costs, firm i increases its output on average and therefore its expected profits increase. This is because of the oligopolistic interdependence. Because firm i enjoys higher quality information, the firm can sustain an increase in expected profits for a longer range of average integration cost, μ. It is noteworthy that an increase in the common shock increases the relative quality of private information since it decreases the relative variance of the private shock (Remark 3.3). But at the same an increase in the common shock means that each firm deals with the added uncertainty coming from the common shock itself. As a result, having sufficiently high quality private information ($z > z_s^{pre}$) helps the firm absorb changes in the common shock as shown in Figure 2.

Proposition 4.1. *(i) In the pre-merger market each firm handles an increase in the common shock effectively for a large range of average integration costs, if the quality of information is sufficiently high. (ii) In the pre-merger market each firm handles an increase in the private shock effectively, if the average integration cost is large enough because of the oligopolistic interdependence.*

4.2 Uncertainty affects post-merger profits

In this sub-section, we examine the effects of the private and common shocks in the post-merger market. We derive an expression analogous to (16) and use Figure 3 to guide our discussion. In particular,

$$dE[\tilde{\pi}^m] = [\mu^2 a_\epsilon^{post} + \mu b_\epsilon^{post} + c_\epsilon^{post}]d\sigma_\epsilon^2 + [\mu^2 a_s^{post} + \mu b_s^{post} + c_s^{post}]d\sigma_s^2$$

where $a_\epsilon^{post} < 0$, $b_\epsilon^{post} > 0$ and $c_\epsilon^{post} > 0$ and $a_s^{post} < 0$, $b_s^{post} > 0$, c_s^{post} for large enough z. As before, these are functions of q^r, z, m and n but not μ.

Private Shock

For a range of average integration cost, μ, each insider is able to enjoy an increase in expected profits by being able to effectively absorb an increase in the private shock. The ability of each insider firm to handle a private shock depends on the size of the merger, m. This is captured by the term c_ϵ^{post}, which is a function of the size of the merger.[1] In particular, for a range

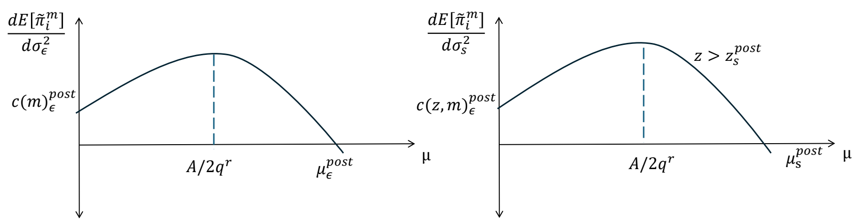
[1]It can be shown that for a range of m, $\partial c_\epsilon^{post}/\partial m > 0$.

of size of merger, each insider is better equipped to absorb an increase in the private shock. But as the average integration cost becomes too large ($\mu > \mu_{\epsilon}^{post}$) it becomes increasingly difficult for each insider to handle the increase in the private shock resulting in a decrease in expected profits. The difference between the private shock in affecting expected profits in the post- and pre-merger markets is that post-merger insiders are better equipped to handle an increase in the private shock for a larger range of average integration costs because of the presence of the size of the merger. That is, being part of a merged entity helps absorb an increase in the private shock. As other firms (outsiders) experience higher average integration costs they are not able to absorb as effectively an increase in the private shock, which decreases their output on average. As a result, each insider reacts strategically by increasing its output on average and therefore its expected profits increase. This is the oligopolistic interdependence effect post-merger. Because insiders enjoy market power as a result of the merger, each insider is better positioned to sustain an increase in expected profits for a longer range of average integration cost.

Common Shock

Similar to the pre-merger case the quality of private information is important when it comes to the analysis of the common shock. If the quality of information is high enough ($z > z_{s}^{post}$), then each insider is better able to handle an increase in the common shock. That is, expected profits likely increase given an increase in the common shock. For example, each firm has better quality information and, as a result, it is better at cushioning any sudden changes. But as before, in the post-merger market the ability of

Figure 3: Post-merger private and common shocks



each insider to handle an increase in the common shock also depends on the size of the merger, m. This is captured by the term c_s^{post}, which depends on z and m. For large enough size of merger (or high enough quality of private information) each insider is better equipped to absorb an increase in the common shock. The role of m is similar to that we identified in the analysis of the private shock.

4.3 Uncertainty affects merger profitability

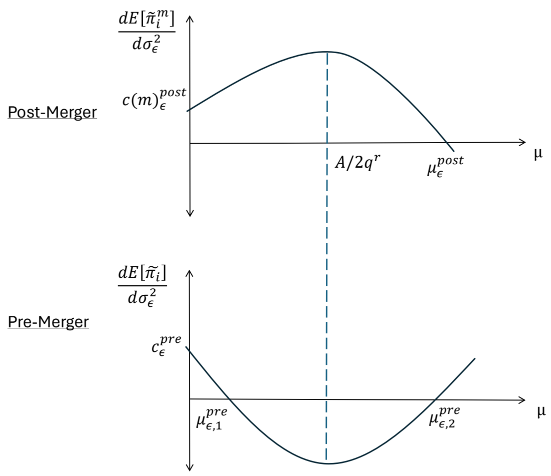
In this sub-section, we examine the effects of a change in the private and common shocks on expected profitability of a merger. We use Figure 4 to guide our discussion on how a private shock affects the expected profitability of a merger, while Figure 5 helps with the analysis of the common shock.

Private Shock

Figure 4 is simply the combination of the left panel in Figure 2 and left panel in Figure 3, where the effects on the post-merger market show on top and the pre-merger effects on the bottom. Using this Figure we identify ranges of average integration costs, μ, where expected profitability increases/decreases.

Based on the definition of expected profitability of a merger (Equation 15), an increase in the expected profits of each insider increases profitability, while an increase in expected profits of a pre-merger firm decreases expected profitability. We breakdown the analysis of how a private shock affects the profitability of a merger into three regions using Figure 4. For

Figure 4: Private shock: Merger profitability



small enough average shock ($\mu < \mu_{\epsilon,1}^{pre}$), each insider and each pre-merger firm absorb the private shock effectively so that each of their respective expected profits increase given an increase in the private shock. In this case, the change in expected profitability due to an increase in the private shock is ambiguous. But if the size of the merger is large enough ($m > \bar{m}$), then the merged entity is able to cushion the effects of a private shock effectively enough, thereby increasing expected profits post-merger and consequently expected profitability of the merger.

Now, for a range of average integration costs (not too large and not too small i.e. $\mu_{\epsilon,1}^{pre} < \mu < \min\{\mu_{\epsilon,1}^{post}, \mu_{\epsilon,2}^{pre}\}$) expected profitability increases given an increase in the private shock. This is because for this range of average integration costs each insider experiences an increase in expected profits (since it can handle private shock relatively more effectively because of the merger), while each pre-merger firm suffers a decrease in expected profits. This is the range of average integration costs for which the oligopolistic interdependence effect is yet not strong enough so expected profits pre-merger decrease.

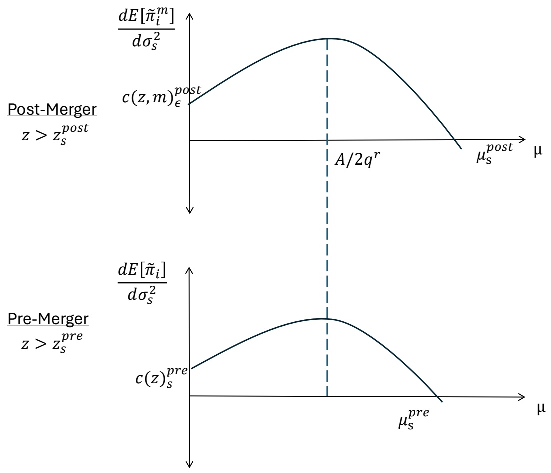
The third region captures the idea that expected profitability decreases with an increase in the private shock for large enough average integration costs both pre-merger and post-merger. In this case expected profits post-merger decrease (insiders can't handle such a large average integration cost), but at the same time the pre-merger firm experiences an increase in expected profits because the oligopolistic interdependence effect is strong enough.

Common Shock

In the case of the common shock the analysis results are less clear-cut. We use Figure 5. The size of the merger and quality of information play an important role. If the size of the merger is large enough (or if the quality of information is higher post-merger relative to the quality of information pre-merger), then expected profitability increases with an increase in the common shock for a relatively large range of average integration costs. Other than this case, we were not able to identify clear-cut cases where we could affirm that expected profitability increases or decreases.

Proposition 4.2. *Consider an increase in the private shock. Then, ex-*

Figure 5: Common shock: Merger profitability



pected profitability of a merger, Δ^u*, (i) increases for a range of average integration costs; (ii) increases for small average integration costs if the size of the merger or quality of information are high enough post-merger; and (iii) decreases for sufficiently large average integration costs.*

5 Conclusion

This study develops an energy merger model to understand the strategic responses of traditional non-renewable energy producers in the face of the emerging distributed renewable energy producers (DREPs) and related uncertainties. We adopt a Cournot model to examine the impact of uncertainty surrounding grid integration costs on the incentives for mergers among energy producers, with a focus on industry-wide shocks and private cost shocks specific to a firm. We consider the case where merging allows firms to observe each other's private costs, which can influence the aggregate expected profit of the merged entity.

The results indicate that uncertainty in the cost of integrating renewable energy into the grid, coupled with private information about each firm's costs, significantly influences the incentives for mergers. Overall, the study finds that the profitability of a merger could be affected by uncertainties in the cost of grid integration as well as the quality of private information each firm has about its private costs. We find that when the average grid integration cost is initially low, uncertainty in private costs increases merger profitability as long as two conditions are fulfilled: the size of the merger is high (many firms consolidate) and the quality of private information is high. Once average grid integration costs exceed a given threshold, merger profitability could start to decline. In addition, we find that when uncertainty in common costs increases, the impact on merger profitability is less clear. The findings suggest that merging could allow firms to better manage uncertainties in private costs by sharing private information, which can enhance their profitability. However, it is not clear whether merging would allow firms to better manage uncertainties in common cost shocks.

Regarding the effect of private shocks on merger profitability, we find the effect is contingent on the average integration costs. We identify three distinct regions based on the size of these costs:

1. Low Average Integration Costs: In this range, both insiders and pre-merger firms effectively absorb the private shock, leading to increased expected profits for each entity. However, the overall change in expected profitability due to an increase in private shocks remains ambiguous unless the merger size is large enough. For substantial mergers (m is large), the merged entity can cushion the shock effectively, thereby increasing post-merger profits and the overall expected profitability.

2. Moderate Average Integration Costs: Here, expected profitability increases with private shocks. This occurs because insiders manage the shocks more efficiently, resulting in higher expected profits post-merger, while pre-merger firms experience decreased profits due to the oligopolistic interdependence effect not being sufficiently strong.

3. High Average Integration Costs: For large integration costs, both pre-merger and post-merger entities are negatively impacted by private shocks. Insiders struggle to handle the high costs, leading to decreased expected profits post-merger. Conversely, pre-merger firms may see an increase in profits due to strong oligopolistic interdependence.

The analysis of common shocks, shows more nuanced results compared to private shocks. The size of the merger and the quality of information available play crucial roles. When the merger size is substantial or the quality of post-merger information is superior to pre-merger information, expected profitability tends to increase across a broad range of integration costs. However, outside of this scenario, clear conclusions on the effect of common shocks on expected profitability are harder to ascertain without imposing further restrictions on the model setup.

This study underscores the complex interplay between uncertainty and energy grid integration costs in determining the profitability of mergers, highlighting the importance of considering both private and common shocks in merger assessments. Our findings provide valuable insights for policymakers, energy producers, and other stakeholders in the energy sector as they navigate the complexities of the energy transition. First, understanding the dynamics of mergers can guide energy companies in making strategic decisions, especially when a merger could help mitigate the risks associated with renewable energy uncertainty. Mergers can potentially pool resources and share private information, which is beneficial in an uncertain environment.

From a regulatory perspective, understanding these dynamics is crucial to ensure a fair and competitive market, especially if mergers become a more common response to renewable energy uncertainty. In addition to the impact of mergers on market competition, policymakers could consider the role of mergers in mitigating risks associated with renewable energy integration. For example, regulatory frameworks could be designed to facilitate mergers that enhance grid stability and efficiency without compromising competition. For investors, understanding these dynamics can affect the company's valuation and their decision to invest. Lastly, from an academic viewpoint, the model presented in this study contributes to the literature on industrial organization, strategic management, and energy economics, providing insights into how firms respond to uncertainty and how these responses affect market structure and performance. However, given the complexity of the energy transition and the role of uncertainty and information asymmetry, further research is needed to fully understand these dynamics.

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Appendices

A Pre-Merger Equilibrium

We characterize the n-firm pre-merger equilibrium. We maximize Equation (7) which yields the following first-order condition:

$$a - 2x_i - \sum E[x_{-i}|\theta_i] - c_{fi} - q^r\theta_i - t\phi_i = 0 \tag{A.1}$$

where the subscript "$-i$" represents all x different from i, where

$$\begin{aligned}\sum E[x_{-i}|\theta_i] &= (\alpha_2 z_1\theta_1 + \alpha_2\lambda_1 + \eta_2) + (\alpha_3 z_1\theta_1 + \alpha_3\lambda_1 + \eta_3) \\ &\quad + \ldots + (\alpha_{n-1}z_1\theta_1 + \alpha_{n-1}\lambda_1 + \eta_{n-1})\end{aligned} \tag{A.2}$$

Then, from (A.1) and (A.2) we get

$$x_i = -\theta_i\frac{(q^r + \sum\alpha_j z_i)}{2} + \frac{A - \sum\eta_j - \sum\alpha_i\lambda_i}{2} \tag{A.3}$$

Then, using the definition of x_i given in Lemma 3.2 we have that for each firm $i = 1, 2, \ldots, n$ we obtain

$$\alpha_i = \frac{-(q^r + \sum\alpha_j z_i)}{2}\forall i \neq j \tag{A.4}$$

$$\eta_i = \frac{A - \sum\eta_j - \sum\alpha_i\lambda_i}{2}\forall i \neq j \tag{A.5}$$

Then, imposing Assumption 3.1 (symmetry) on (A.4) and (A.5) we obtain

$$2\alpha = -(q^r + (n-1)\alpha z) \Rightarrow \alpha = -q^r/(2 + (n-1)z) \tag{A.6}$$

$$2\eta = A - (n-1)\eta - (n-1)\alpha(1-z)\mu \Rightarrow \eta = \frac{A - (n-1)\alpha(1-z)\mu}{n+1} \tag{A.7}$$

Hence, using the expressions for α and η given in (A.6) and (A.7) we obtain Equation (9). We then substitute Equation (9) into the definition of profits, and use Lemma 3.2 along with the expectations operator to derive the expression in Equation (10); some of the expectations operators used are the following:

$$
\begin{aligned}
E[x_i^2] &= Var(x_i) + (\bar{x}_i)^2 \\
\sum E[x_i x_j] &= \sum [Cov(x_i, x_j) + \bar{x}_i \bar{x}_j] \\
Cov(x_i, x_j) &= Cov(\alpha_i \theta_i^2 + \eta_i, \alpha_j \theta_j^2 + \eta_j) = \alpha_i \alpha_j Cov(\theta_i, \theta_j) = \alpha_i \alpha_j \sigma_i^2 \\
Var(x_i) &= Var(\alpha_i \theta_i + \eta_i) = (\alpha_i)^2 Var(\theta_i) = (\alpha_i)^2 (\sigma_i)^2 \\
E[\theta_i x_i] &= E[\alpha_i (\theta_i)^2 + \eta_i \theta_i] = \alpha_i (Var(\theta_i) + (\theta_i)^2) + \eta_i \mu_i \\
&= \alpha_i (\sigma_i)^2 + \alpha_i \mu_i^2 + \eta_i \mu_i
\end{aligned}
$$

B Post-Merger Equilibrium

We characterize the post-merger equilibrium (insiders and outsiders) when m firms merge. In this case we have two sets of firms, namely, the insiders and outsiders. Insiders maximize joint profits; that is, the profit maximizing problem for the m-plant merged entity ($i = 1, 2, \ldots, m$) is given by:

$$
max_{x_1, \ldots x_m} E\left[\sum_{i=1}^{m} \pi_i^m | \tilde{\theta}\right] \tag{B.1}
$$

where

$$
E\left[\sum_{i=1}^{m} \pi_i^m | \tilde{\theta}\right] = E\left[\sum p_i x_i - (c_{f_i} \sum x_i + c_{r_i} m q^r + \theta q^r \sum x_i) - t(\sum \phi_i x_i - m q^r) | \tilde{\theta}\right] \tag{B.2}
$$

where $A = a - n\gamma q^r - c_f - t\phi > 0$.

Following the same computational steps shown in Appendix A for the pre-merger case (i.e., imposing symmetry and using Lemma 3.2) the equilibrium output for each insider is given by:

$$
x_i^m(\tilde{\theta}) = \frac{-q^r \tilde{\theta} n + A\,(2 + (n-m)z) + q^r \mu (n-m)(1-z)}{mn(2 + (n-m)z)} \tag{B.3}
$$

where as before under symmetry we have $\alpha^m = -q^r/m\,((n-m)z+2)$, $\lambda^m = (1-z)\mu$, $\eta^m = (Am-(n-m)\alpha^m\lambda)/(nm)$, for for all i.

As for the $n-m$ outsiders, each firm maximizes profits individually as in the pre-merger case. As before, we use Lemma 3.2 and the symmetry condition to characterize each outsiders output. Since our focus is on the insider's output we don't present the outsider's output.

Then, we substitute Equation (B.3) in the definition of profits for each insider, and use Lemma 3.2 and the expectations operator to obtain Equation (14).

C Comparative Statics

Pre-Merger Market

We first derive the comparative statics for the expected profits for each firm in the pre-merger market. Collecting terms in (10) and simplifying using the definition of $\bar{x} = \alpha E[\theta] + \eta$ gives

$$\begin{aligned} E[\tilde{\pi}_i] &= A\,(\alpha E[\theta]+\eta) - n\left(\alpha^2(E[\theta])^2 + 2\alpha\eta E[\theta] + \eta^2\right) + \alpha^2\sigma^2 \\ &\quad -\alpha(E[\theta])^2 q^r - \eta q^r E[\theta] + (t-c_r)q^r \end{aligned} \tag{C.1}$$

where $E[\theta]=\mu$, $A = a - n\gamma q^r - c_f - t\phi > 0$ and $\alpha^2\sigma^2 = -\alpha^2\sigma^2(1+(n-1)z) - \alpha\sigma^2 q^r$.

Then, we obtain changes in expected profits with respect to the shocks σ_s^2, σ_ϵ^2. Total differentiation with respect to α, η and σ^2 gives Equation (16):

$$\begin{aligned} dE[\tilde{\pi}_i] &= [A\mu - 2n\alpha\mu^2 - 2n\mu\eta + 2\sigma^2\alpha - q^r\mu^2]d\alpha + [A - 2n\mu\alpha - 2n\eta - \mu q^r]\,d\eta \\ &\quad +\alpha^2 d\sigma^2 \end{aligned} \tag{C.2}$$

where $\sigma^2 = \sigma_s^2 + \sigma_\epsilon^2$ and

$$\begin{aligned}
z &= \frac{\sigma_s^2}{\sigma_s^2 + \sigma_\epsilon^2} \Rightarrow dz = \frac{(1-z)d\sigma_s^2 + zd\sigma_\epsilon^2}{\sigma_s^2 + \sigma_\epsilon^2} \\
\alpha &= \frac{-q^r}{2+(n-1)z} \Rightarrow d\alpha = \frac{-\alpha(n-1)}{(\sigma_s^2 + \sigma_\epsilon^2)(2+(n-1)z)}\left((1-z)d\sigma_s^2 - zd\sigma_\epsilon^2\right) \\
\eta &= \frac{A-(n-1)\alpha(1-z)\mu}{n+1} \\
&\Rightarrow \; d\eta = \frac{(n-1)}{(n+1)}\left[\frac{\alpha\mu(1-z)}{\sigma_s^2+\sigma_\epsilon^2} + \frac{\alpha\mu(n-1)(1-z)^2}{(2+(n-1)z)(\sigma_s^2+\sigma_\epsilon^2)}\right] d\sigma_s^2 \\
&\quad -\frac{(n-1)}{(n+1)}\left[\frac{\alpha\mu z}{\sigma_s^2+\sigma_\epsilon^2} + \frac{\alpha\mu(n-1)(1-z)z}{(2+(n-1)z)(\sigma_s^2+\sigma_\epsilon^2)}\right] d\sigma_\epsilon^2
\end{aligned}$$

where $dA = 0$, $d\mu = 0$, $dq^r = 0$. Substitution $d\sigma^2$, $d\alpha$ and $d\eta$ into Equation (C.2) gives changes in expected profits for firm i with respect to changes in the shocks $d\sigma_\epsilon^2$, $d\sigma_s^2$:

$$\begin{aligned}
dE[\tilde{\pi}_i] &= \left[\mu^2 a_\epsilon^{pre} + \mu 2 b_\epsilon^{pre} + c_\epsilon^{pre}\right] d\sigma_\epsilon^2 \\
&\quad + \left[\mu^2 a_s^{pre} + \mu 2 b_s^{pre} + c_s^{pre}\right] d\sigma_s^2
\end{aligned} \tag{C.3}$$

where

$$\begin{aligned}
a_\epsilon^{pre} &= \rho_1 \frac{(n-1)}{(n+1)} q^r - \rho_2 \frac{(n-1)}{(n+1)} q^r > 0 \\
b_\epsilon^{pre} &= -\rho_1 \frac{(n-1)}{(n+1)} A + \rho_2 \frac{(n-1)}{(n+1)} A < 0 \\
c_\epsilon^{pre} &= \rho_1 2\sigma^2\alpha + \alpha^2 > 0 \\
a_s^{pre} &= \frac{(n-1)}{(n+1)} q^r \left(-\tilde{\rho}_1 + \tilde{\rho}_2/(n+1)\right) < 0 \\
b_s^{pre} &= \frac{(n-1)}{(n+1)} A \left(\tilde{\rho}_1 - \tilde{\rho}_2/(n+1)\right) > 0 \\
c_s^{pre} &= -\tilde{\rho}_1 2\sigma^2\alpha + \alpha^2 > 0 \Leftrightarrow z > (2(n-2))/(3(n-1))
\end{aligned} \tag{C.4}$$

and where

$$\begin{aligned}
\rho_1 &= \frac{\alpha(n-1)z}{\sigma^2(2+(n-1)z)} < 0 \\
\rho_2 &= \left(1+\frac{(n-1)(1-z)}{2+(n-1)z}\right) < 0 \\
\tilde{\rho}_1 &= \frac{\alpha(n-1)(1-z)}{\sigma^2(2+(n-1)z)} < 0 \\
\tilde{\rho}_2 &= \frac{(n-1)\alpha(1-z)}{\sigma^2}\left(1+\frac{(n-1)(1-z)}{2+(n-1)z}\right) < 0 \qquad \text{(C.5)}
\end{aligned}$$

Post-Merger Market

Next, derive the comparative statics for the expected profits for each insider firm in the post-merger market. Collecting terms in (14) and simplifying using the definition of $\bar{x}^m = \alpha^m E[\theta] + \eta^m$ gives

$$\begin{aligned}
dE[\pi^m] &= [A\mu - m(n-m+1)2\alpha^m\mu^2 - m(n-m+1)2\mu\eta^m + m2\sigma^2\alpha^m - q^r\mu^2]d\alpha^m \\
&\quad + [A - m(n-m+1)2\mu\alpha^m - m(n-m+1)2\eta^m - \mu q^r]\, d\eta^m \\
&\quad + m(\alpha^m)^2 d\sigma^2 \qquad \text{(C.6)}
\end{aligned}$$

where $\sigma^2 = \sigma_s^2 + \sigma_\epsilon^2$ and

$$\begin{aligned}
z &= \frac{\sigma_s^2}{\sigma_s^2+\sigma_\epsilon^2} \Rightarrow dz = \frac{(1-z)d\sigma_s^2 + zd\sigma_\epsilon^2}{\sigma_s^2+\sigma_\epsilon^2} \\
\alpha^m &= \frac{-q^r}{m(2+(n-m)z)} \Rightarrow d\alpha^m = \frac{-\alpha^m(n-m)}{(\sigma_s^2+\sigma_\epsilon^2)(2+(n-m)z)}\left((1-z)d\sigma_s^2 - zd\sigma_\epsilon^2\right) \\
\eta^m &= \frac{Am-(n-m)\alpha^m(1-z)\mu}{nm} \\
&\Rightarrow d\eta^m = \frac{(n-m)}{(nm)}\left[\frac{\alpha^m\mu(1-z)}{\sigma_s^2+\sigma_\epsilon^2} + \frac{\alpha^m\mu(n-m)(1-z)^2}{(2+(n-m)z)(\sigma_s^2+\sigma_\epsilon^2)}\right]d\sigma_s^2 \\
&\quad -\frac{(n-m)}{(nm)}\left[\frac{\alpha^m\mu z}{\sigma_s^2+\sigma_\epsilon^2} + \frac{\alpha^m\mu(n-m)(1-z)z}{(2+(n-m)z)(\sigma_s^2+\sigma_\epsilon^2)}\right]d\sigma_\epsilon^2
\end{aligned}$$

where $dA = 0$, $d\mu = 0$, $dq^r = 0$. Substitution $d\sigma^2$, $d\alpha^m$ and $d\eta^m$ into Equation (C.6) gives changes in expected profits for each insider with respect to changes in the shocks $d\sigma_\epsilon^2$, $d\sigma_s^2$:

$$\begin{aligned} dE[\pi^m] &= \left[\mu^2 a_\epsilon^{post} + \mu 2 b_\epsilon^{post} + c_\epsilon^{post}\right] d\sigma_\epsilon^2 \\ &\quad + \left[\mu^2 a_s^{post} + \mu 2 b_s^{post} + c_s^{post}\right] d\sigma_s^2 \end{aligned} \tag{C.7}$$

where

$$\begin{aligned} a_\epsilon^{post} &= \frac{(n-m)}{(n-m+2)} q^r(\rho_1^m - \rho_2^m) < 0 \\ b_\epsilon^{post} &= \frac{(n-m)}{(n-m+2)} A(-\rho_1^m + \rho_2^m) > 0 \\ c_\epsilon^{post} &= m(\rho_1^m 2\sigma^2\alpha^m + (\alpha^m)^2) > 0 \\ a_s^{post} &= \frac{(n-m)}{(n-m+2)} q^r(\tilde{\rho}_2^m - \tilde{\rho}_1^m) < 0 \\ b_s^{post} &= \frac{(n-m)}{(n-m+2)} A(-\tilde{\rho}_2^m + \tilde{\rho}_1^m) > 0 \\ c_s^{post} &= m(-\rho_1^m\sigma^2\alpha^m + (\alpha^m)^2) > 0 \Leftrightarrow z > (n-m-2)/(2(n-m)) \end{aligned}$$

and where

$$\begin{aligned} \rho_1^m &= \frac{\alpha^m(n-m)z}{\sigma^2(2+(n-m)z)} < 0 \\ \rho_2^m &= \frac{\alpha^m(n-m)z}{nm\sigma^2}\left(1 + \frac{(n-m)(1-z)}{2+(n-m)z}\right) < 0 \\ \tilde{\rho}_1^m &= \frac{\alpha^m(n-m)(1-z)}{\sigma^2(2+(n-m)z)} < 0 \\ \tilde{\rho}_2^m &= \frac{\alpha^m(n-m)(1-z)}{nm\sigma^2}\left(1 + \frac{(n-m)(1-z)}{2+(n-m)z}\right) < 0 \end{aligned} \tag{C.8}$$